

On soft semi compact space

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Abstract: The concept of soft set theory is introduced by Molodtsov which can be used as a mathematical tool for dealing with uncertainty. In this paper the basic properties of soft sets are introduced and the concepts of soft compact space and soft semi compact space are discussed. Further some results in soft compact space and soft semi compact space are established.

Keywords: Soft set, soft topology, soft open (closed) sets, soft semi open(closed) sets, soft compact space, soft semi compact space.

1. INTRODUCTION

Molodtsov[3] in the year 1999 introduced soft set theory as a mathematical tools for dealing uncertainty and vagueness. The development in the field of soft set theory and its applications has been taking place in a rapid pace. Soft topological spaces were initially introduced by shabir and Naz[6]. In the year 2013 Chen.B[2] has contributed to soft semi open sets in soft topological spaces. Some properties of semi compact spaces are presented by Chandrasekhara Rao[1] in the year 2014 and Sai and Srinivasa kumar[4] discussed some properties of soft semi open sets and soft semi topology. Soft regular generalized closed sets in soft topological spaces are studied by Saziye Yuksel[5]. In the present study soft compact spaces, soft semi compact spaces are introduced and some results are established.

2. PRELIMINARIES : In this section the concept of soft sets and several operations among them are presented.

Let X be an initial universal set and E be a set. $P(X)$ denotes power set of X and $A \subseteq E$.

2.1 Definition: A pair (F, A) is called a soft set over X where F is a mapping $F: A \rightarrow P(X)$. That is soft set over X is a parameterized family of subsets of the universal

set X . For $e \in A$, $F(e)$ may be considered as the set of e-elements of the soft set (F, A) . Let us denote collection of all soft sets over X by $SS(X)$.

A soft set is not just a set, for illustration an example from Molodsov [3] is given below.

Example: Let a soft set (F, E) describes the attractiveness of the houses which Mr. X is going to buy.

U : Set of houses under consideration.

E : Set of parameters = {Expensive, beautiful, wooden, cheap, in the green surrounding, modern, in good repair, in bad repair}

Here to define a soft set means to point out expensive houses, beautiful houses etc. Here $F(e)$ is arbitrary some of them may be empty or may have non-empty intersections.

2.2 Definition: Let $(F, A) \in SS(X)$. If $F(x) = \phi \forall x \in A$ then (F, A) is called an empty soft set and is denoted by F_ϕ or Φ . If $F(x) = X \forall x \in A$ then (F, A) is called an A-universal soft set and if $A=E$ then it is called an universal soft set on X denoted by \bar{X} .

2.3 Definition: Let $(F, A), (G, B) \in SS(X)$. Then (F, A) is said to be a soft subset of (G, B) denoted by $(F, A) \subseteq (G, B)$ if $A \subseteq B$ and $F(x) \subseteq G(x) \forall x \in A$. Further $(F, A), (G, B)$ are said to be soft equal if (F, A) is a soft subset of (G, B) and vice-versa.

2.4 Definition: For $(F, A), (G, B) \in SS(X)$ the union of $(F, A), (G, B)$ denoted by $(F, A) \cup (G, B)$ is defined as a soft set (H, C) where $C = A \cup B$ and $\forall x \in C$

$$\begin{aligned} H(x) &= F(x) \text{ Whenever } x \in A - B \\ &= G(x) \text{ Whenever } x \in B - A \\ &= F(x) \cup G(x) \text{ Whenever } x \in A \cap B \end{aligned}$$

2.5 Definition: The soft intersection of two soft sets (F, A) and (G, B) denoted by $(F, A) \cap (G, B)$ is defined as a soft set (H, C) where $C = A \cap B$ and $H(x) = F(x) \cap G(x) \forall x \in C$.

2.6 Definition: The difference (H, A) of two soft sets (F, A) and (G, A) over X denoted by $(F, A) / (G, A)$ is defined as $H(x) = F(x) - G(x)$.

2.7 Definition: The relatively complement of a soft set (F, A) denoted by $(F, A)^C$ is defined by $(F, A)^C = (F^C, A)$ where $F^C : A \rightarrow P(X)$ is a mapping given by $F^C(x) = X - F(x) \quad \forall x \in A$.

2.8 Definition: If (F, A) and (G, B) are any two soft sets then “ (F, A) AND (G, B) ” denoted by $(F, A) \wedge (G, B)$ is defined by $(F, A) \wedge (G, B) = (H, A \times B)$, where $H(\alpha, \beta) = F(\alpha) \cap G(\beta), \forall (\alpha, \beta) \in A \times B$.

2.9 Definition: If Λ is an index set and $\{(F_i, A)\}_{i \in \Lambda}$ is a family of soft sets over X . Then their unions and intersections denoted by $\bigcup_{i \in \Lambda} (F_i, A)$ and $\bigcap_{i \in \Lambda} (F_i, A)$ are the soft sets (G, A) and (H, A) given by

$$G(a) = \bigcup_{i \in \Lambda} F_i(a) \quad \forall a \in A \quad \text{and} \quad H(a) = \bigcap_{i \in \Lambda} F_i(a) \quad \forall a \in A$$

We also write $\bigcup_{i \in \Lambda} (F_i, A) = (G, A)$ and $\bigcap_{i \in \Lambda} (F_i, A) = (H, A)$

2.10 Definition: Let $\tilde{\tau}$ be the collection of soft sets over X then $\tilde{\tau}$ is said to be a soft topology on X is (i) $\Phi, \tilde{X} \in \tilde{\tau}$ (ii) For an arbitrary collection $\{(F_i, A)\}_{i \in \Lambda}$ of soft sets in X , $\bigcup_{i \in \Lambda} (F_i, A) \in \tilde{\tau}$ and (iii) soft intersection of two soft sets in $\tilde{\tau}$ is a member of $\tilde{\tau}$. The pair $(X, \tilde{\tau})$ is called a soft topological space and every member of $\tilde{\tau}$ is called a soft open set.

2.11 Definition: A soft set (F, E) is called soft closed in X if $(F, E)^C \in \tilde{\tau}$.

2.12 Definition: Let $(X, \tilde{\tau})$ be a soft topological space over X and (F, E) be a soft set over X , then the soft closure of (F, E) denoted by $\overline{(F, E)}^s$ is defined as the intersection of all soft closed supersets of (F, E) .

Remark: $\overline{(F, E)}^s$ is the smallest soft closed super set of (F, E) .

2.13 Definition: The soft interior of a soft set (F, E) is defined as the soft union of all soft open subsets of (F, E) and is denoted by $(F, E)_s^o$.

Remark: $(F, E)_s^o$ is the largest soft open set contained in (F, E) .

2.14 Definition: A soft open subset (F, B) of (F, E) is said to be a soft neighborhood of $\alpha \in (F, E)$ if $\alpha \in (F, B)$.

2.15 Definition: Let (F, B) be a soft subset of (F, E) . A point $x \in (F, E)$ is called a soft limit point of (F, B) if every soft neighborhood of x contains a point of (F, B) other than x .

2.16 Definition: Let $(X, \tilde{\tau})$ be a soft topological space then a soft subset (F, A) of X is said to be soft semi open if $(F, A) \subseteq \overline{\left(\frac{s}{(F, B)} \right)}_s^o$.

2.17 Definition: A soft subset (F, B) of X is said to be soft semi closed if

$$(F, B) \subseteq \left(\frac{s}{(F, B)} \right)_s^o$$

3. Soft compact space/ Soft semi compact space and related results.

3.1 Definition: Let $(X, \tilde{\tau})$ be a soft topological space and $(F, A) \subseteq \tilde{X}$ then

$\tilde{\tau}_A = \{ (F_i, L) \tilde{\cap} (F, A) / (F_i, L) \in \tilde{\tau} \}$ is a soft topology on (F, A) and (F, A) together with $\tilde{\tau}_A$ is called a **soft topological subspace** of the soft topological space $(X, \tilde{\tau})$.

3.2 Definition: A soft topological space $(X, \tilde{\tau})$ is said to be a **soft compact space** if every soft open cover of X has a finite sub cover for X .

3.3 Definition: A soft topological space $(X, \tilde{\tau})$ is said to be a **soft semi compact space** if every soft semi open cover of X has a finite sub cover for X .

Remark: Every soft compact topological space is soft semi compact topological space.

3.4 Theorem: *Soft closed subspaces of a soft compact space are always soft compact.*

3.5 Theorem: *Soft semi closed subspaces of a soft semi compact space are soft semi compact.*

3.6 Theorem: *If (F, A) and (G, B) are any two soft semi compact subspaces of a soft semi compact space then $(F, A) \wedge (G, B)$ is also soft semi compact.*

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