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A problem of forced convective heat flow of a viscous liquid in a porous medium over a fixed horizontal impermeable plate for different depths of the channel

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Abstract

A steady flow of a viscous fluid of finite depth in a porous medium over a fixed horizontal, impermeable bottom is considered in this paper. Exact solutions of Momentum and Energy equations are obtained when the temperatures on the fixed bottom and on the free surface are prescribed. Flow rate ,Mean velocity , Temperature , Mean Temperature , Mean Mixed Temperature in the flow region and the Nusselt number on the boundaries have been obtained. The cases of large depth (deep fluid) small depth(shallow fluid) are discussed. The results are illustrated graphically.

Introduction:

Forced convective heat flow of a viscous liquid of finite depth in a porous medium over a fixed horizontal impermeable plate is studied by Moinuddin.K and Pattabhi ramacharyulu N.Ch[2] in the year 2011.Three dimensional convective flows in a porous media were studied theoretically and experimentally by AZIZ .K and HOLST P.H [1] in the year 1971 . Later Sharma.R.C, Veena Kumari, Mishra [4] examined thermo solute convection flow in a porous medium. Forced convective flows through porous and non porous channels of a variety of geometries was examined by Raghava charyulu [3]in 1984 and G.V.Satya narayana Raju [5] in the year 1989.

In this paper the steady forced convective flow of a viscous liquid of viscosity μ and of finite depth H through a porous medium of porosity coefficient ‘k*’ over a fixed impermeable bottom is investigated.

The flow is generated by a constant pressure gradient parallel to the fixed bottom plate. The momentum equation considered is the generalized Darcy’s law proposed by Yama Moto and

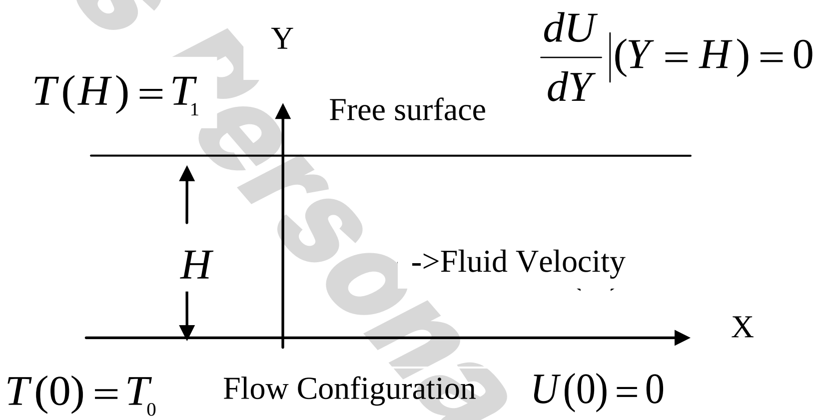
Iwamura[6] which takes into account the convective acceleration and the Newtonian viscous stresses in addition to the classical Darcy force.

The basic equations of momentum and energy are solved to give exact expressions for velocity and temperature distributions. Employing the flow rate , mean velocity, mean temperature, mean mixed temperature and the nusselt numbers at the fluid boundaries have been obtained and illustrated graphically.

The cases of 1. Large depths (large H) and 2. Shallow depths (small H) are also discussed.

Mathematical Formulation

Consider the steady forced convective flow of a viscous liquid through a porous medium of viscosity coefficient μ and of finite depth (H) over a fixed horizontal impermeable bottom. The flow is generated by a constant horizontal pressure gradient parallel to the bottom. Further the bottom is kept at a constant temperature T_0 and the free surface is exposed to atmospheric temperature T_1.



Flow Configuration

With reference to a rectangular Cartesian co-ordinates system with the origin 'O' on the bottom, X-axis in the flow direction (that is parallel to the applied pressure gradient).The Y-axis vertically upwards, the bottom is represented as Y=0 and the free surface as Y=H. Let the flow be characterized by a velocity U= (U(Y), 0, 0).This choice of velocity evidently satisfies the continuity equation $\nabla.U=0$ where U is the fluid velocity vector. Further let T(Y) denotes the temperature distribution.

Basic Equations:

Let the convective flow be calculated by the velocity field U=(U(Y), 0, 0) and the temperature T(Y). This choice of the velocity satisfies the continuity equation $\nabla.U=0$ ---- (1)

The Momentum Equation is

$$-\frac{\partial P}{\partial X}+\mu\frac{d^2U}{dY^2}-\mu\frac{U}{k^*}=0$$ ---- (2)

and the Energy Equation is

$$\rho cU\frac{\partial T}{\partial X}=K\frac{d^2T}{dY^2}+\mu\left(\frac{dU}{dY}\right)^2$$ ---- (3)

In the above equations ρ is the fluid density, k^* the coefficient of porosity of the medium, c is the specific heat, K the thermal conductivity of the fluid and P the fluid pressure.

Boundary Conditions:

Since the bottom is fixed,

$$U(0)=0$$ ---- (4a)

At the free surface shear stress vanishes

$$\mu\frac{dU}{dY}=0 \text{ at } Y=H.$$ ---- (4b)

Also $T(0) = T_0$ -----(5a) and $T(H) = T_1$ ---- (5b)

where T_0 is the bottom temperature and T_1 is the atmosphere temperature.

In terms of the non-dimensional variables defined as:

$$Y=ay;\ X=ax;\ H=ah;\ U=\frac{\mu u}{\rho a^2};\ P=\frac{\mu^2 p}{\rho a^2};\ T=T_0+(T_1-T_0)\theta;\ \Pr=\frac{\mu c}{k};\ k^*=\frac{a^2}{\alpha^2};$$

$$E=\frac{\mu^3}{\rho^2a^2K(T_1-T_0)};-\frac{\partial P}{\partial X}=\frac{\mu^2c_1}{\rho a^3}\left(c_1=-\frac{\partial p}{\partial x}\right) \text{ and } \frac{\partial T}{\partial X}=\frac{(T_1-T_0)}{a}c_2 \text{ where } c_2=\frac{\partial\theta}{\partial x}$$ ----(6)

where 'a' is some standard length, the basic field equations (2) , (3) can be rewritten as follows:

Momentum Equation:

$$\frac{d^2u}{dy^2}-\alpha^2u=-c_1$$ ---- (7) and

Energy Equation:

$$\frac{d^2\theta}{dy^2}=P_rc_2u-E\left(\frac{du}{dy}\right)^2$$ ----- (8)

together with the boundary conditions

For the velocity u (0) =0 and $\frac{du}{dy}=0$ at y=h ----- (9)

and for the temperature $\theta(0)$ =0 and $\theta(h)=1$ ---- (10)

The momentum equation together with the related boundary conditions yields the velocity distribution:

$$u(y)=\frac{c_1}{\alpha^2}\left(1-\frac{\cosh\alpha(h-y)}{\cosh\alpha h}\right) \qquad \text{-----(11)}$$

The energy equation satisfying the boundary conditions yields the temperature distribution:

$$\theta(y)=\frac{y}{h}+\frac{P_r c_1 c_2}{\alpha^2}\left[\frac{(h-y)}{h\alpha^2}-\frac{y(h-y)}{2}+\frac{1}{\alpha^2\cosh\alpha h}\left(\frac{y}{h}-\cosh\alpha(h-y)\right)\right]+ \frac{Ec_1^2}{2\alpha^2\cosh^2\alpha h}\left[\frac{(h-y)\cosh(2\alpha h)}{4\alpha^2 h}-\frac{y(h-y)}{2}+\frac{1}{4\alpha^2}\left(\frac{y}{h}-\cosh 2\alpha(h-y)\right)\right] \qquad \text{-----(12)}$$

The flow rate in the non-dimensional form is

$$q=\int_0^h u(y)dy=\frac{c_1}{\alpha^2}\left(h-\frac{\tanh\alpha h}{\alpha}\right)$$

The mean velocity in the non-dimensional form is

$$\frac{1}{h}\int_0^h u(y)dy=\frac{c_1}{h\alpha^2}\left(h-\frac{\tanh\alpha h}{\alpha}\right) \qquad \text{--- (13)}$$

Further the mean temperature in non-dimensional form is given by

$$\begin{aligned}\bar{\theta} &= \frac{1}{h}\int_0^h \theta dy\\ &= \frac{1}{2}+\frac{P_r c_1 c_2}{\alpha^2}\left(\frac{-h^2}{12}+\frac{1}{2\alpha^2}+\frac{1}{2\alpha^2\cosh\alpha h}-\frac{\tanh\alpha h}{h\alpha^3}\right)-\\ &\frac{Ec_1^2}{2\alpha^2\cosh^2\alpha h}\left(\frac{-1}{8\alpha^2}+\frac{h^2}{12}-\frac{\cosh 2\alpha h}{8\alpha^2}+\frac{\sinh 2\alpha h}{8h\alpha^3}\right)\end{aligned} \qquad \text{---(14)}$$

The mean mixed temperature in the dimensionless form is :

$$\frac{\int_0^h \theta u dy}{\int_0^h u dy} = \frac{1}{\left(h - \frac{\tanh \alpha h}{\alpha}\right)} \left[\begin{array}{l} \left(\frac{h}{2} + \frac{1}{\alpha^2 h \cosh \alpha h} - \frac{1}{\alpha^2 h}\right) + \\ \frac{P_r c_1 c_2}{\alpha^2} \left\{ \begin{array}{l} \frac{-h^3}{12} + \frac{h}{\alpha^2} + \frac{1}{h\alpha^4} + \frac{h}{\alpha^2 \cosh \alpha h} \\ -\frac{5 \tanh \alpha h}{2\alpha^3} + \frac{h}{2\alpha^2 \cosh^2 \alpha h} - \frac{2}{h\alpha^4 \cosh \alpha h} + \frac{1}{h\alpha^4 \cosh^2 \alpha h} \end{array} \right\} + \\ \frac{Ec_1^2}{2\alpha^2 \cosh^2 \alpha h} \left\{ \begin{array}{l} \frac{5h}{8\alpha^2} - \frac{h^3}{12} - \frac{1}{4\alpha^4 h} - \frac{\sinh 2\alpha h}{8\alpha^3} + \frac{h \cosh 2\alpha h}{8\alpha^2} + \\ \frac{h}{2\alpha^2 \cosh \alpha h} - \frac{7 \tanh \alpha h}{8\alpha^3} + \frac{\sinh 3\alpha h}{24\alpha^3 \cosh \alpha h} - \\ \frac{\sinh \alpha h \tanh \alpha h}{2\alpha^4 h} + \frac{\cosh 2\alpha h}{4\alpha^4 h} - \frac{\cosh 2\alpha h \tanh \alpha h}{4\alpha^3} \end{array} \right\} \end{array} \right] \quad \text{----(15)}$$

Heat transfer coefficient (Nusselt Number):

On the bottom:

$$\frac{d\theta}{dy}\Big|_{y=0} = \frac{1}{h} + \frac{P_r c_1 c_2}{\alpha^2}\left(\frac{\tanh \alpha h}{\alpha} - \frac{h}{2} + \frac{1}{h\alpha^2 \cosh \alpha h} - \frac{1}{h\alpha^2}\right) - \frac{Ec_1^2}{2\alpha^2 \cosh^2 \alpha h}\left(\frac{\sinh 2\alpha h}{-2\alpha} - \frac{1}{4h\alpha^2} + \frac{h}{2} + \frac{\cosh 2\alpha h}{4h\alpha^2}\right) \quad \text{-----(16)}$$

On the free surface :

$$\frac{d\theta}{dy}\Big|_{y=h}=\frac{1}{h}+\frac{P_r c_1 c_2}{\alpha^2}\left(\frac{h}{2}+\frac{1}{h\alpha^2\cosh\alpha h}-\frac{1}{h\alpha^2}\right)-\frac{Ec_1^{\,2}}{2\alpha^2\cosh^2\alpha h}\left(-\frac{h}{2}-\frac{1}{4h\alpha^2}+\frac{\cosh 2\alpha h}{4h\alpha^2}\right)$$

---- (17)

1. Flow for large depths (i.e for large h):

For large h $\sinh\alpha h\approx\frac{e^{\alpha h}}{2}$; $\cosh\alpha h\approx\frac{e^{\alpha h}}{2}$; $\tanh\alpha h\approx 1$ and neglecting terms of $O(\frac{1}{h^3})$.

Velocity: $u(y)=\frac{c_1}{\alpha^2}(1-e^{-\alpha y})$ --- (18)

$$=\frac{c_1}{\alpha^2}\left(1-e^{-\frac{y}{\delta}}\right) \text{ where y>>} \delta=\frac{1}{\alpha}=\frac{\sqrt{k^*}}{a}$$

It is noted that the porosity effect on the velocity is confined to a narrow region of thickness of the order $\sqrt{k^*}$ above the bottom. Beyond this there would be a plug flow with velocity

$$\frac{c_1}{\alpha^2}=\frac{c_1 a^2}{k^*}$$

This velocity is the same at that when viscous term is not present in the momentum equation (2 or 7)

Therefore $c_1-\alpha^2 u=0 \Rightarrow u=\frac{c_1}{\alpha^2}=\frac{c_1 a^2}{k*}$ (Darcian velocity)

Mean velocity:

$$u(y)=\frac{c_1}{h\alpha^2}(h-\frac{1}{\alpha})$$ --- (19)

Temperature:

$$\theta=\frac{y}{h}+\frac{P_r c_1 c_2}{\alpha^2}\left[\frac{1}{h\alpha^2}(h-y)-\frac{y}{2}(h-y)-\frac{e^{-\alpha y}}{\alpha^2}\right]+\frac{Ec_1^{\,2}}{\alpha^2}\left[\frac{h-y}{4\alpha^2 h}-\frac{e^{-2\alpha y}}{4\alpha^2}\right]$$

---(20)

Mean temperature

$$\bar{\theta}=\frac{1}{2}+\frac{P_r c_1 c_2}{\alpha^2}\left\{\frac{1}{2\alpha^2}-\frac{h^2}{12}-\frac{1}{h\alpha^3}\right\}+\frac{Ec_1^{\,2}}{\alpha^2}\left\{\frac{1}{8\alpha^2}-\frac{1}{8h\alpha^3}\right\}$$ ---(21)

Mean mixed temperature:

$$\frac{\int_0^h \theta u dy}{\int_0^h u dy} = \frac{1}{\left(h-\frac{1}{\alpha}\right)}\left[\begin{array}{l}\left(\frac{h}{2}-\frac{1}{\alpha^2 h}\right)+\frac{P_r c_1 c_2}{\alpha^2}\left\{\frac{-h^3}{12}+\frac{h}{\alpha^2}+\frac{1}{h\alpha^4}-\frac{5}{2\alpha^3}\right\}+ \\ \frac{Ec_1^2}{\alpha^2}\left\{\frac{h}{8\alpha^2}-\frac{7}{24\alpha^3}+\frac{1}{4\alpha^4 h}\right\}\end{array}\right] \quad \text{--- (22)}$$

Nusselt Number:

On the bottom:

$$\frac{d\theta}{dy}\Big|_{y=0} = \frac{1}{h}+\frac{P_r c_1 c_2}{\alpha^2}\left[\frac{-1}{\alpha^2 h}-\frac{h}{2}+\frac{1}{\alpha}\right]+\frac{Ec_1^2}{\alpha^2}\left[\frac{-1}{4\alpha^2 h}+\frac{1}{2\alpha}\right] \quad \text{---- (23)}$$

On the top:

$$\frac{d\theta}{dy}\Big|_{y=h} = \frac{1}{h}+\frac{P_r c_1 c_2}{\alpha^2}\left[\frac{-1}{\alpha^2 h}+\frac{h}{2}\right]+\frac{Ec_1^2}{\alpha^2}\left[\frac{-1}{4\alpha^2 h}\right] \quad \text{---(24)}$$

2. Flow for shallow fluids. (i.e, for small h)

Retaining terms up to the $o\left(h^2\right)$ and neglecting its higher powers we get

Velocity: $$u(y)=c_1\left[\frac{\left(2hy-y^2\right)}{2}-\frac{\alpha^2}{24}\left(-4hy^3+y^4\right)\right] \quad \text{--- (25)}$$

Mean velocity: $$\bar{u}=\frac{c_1 h^2}{3} \quad \text{---(26)}$$

Temperature:

$$\theta(y)=\frac{y}{h}+\frac{P_r c_1 c_2}{720}[(120hy^3-30y^4)-\alpha^2(-6hy^5+y^6)]+$$
$$\frac{Ec_1^2}{180}\left[(60hy^3-90h^2y^2-15y^4)-\alpha^2(15h^2y^4-12hy^5+2y^6)\right] \quad \text{--- (27)}$$

Mean temperature:

$$\bar{\theta} = \frac{1}{h}\int_0^h \theta dy = \frac{1}{2} \quad \text{---(28)}$$

Mean mixed temperature:

$$\frac{\int_0^h \theta u dy}{\int_0^h u dy} = \frac{(150 - 31\alpha^2 h^2)}{240} \quad ---(29)$$

Nusselt Number:
On the bottom

$$\frac{d\theta}{dy}\Big|_{y=0} = \frac{1}{h} \quad ---(30)$$

On the top $$\frac{d\theta}{dy}\Big|_{y=h} = \frac{1}{h} \quad ---(31)$$

For shallow fluids the heat transfer rate on the fluid boundaries is the reciprocal of the depth of the channel.

Conclusions:

1. It is noticed that the velocity profiles are more steep for large values of α that is the velocity of the fluid decreases with the increase in the value of α (fig 1)..It can be observed from figure 2 thickness of the boundary layer decreases as the porosity parameter α increases for large h. In case of shallow fluids it can be observed from fig 3 that the velocity of the flow region increases with the increase in the porosity parameter α. .

2. It is evident from the fig 4 that for the increasing values of the pressure gradient (c_1) the mean velocity increases and appears to be decreasing with the increase in the values of α. Figure 5 illustrates that the mean velocity decreases with the increase of the porosity parameter α in the case of large h. Figure 6 illustrates that the mean velocity of the flow region increases with the increasing values of the pressure gradient c_1 and depth of the channel 'h' for shallow fluids.

3. It is clearly illustrated in the fig7 that the temperature profiles gradually decreases with the increase in the values of α. An increase in the porosity parameter α increases the temperature profile of the flow region. (Fig.8) for large depths and for shalloe depths an increase in the porosity parameter α reduces the temperature of the flow region .Fig 9.

4. It is noticed that the mean temperature decreases as the prandtl number P increases along with the increase in the values of the porosity parameter α (Fig .10). Figure 11 illustrates that the mean temperature decreases with increase of prandtl number p for large depths whereas it remains constant for shallow depths.

5. Fig 12 illustrates that with an increase in the prandtl number the mean mixed temperature decreases in the flow region. Mean mixed temperature remains unaltered for different smaller values of $\alpha \geq 0.2$ but increases with the increase in the values of the prandtl number p (Figure 13)for large depths. Mean mixed temperature of the flow region decreases as the porosity parameter α increases. Fig 14 in the case of shallow fluids .

6. The rate of heat transfer (Nusselt Number) decreases with the increase in the values of α and the prandtl number 'p'.Fig.15.The rate of heat transfer on the bottom decreases with the increase in the values of the prandtl number p (Fig 16) for large depths.

7. Fig.17 illustrates that the rate of heat transfer towards the free surface increases with the increase in the values of α as well as the prandtl number 'p'. Figure18 illustrates that the Nusselt number increases on the free surface with the increase of the prandtl number p for large depths.

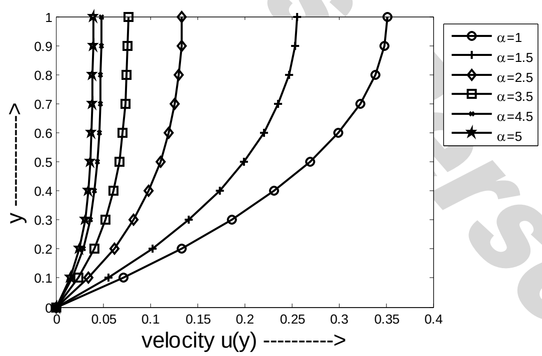


fig.1 velocity profile for c1=1 and h=1

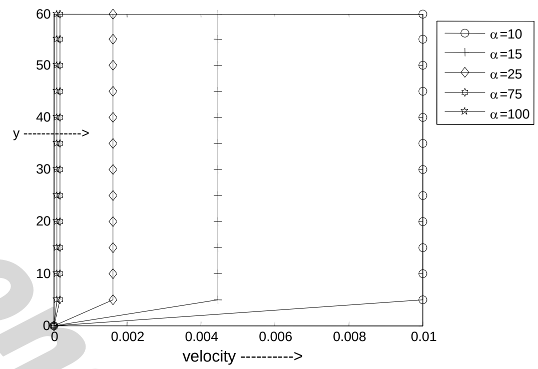


fig.2 velocity profile for large h=60

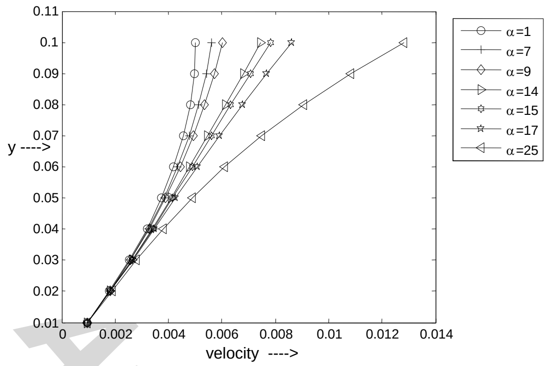


fig.3 velocity for small h=.1

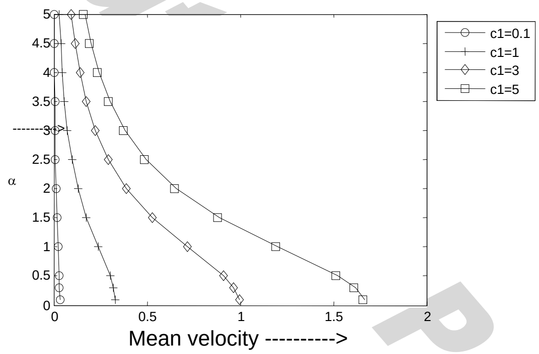


fig.4 mean velocity for h=1

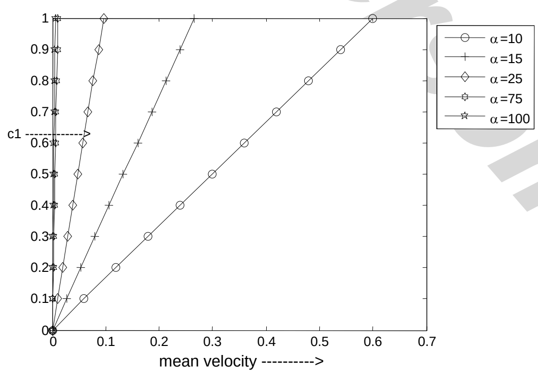


fig.5 mean velocity profile for large h=60

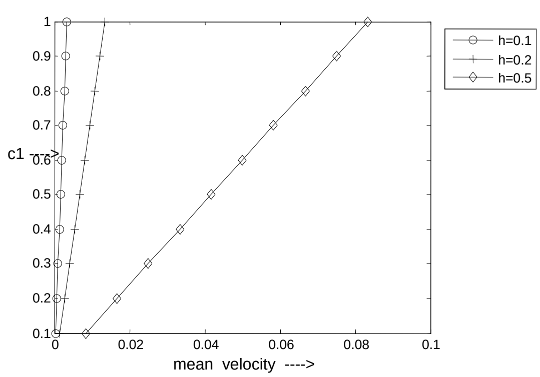


fig.6 mean velocity for small h=.1,.2,.3

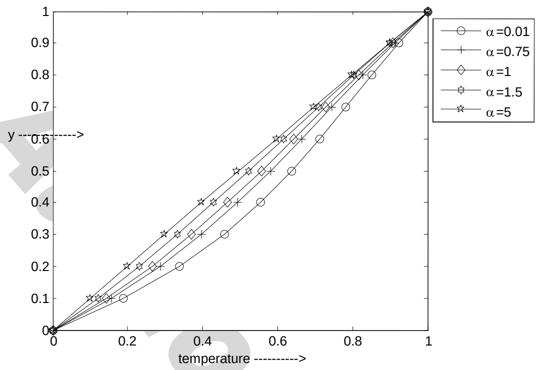


fig.7 Temperature distribution for h=1,E=5,c1=1,c2=1,p=1

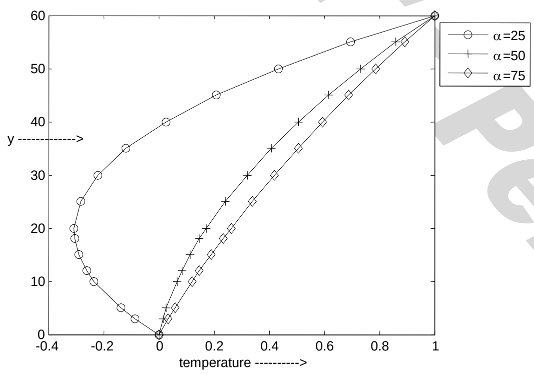


fig. 8 Temperature distribution for large h=30,E=5,c1=1,c2=1,p=1

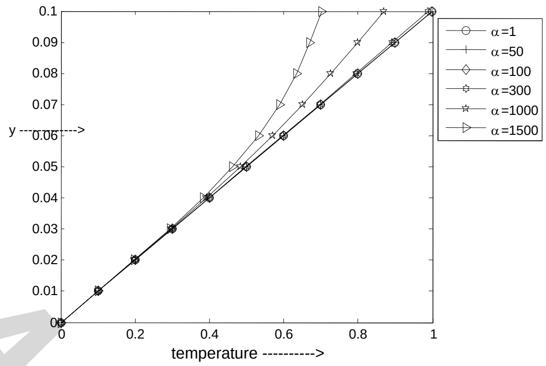


fig.9 Temperature distribution for p=1 ,h=.1,E=5,c1=1,c2=1

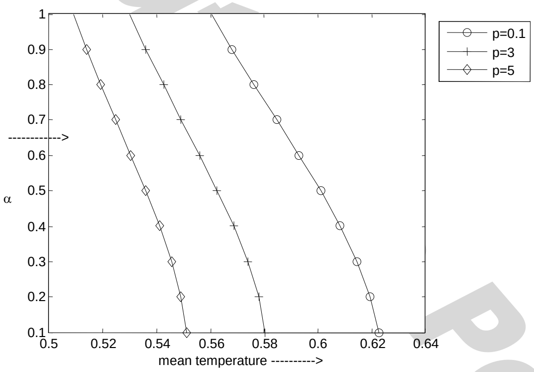


fig.10 Mean Temperature for h=1,E=5,c1=1,c2=.5

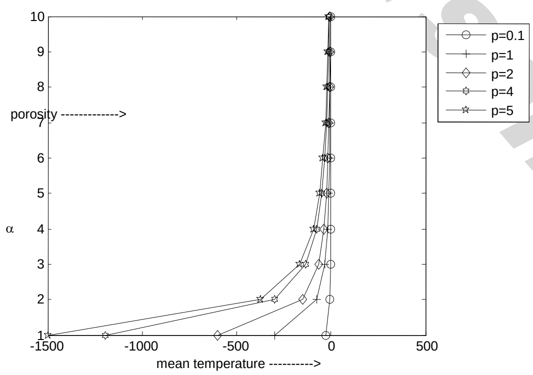


fig. 11 mean temperature distribution for large h=60,E=5,c1=1,c2=1

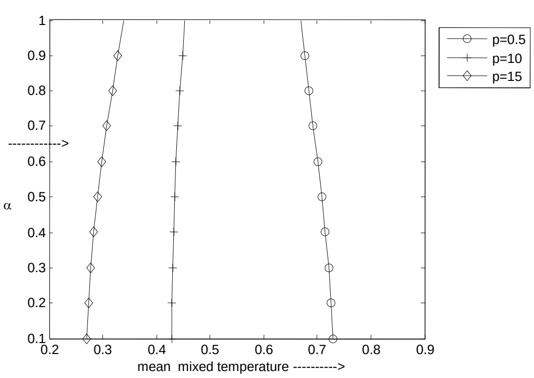


fig.12 Mean Mixed Temperature for h=1,E=5,c1=1,c2=1

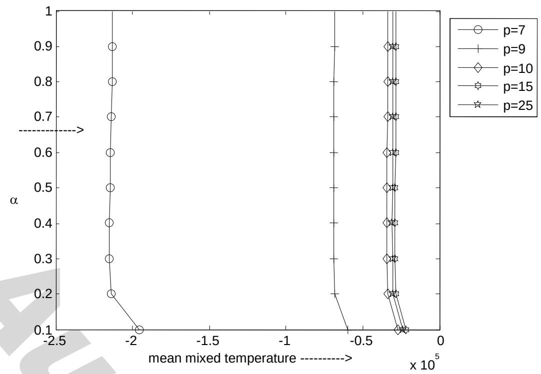


fig.13 mean mixed temperature for h=60,E=5,c1=1,c2=1

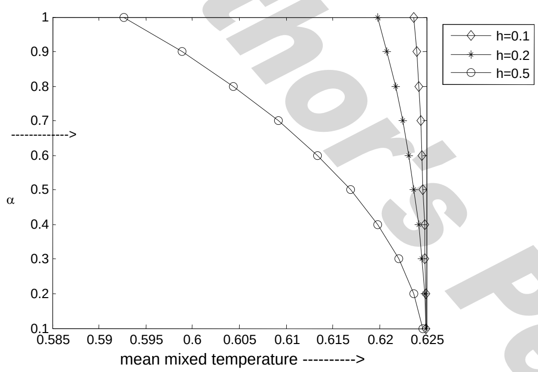


fig.14 mean mixed temperature

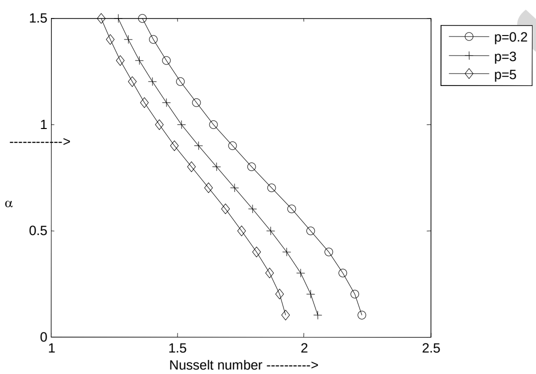


fig.15 Nusselt number on the bottom for h=1,E=5,c1=1,c2=.5

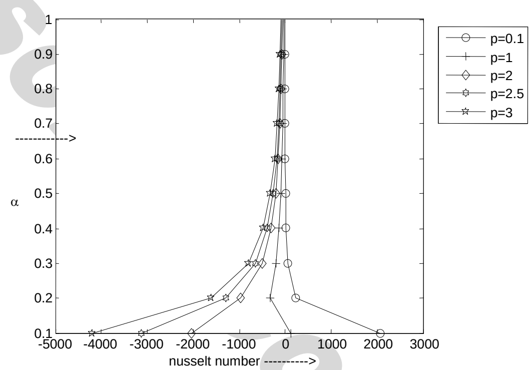


fig.16 nusselt number for h=60,E=5,c1=1,c2=1

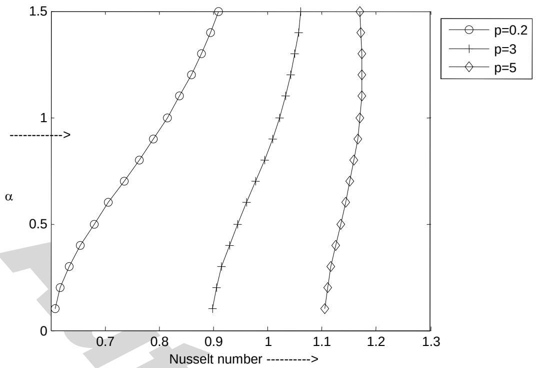


fig.17 Nusselt number on the top for h=1,E=5,c1=1,c2=.5

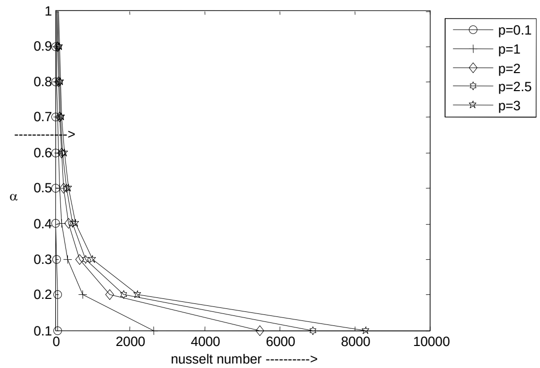


fig.18 nusselt number for h=60,E=5,c1=1,c2=1

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