

MHD Flow of an Incompressible Viscous Fluid Through Porous Medium Bounded by Two Semi Infinite Parallel Plates

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ABSTRACT

The aim of the present investigation is to study the unsteady flow of an incompressible viscous fluid through porous medium bounded between two semi infinite parallel plates under the influence of transverse magnetic field applied perpendicular to the length of the plates. The whole system is under rotation about an axis perpendicular to the length of the plates. The fluid is set in motion due to impulsive moment of upper plate, while the lower plate is fixed. The effect of rotation, magnetic parameter and permeability parameter on the velocity of the fluid is examined at length. The effect of physical parameters on shear stress is also studied.

Keywords : Magnetic parameter, rotation parameter, permeability parameter, porous medium, shear stress.

1. INTRODUCTION

Green span [1] proposed theory of rotating fluids, which is considered to be highly important in various natural phenomena and for its applications in various technological situations. The study of rotating fluids relates to the oceanography, meteorology, atmospheric science and limnology etc. Many investigators such as Greenspan and Howard [2], Holton [3], Walin [4], Siegman [5], Puri and Kulshretha [6], Mazumder [7], Ganapathy [8], Hayat and Hutter [9], Singh *et al.* [10] and Guria *et al.* [11] have studied viscous incompressible fluid in a rotating system considering various situations. The problem of flow of viscous fluid under magnetic field, which is electrically conducting in a rotating medium is

studied by many investigators Seth and Jana [12], Setha and Maiti [13], Singh *et al.* [14], Ghosh and Pop [15], Wang and Hayat [16].

The study of flow through porous medium assumed importance because of it's interesting applications in diverse fields of Science, Engineering and Technology. The practical applications are in the percolation of water through soil, extraction and filtration of oils from wells, the drainage of water, irrigation and sanitary engineering and also in the inter-disciplinary fields such as biomedical engineering etc. The lung alveolar is an example that finds application in an animal body. The classical Darcy's law states that the pressure gradient pushes the fluid against body forces exerted by the medium which can be expressed as

$$\vec{V} = -\left(\frac{k}{\mu}\right)\nabla P$$

Modifications for the Classical Darcy's law were considered by the Beverse and Joseph [17], Saffmann [18] and others. A generalized Darcy's law proposed by Brinkman [19] is given by

$$O = -\nabla P - \left(\frac{\mu}{K}\right)\vec{V} + \mu\nabla^2\vec{V}$$

where μ and k are coefficients of viscosity of the fluid and permeability of the porous medium.

The non-Darcian approach is employed to study the problem of flow through highly porous medium by several investigators. Narasimha Charyulu and Pattabhi Rama Charyulu [20, 21], Narasimha Charyulu [22] and Singh [23] etc. studied the flow employing Brinkman law [19] for flow through highly porous medium.

The problem of fluid flow through rotating system studied by Guria *et al.* [24], Chandran *et al.* [25], Das *et al.* [26] for the classical flow of viscous fluid. Narasimha Charyulu *et al* [27] study the non-Darcian MHD flow through rotating porous duct.

In the present problem the unsteady flow of viscous incompressible electrically conducting fluid in a rotating system in the presence of uniform transverse magnetic field through porous medium is studied. The flow is induced due to the impulsive moment of

upper plate. The effect of permeability parameter, magnetic parameter are studied and the results are graphically represented.

2. Formulation of the problem :

Consider the unsteady flow of Newtonian fluid through porous medium bounded by two semi-infinite parallel plates $z=0$ and $z=h$. The fluid is electrically conducting, uniform transverse magnetic field B_0 is applied along Z-axis. The plates and the fluid are in the state of rigid body rotation about Z-axis with uniform angular velocity Ω . The whole system is at rest initially for $t \leq 0$. When $t > 0$, the upper plate moves with impulsive velocity U_0 in it's own plane in X direction, while the lower plate is at rest. The physical quantities depend upon only z and t . It is assumed that no applied voltage exists, as there will be no energy being added (or) extracted from the fluid by electrical means [28].

Since magnetic Reynolds number is very small for metallic liquids and partially ionized fluids, the induced magnetic field is neglected, in comparison to the applied magnetic field [29].

The velocity of the fluid is choosen to be $(u, v, 0)$ and magnetic field $\bar{B} = (0, 0, B_0)$

The equation of continuity is satisfied by the choice of the velocity and the equation of motion of the fluid is given by

$$\frac{\partial u}{\partial t} - 2\Omega v = \nu \frac{\partial^2 u}{\partial z^2} - \frac{\nu}{P} u - \frac{\sigma B_0^2}{\rho} u \quad \dots \quad (2.1)$$

$$\frac{\partial v}{\partial t} + 2\Omega v = \nu \frac{\partial^2 v}{\partial z^2} - \frac{\nu}{P} v - \frac{\sigma B_0^2}{\rho} v \quad \dots \quad (2.2)$$

$$-\frac{1}{\rho} \frac{\partial P}{\partial z} = 0 \quad \dots \quad (2.3)$$

The equation (2.3) implies the pressure is absent along the axis of rotation.

The equation (2.2) implies that there is a net cross flow in y direction because of absence of pressure.

The flow of the fluid is induced due to moment of upper plate in X-direction

The initial and boundary conditions for the problem are

$$\left. \begin{aligned} u = 0, v = 0; & 0 \leq z \leq h \text{ for } t \leq 0 \\ u = 0, v = 0; & \text{ at } z = 0 \text{ for } t > 0 \\ u = U_0, v = 0; & \text{ at } z = h \text{ for } t > 0 \end{aligned} \right\} \dots (2.4)$$

Introducing the non-dimensional variables.

$$z^* = \frac{z}{h}, u^* = \frac{u}{U_0}, v^* = \frac{v}{U_0}, t^* = \frac{tv}{h^2}, M^2 = \frac{\sigma B_0^2 h^2}{\rho\nu}, \beta^2 = \frac{h^2}{p}$$

$$\text{and } K^2 = \frac{\Omega h^2}{\nu} \dots (2.5)$$

Removing (*), the equations (2.1) & (2.2) become

$$\frac{\partial u}{\partial t} - 2K^2 v = \frac{\partial^2 u}{\partial z^2} - (M^2 + \beta^2)u \dots (2.6)$$

$$\frac{\partial v}{\partial t} + 2K^2 u = \frac{\partial^2 v}{\partial z^2} - (M^2 + \beta^2)v \dots (2.7)$$

Where M magnetic parameter, β permeability parameter and k rotation parameter which is reciprocal to Ekman number. The non dimensional initial and boundary conditions are

$$\left. \begin{aligned} u = 0, v = 0; & 0 \leq z \leq 1 \text{ for } t \leq 0 \\ u = 0, v = 0; & \text{ at } z = 0 \text{ for } t > 0 \\ u = 1, v = 0; & \text{ at } z = 1 \text{ for } t > 0 \end{aligned} \right\} \dots (2.8)$$

Combining the equations (2.6) and (2.7)

$$\frac{\partial q}{\partial t} = \frac{\partial^2 q}{\partial z^2} - (M^2 + \beta^2 + 2iK^2)q \dots (2.9)$$

Where $q = u + iv$ and $i = \sqrt{-1}$

The new boundary conditions (2.8) become

$$\left. \begin{aligned} q = 0; & 0 \leq z \leq 1, t \leq 0 \\ q = 0; & \text{ at } z = 0, t > 0 \\ q = 1; & \text{ at } z = 1, t > 0 \end{aligned} \right\} \dots (2.10)$$

Solution of the problem

The velocity of the fluid is represented as

$$q(z, t) = q_1(z) + q_2(z, t)$$

Where $q_1(z)$ is steady state solution and $q_2(z, t)$ is unsteady state solution.

Substituting in the equation (2.9), the equation satisfied by $q_1(t)$ is

$$\frac{d^2 q(z)}{dz^2} - (M^2 + \beta^2 + 2iK^2)q_1(t) = 0 \quad \dots \quad (2.11)$$

The solution of the equation (2.11) subject to the B.C's (2.10) is

$$q_1(z) = \frac{\sin h(A+iB)z}{\sin h(A+iB)} \quad \dots \quad (2.12)$$

Where

$$\left. \begin{aligned} A &= \frac{1}{\sqrt{2}} \left\{ \left[\sqrt{(M^2 + \beta^2)^2 + 4K^4} + (M^2 + \beta^2) \right]^{\frac{1}{2}} \right\} \\ B &= \frac{1}{\sqrt{2}} \left\{ \left[\sqrt{(M^2 + \beta^2)^2 + 4K^4} - (M^2 + \beta^2) \right]^{\frac{1}{2}} \right\} \end{aligned} \right\} \quad \dots \quad (2.13)$$

The unsteady state solution $q_2(z, t)$ is obtained from solving the equation (2.9)

Now $q_2(z, t)$ satisfies the differential equation

$$\frac{\partial q_2}{\partial t} + (M^2 + \beta^2 + 2iK^2)q_2 = \frac{\partial^2 q_2}{\partial z^2} \quad \dots \quad (2.14)$$

With the conditions

$$q_2(0, t) = 0, \quad q_2(1, t) = 0, \quad q_2(z, 0) = -\frac{\sin h(A+iB)z}{\sin h(A+iB)} \quad \dots \quad (2.15)$$

The solution of equation (2.14) subject to the conditions (2.15) is given by

$$q_2(z, t) = \sum_{n=1}^{\infty} A_n e^{-\lambda_n^2 t} \sin(n\pi z) \quad \dots \quad (2.16)$$

where $\lambda_n^2 = n^2 \pi^2 + (A+iB)^2 \quad \dots \quad (2.17)$

and $A_n = -2 \int_0^1 \frac{\sin h(A+iB)z}{\sin h(A+iB)} \sin(n\pi z) dz \quad \dots \quad (2.18)$

making use of (2.16), (2.17), (2.18), the fluid velocity is given by

$$q(z, t) = \frac{\sin h(A+iB)z}{\sin h(A+iB)} + 2 \sum_{n=1}^{\infty} \frac{n\pi(-1)^n e^{-\lambda_n^2 t}}{n^2 \pi^2 + (A+iB)^2} \sin(n\pi z) \quad \dots \quad (2.19)$$

The velocity components will be

$$u = \frac{(\sin hAZ \cos BZ)(\sin hA \cos B) + (\cos hAZ \sin BZ)(\cos hA \sin B)}{\sin h^2 A \cos^2 B + \cos h^2 A \sin^2 B} +$$

$$2 \sum_{n=1}^{\infty} \frac{n\pi(-1)^n e^{-(n^2\pi^2 + A^2 - B^2)t} \sin(n\pi z)}{(n^2\pi^2 + A^2 - B^2)^2 + 4A^2B^2} \times \left\{ (n^2\pi^2 + A^2 - B^2) \cos 2ABt - 2AB \sin 2ABt \right\}$$

... (2.20)

$$v = \frac{(\cos hAZ \sin BZ)(\sin hA \cos B) - (\sin hAZ \cos BZ)(\cos hA \sin B)}{\sin h^2 A \cos^2 B + \cos h^2 A \sin^2 B} -$$

$$2 \sum_{n=1}^{\infty} \frac{n\pi(-1)^n e^{-(n^2\pi^2 + A^2 - B^2)t} \sin(n\pi z)}{(n^2\pi^2 + A^2 - B^2)^2 + 4A^2B^2} \times \left\{ 2AB \cos 2ABt + (n^2\pi^2 + A^2 - B^2) \sin 2ABt \right\}$$

... (2.21)

Shearstress :

Shear stress near the plate $z = 0$ is given by

$$(\tau_{x_0} + i\tau_{y_0}) = \frac{(A + iB)}{\sin h(A + iB)} + 2 \sum_{n=1}^{\infty} \frac{n^2\pi^2(-1)^n e^{-[n^2\pi^2 + (A + iB)^2]t}}{[n^2\pi^2 + (A + iB)^2]} \dots \quad (2.22)$$

On separating real and imaginary parts in equation (2.22), the shear stress components τ_{x_0} and τ_{y_0} are given by

$$\tau_{x_0} = \frac{2(A \sin hA \cos B + \cos hA \sin B)}{\cos h 2A - \cos 2B}$$

$$+ 2 \sum_{n=1}^{\infty} \frac{n^2\pi^2(-1)^n e^{-[n^2\pi^2 + A^2 - B^2]t}}{[n^2\pi^2 + A^2 - B^2]^2 + 4A^2B^2} \times \left\{ (n^2\pi^2 + A^2 - B^2) \cos 2ABt - 2AB \sin 2ABt \right\}$$

... (2.23)

$$\tau_{y_0} = \frac{2(B \sin hA \cos B - A \cos hA \sin B)}{\cos h 2A - \cos 2B}$$

$$- 2 \sum_{n=1}^{\infty} \frac{n^2\pi^2(-1)^n e^{-[n^2\pi^2 + A^2 - B^2]t}}{(n^2\pi^2 + A^2 - B^2)^2 + 4A^2B^2} \times \left\{ (n^2\pi^2 + A^2 - B^2) \sin 2ABt + 2AB \cos 2ABt \right\}$$

... (2.24)

The general solution, given by (2.20) and (2.21) for the fluid velocity is valid for every value of time t. But it converges slowly for small values of time t [30].

Special cases :

Case 1 : The flow of the fluid through porous medium in a rotating system in the absence of magnetic field is obtained by taking $M = 0$.

The fluid velocity is given by

$$q(z,t) = \frac{\sin h(A+iB)z}{\sin h(A+iB)} + 2 \sum_{n=1}^{\infty} \frac{n\pi(-1)^n e^{-\lambda_n^2 t}}{n^2\pi^2 + (A+iB)^2} \sin(n\pi z) \quad \dots \quad (2.25)$$

Sheary stress near the plate $z = 0$ is given by

$$\tau_{xo} + \tau_{yo} = \frac{(A+iB)}{\sin h(A+iB)} + 2 \sum_{n=1}^{\infty} \frac{n^2\pi^2(-1)^n e^{-[n^2\pi^2+(A+iB)^2]t}}{[n^2\pi^2 + (A+iB)^2]} \quad \dots \quad (2.26)$$

Where

$$\left. \begin{aligned} A &= \frac{1}{\sqrt{2}} \left\{ \left[\sqrt{\beta^4 + 4K^4} + \beta^2 \right]^{\frac{1}{2}} \right\} \\ B &= \frac{1}{\sqrt{2}} \left\{ \left[\sqrt{\beta^4 + 4K^4} - \beta^2 \right]^{\frac{1}{2}} \right\} \end{aligned} \right\} \quad \dots \quad (2.27)$$

Case 2 : The flow of the fluid through porous medium under magnetic field when system is non rotating about Z-axis i.e. $K = 0$.

The fluid velocity is given by

$$q(z,t) = \frac{\sin h\left(\sqrt{M^2 + \beta^2}\right)z}{\sin h\left(\sqrt{M^2 + \beta^2}\right)} + 2 \sum_{n=1}^{\infty} \frac{n\pi(-1)^n e^{-\lambda_n^2 t}}{(n^2\pi^2 + M^2 + \beta^2)^2} \sin(n\pi z) \quad \dots \quad (2.28)$$

Sheary stress near the plate $z = 0$ is given by

$$\tau_{xo} + i\tau_{yo} = \frac{\sqrt{M^2 + \beta^2}}{\sin h\left(\sqrt{M^2 + \beta^2}\right)} + 2 \sum_{n=1}^{\infty} \frac{n^2\pi^2(-1)^n e^{-[n^2\pi^2+M^2+\beta^2]t}}{[n^2\pi^2 + M^2 + \beta^2]} \quad \dots \quad (2.29)$$

Case 3 : Flow of the fluid through porous medium in the absence of magnetic field and non rotation of the system about Z-axis.

$$q(z, t) = \frac{\sin h \beta z}{\sin h \beta} + 2 \sum_{n=1}^{\infty} \frac{n \pi (-1)^n e^{-\lambda_n^2 t}}{n^2 \pi^2 + \beta^2} \sin(n \pi z) \quad \dots \quad (2.30)$$

Sheary stress near the plate $z = 0$ is given by

$$\tau_{x_0} + i \tau_{y_0} = \frac{\beta}{\sin h \beta} + 2 \sum_{n=1}^{\infty} \frac{n^2 \pi^2 (-1)^n e^{-[n^2 \pi^2 + \beta^2] t}}{(n^2 \pi^2 + \beta^2)} \quad \dots \quad (2.31)$$

Case 4 : Flow of the fluid through clear region under the influence of magnetic field and rotation of the system about z-axis.

The fluid velocity is given by

$$q(z, t) = \frac{\sin h(A + iB)z}{\sin h(A + iB)} + 2 \sum_{n=1}^{\infty} \frac{n \pi (-1)^n e^{-\lambda_n^2 t}}{n^2 \pi^2 + (A + iB)^2} \sin(n \pi z) \quad \dots \quad (2.32)$$

Sheary stress near the plate $z = 0$ is given by

$$\tau_{x_0} + i \tau_{y_0} = \frac{(A + iB)}{\sin h(A + iB)} + 2 \sum_{n=1}^{\infty} \frac{n^2 \pi^2 (-1)^n e^{-[n^2 \pi^2 + (A + iB)^2] t}}{[n^2 \pi^2 + (A + iB)^2]} \quad \dots \quad (2.33)$$

Where

$$\left. \begin{aligned} A &= \frac{1}{\sqrt{2}} \left\{ \sqrt{M^4 + 4K^4 + M^2} \right\}^{\frac{1}{2}} \\ B &= \frac{1}{\sqrt{2}} \left\{ \sqrt{M^4 + 4K^4 - M^2} \right\}^{\frac{1}{2}} \end{aligned} \right\} \quad \dots \quad (2.34)$$

Conclusion :

The flow of viscous incompressible fluid through porous medium under magnetic field is examined. When the lower plate is at rest and the upper plate is given impulsive velocity U_0 at $t > 0$, the whole system is under rotation with angular velocity Ω . As $t \rightarrow \infty$ the flow becomes only steady state, the transient effect is negligible. The velocity components u and v are given by (2.20), (2.21). For large values of rotation parameter K^2 boundary layer flow is expected near upper plate i.e. $z = 1$.

The velocity components for the Newtonian fluid through porous medium under magnetic field are.

$$u = e^{-A\xi} \cos B\xi, \quad v = e^{-A\xi} \sin B\xi$$

Where ξ is boundary layer coordinate given by $\xi = 1 - z$

where
$$A = K \left[1 + \frac{M^2}{4K^2} + \frac{\beta^2}{4K^2} \right]$$

$$B = K \left[1 - \frac{M^2}{4K^2} - \frac{\beta^2}{4K^2} \right]$$

velocity components near the boundary for the Newtonian fluid in clear region i.e., $\beta = 0$

$$u = e^{-A\xi} \cos B\xi, \quad v = e^{-A\xi} \sin B\xi$$

where

$$A = K \left\{ 1 + \frac{M^2}{4K^2} \right\}$$

$$B = K \left\{ 1 - \frac{M^2}{4K^2} \right\}$$

The boundary layer thickness is of order $O(A^{-1})$ near the moving plate representing the modified Ekman boundary layer.

The velocity components of the Newtonian fluid tends to zero as ξ increases i.e., $\xi \geq \frac{1}{A}$ outside the boundary layer region.

Therefore it is observed that incase of rotating system the flow is confined to the boundary layer region only. The effect of permeability parameter and magnetic parameter will influence the boundary layer flow.

Fig 1 and Fig. 2 shows that the effect of magnetic parameter is to decrease the velocity components u and v of the fluid. As M increases, velocity profiles decreases.

From Fig. 3 and 4 it is observed that as the rotation parameter K increases the primary velocity u decreases but also the secondary velocity v increases. This is due to the fact that coriolis force induce secondary flow. As the distance moves from lower plate to upper plate the primary velocity increases from

$z = 0$ to $z = 1$.

From Fig. 5 and Fig. 6 we can conclude that as the permeability parameter P decreases i.e., β increases, the velocity of the fluid is decreasing in both cases of primary and secondary flow. This is due to the increasing resistance of porous medium.

From 7 to 10, it is observed that as the magnetic parameter (or) rotation parameter increases, the values of primary shear stress (τ_{xo}) show decrease. But the secondary shear stress (τ_{yo}) increases with increasing in magnetic (or) rotation parameter values.

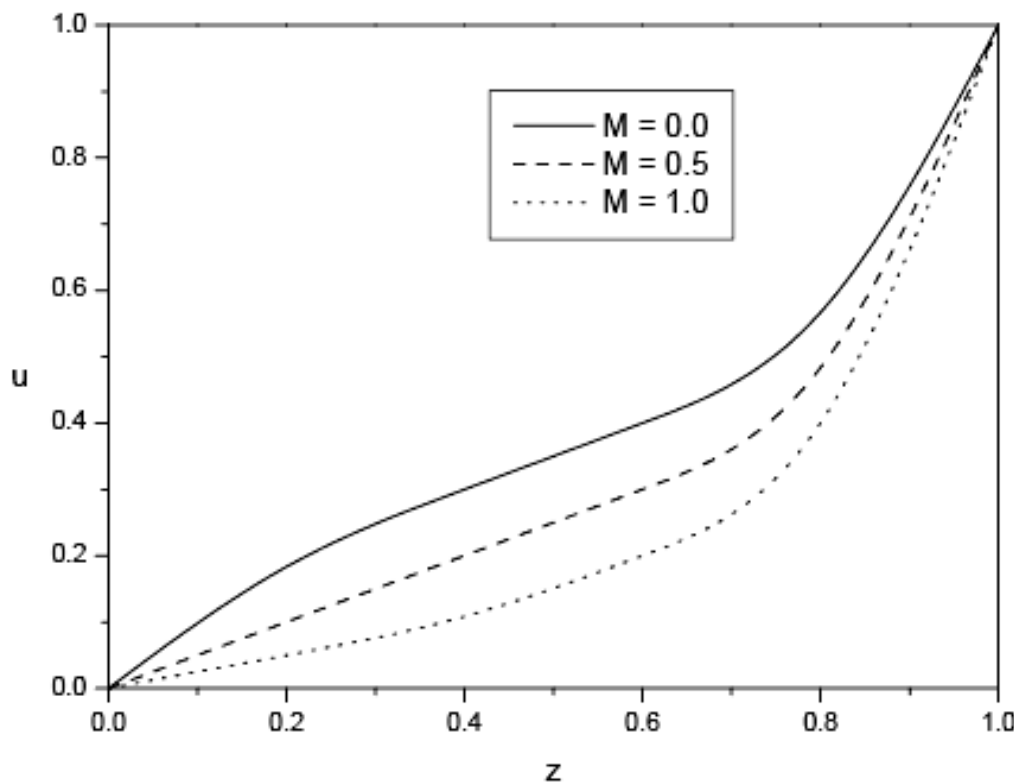


Fig. 1 : Variation of primary velocity with Magnetic parameter

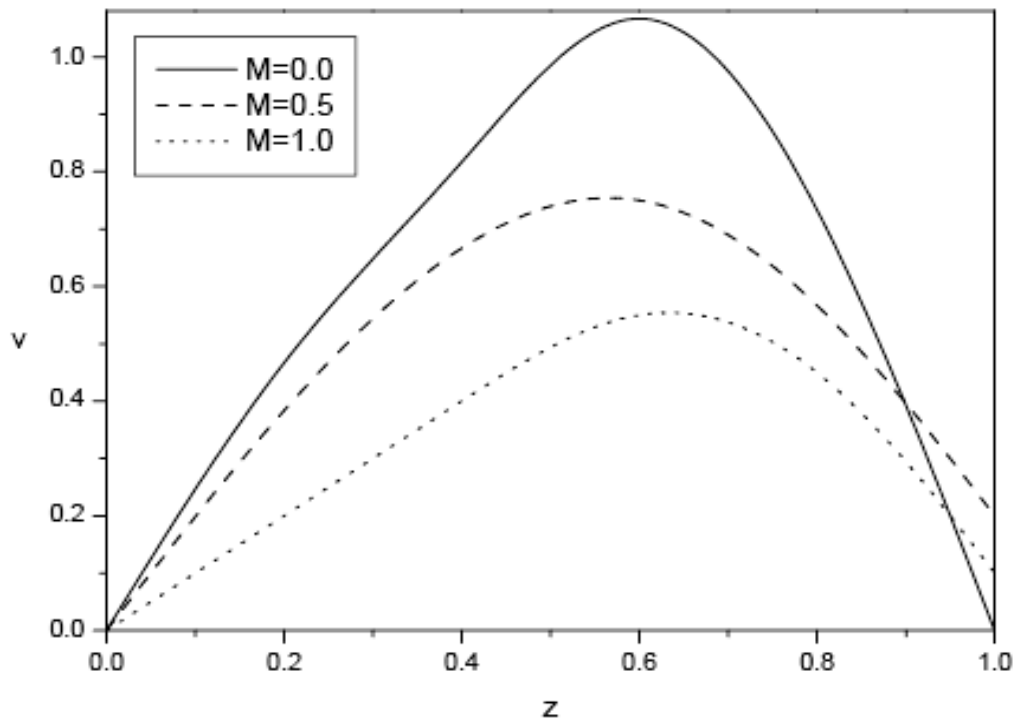


Fig. 2 : Variation of secondary velocity with Magnetic parameter

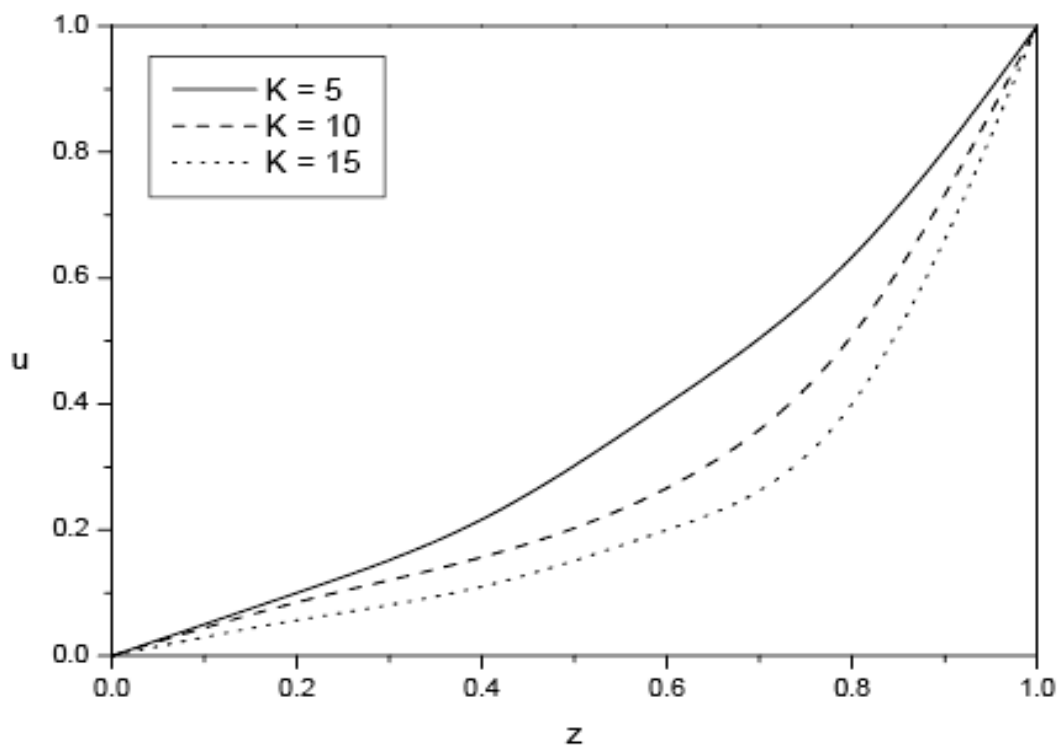


Fig. 3 : Variation of primary velocity with Rotation parameter

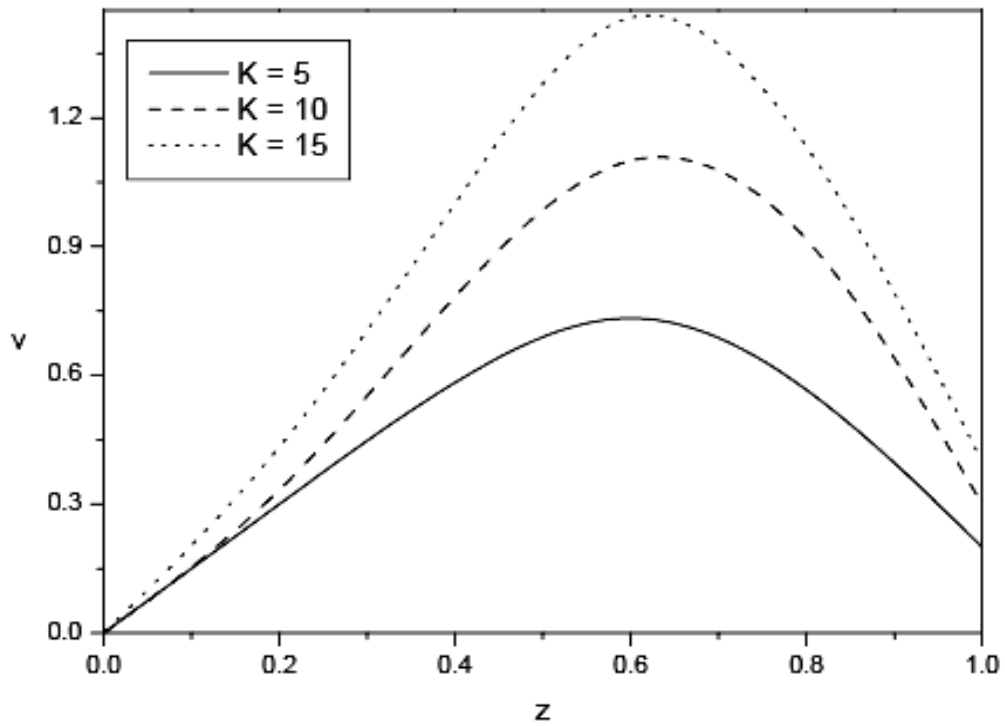


Fig. 4 : Variation of secondary velocity with Rotation parameter

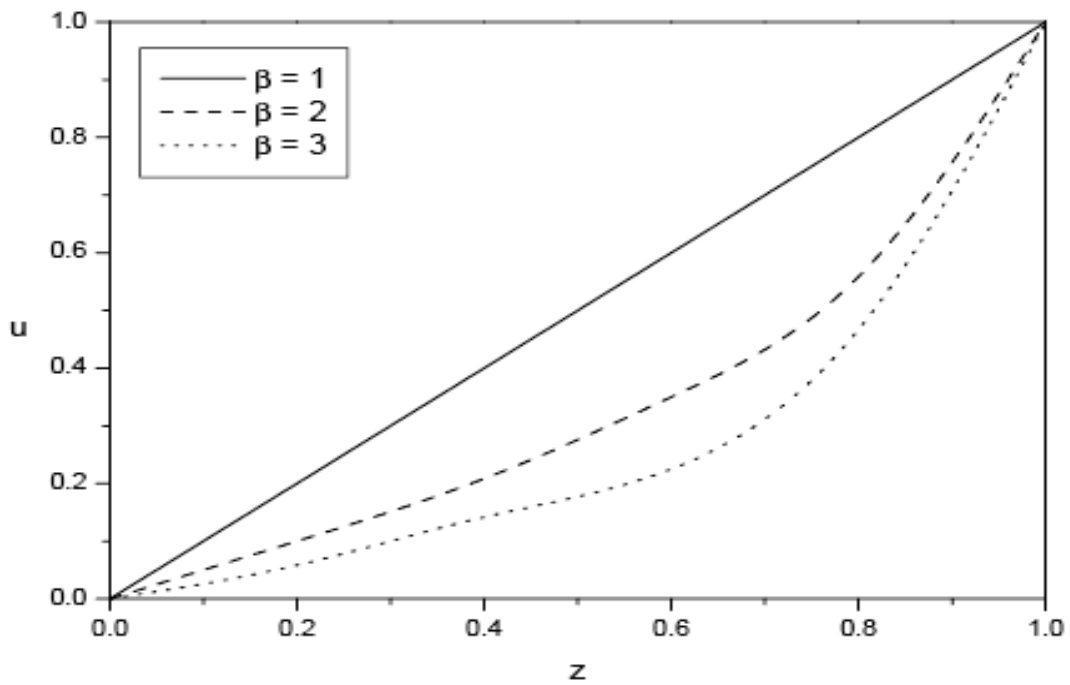


Fig. 5 : Variation of primary velocity with Permeability parameter

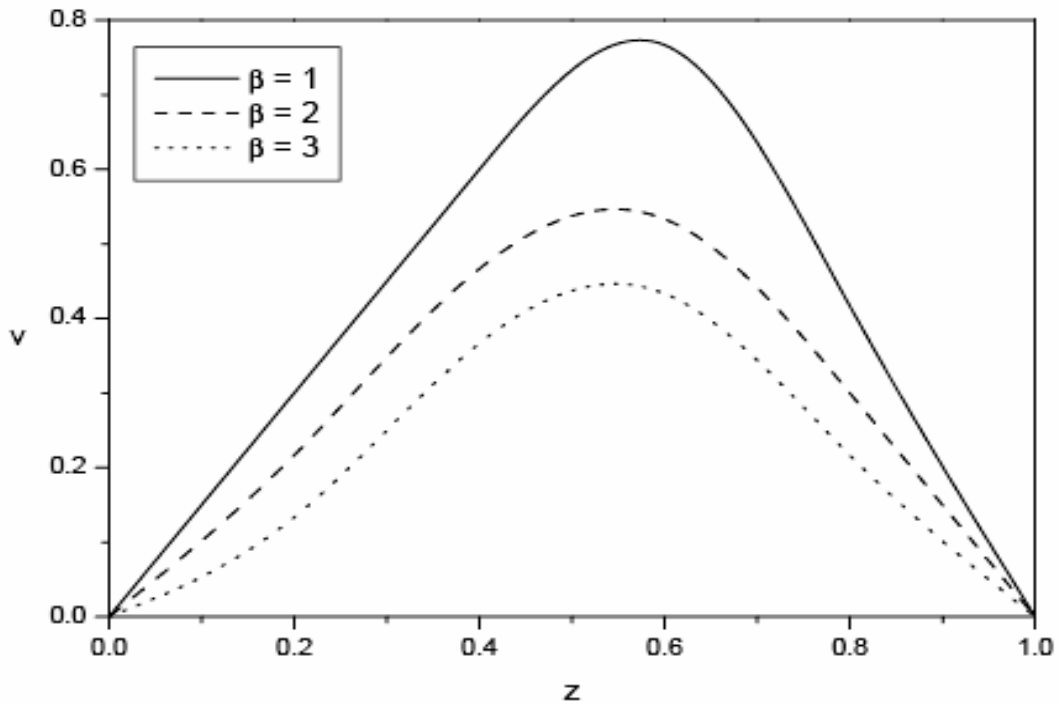


Fig. 6 : Variation of secondary velocity with Permeability parameter

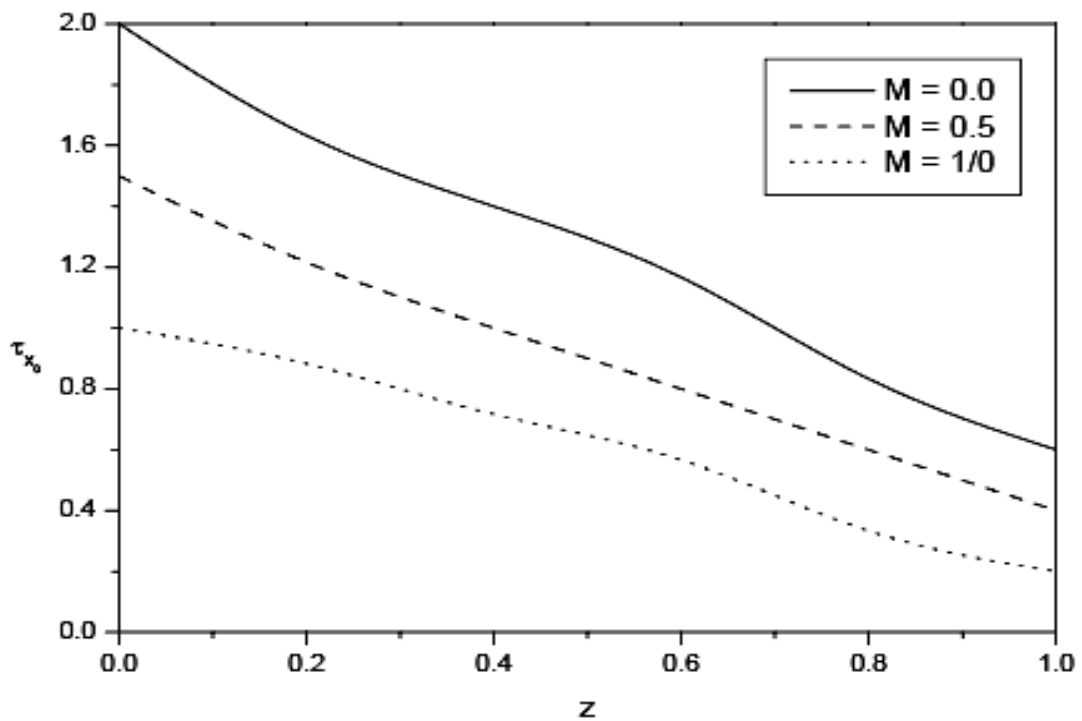


Fig. 7 : Variation of primary shear stress with Magnetic parameter

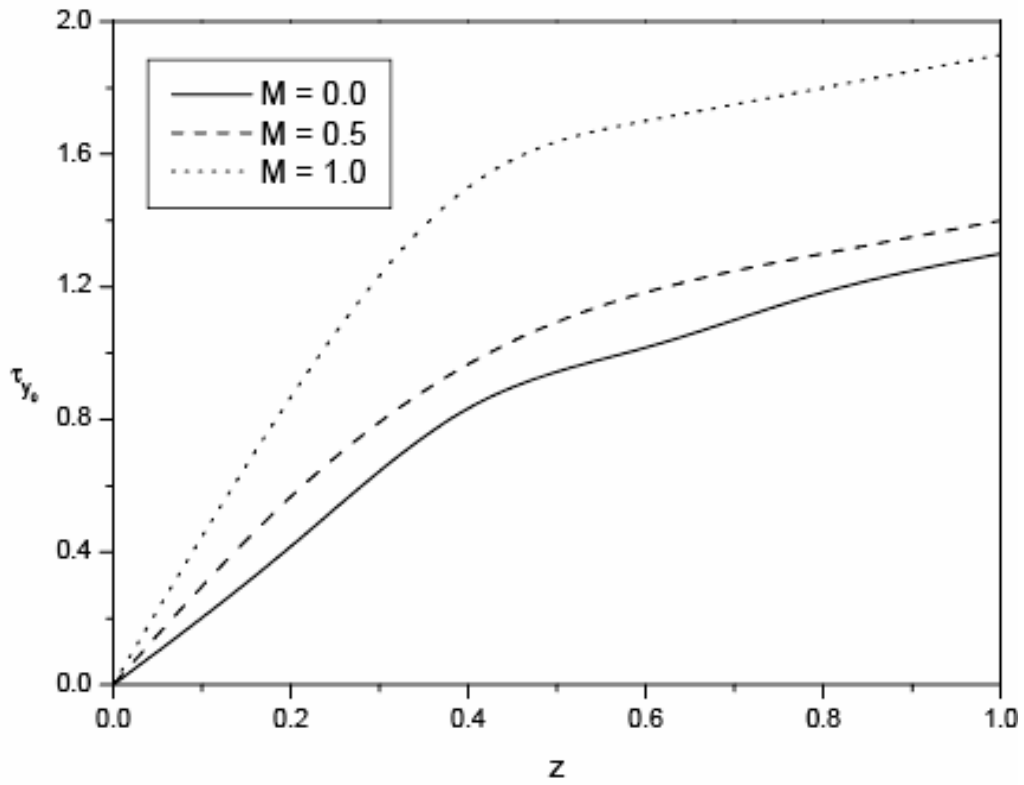


Fig. 8 : Variation of secondary shear stress with Magnetic parameter

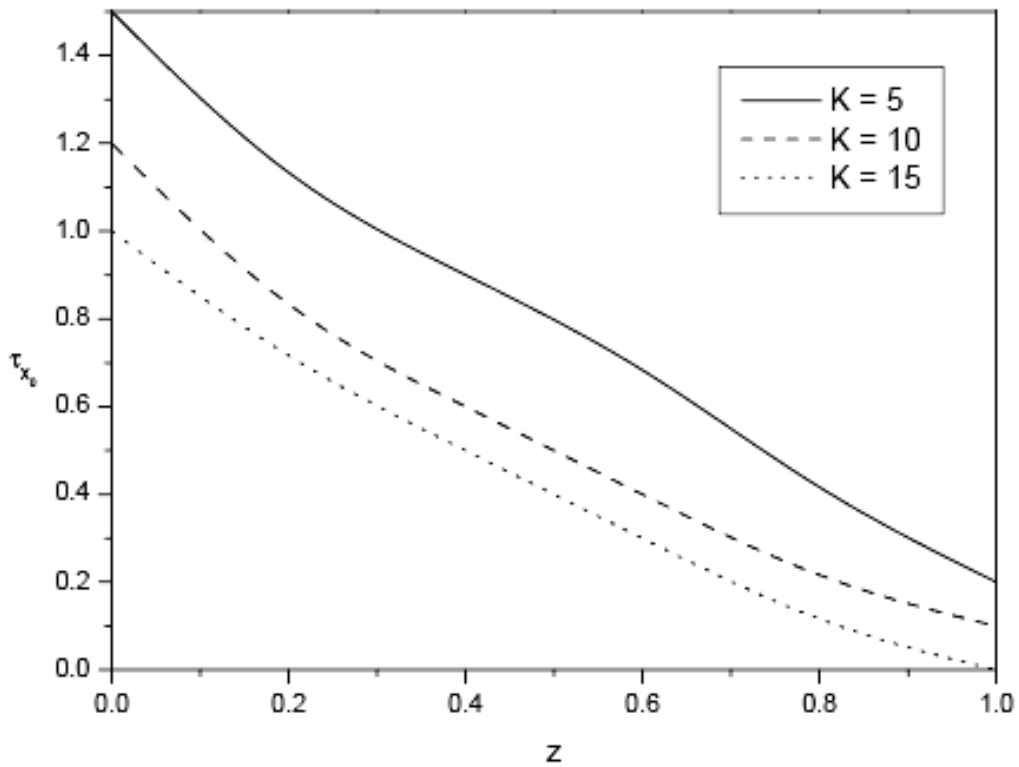


Fig. 9 : Variation of primary shear stress with Rotation parameter

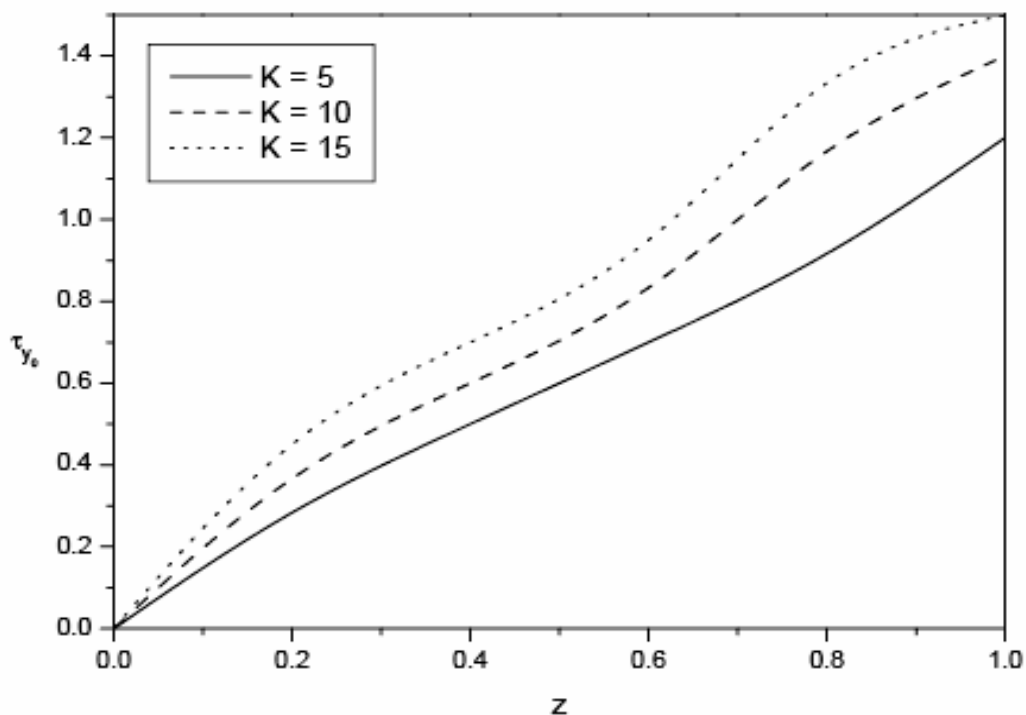


Fig. 10 : Variation of secondary shear stress with Rotation parameter

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