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Global Stability Analysis of A Three Level Ecological Ammensalism with Four Species

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ABSTRACT

This paper INTENDS to investigate the global stability of a mathematical model of an ecological Ammensal model with Four Species $(\mathcal{A},\mathcal{P},\mathcal{E},\mathcal{H})$Eco-System of Ammensal-Prey($\mathcal{A}$),Predator-Ammensal($\mathcal{P}$),Enemy-Ammensal($\mathcal{E}$) and malice($\mathcal{M}$) species.This model is formed by establishing the System of an Ammensal –prey($\mathcal{A}$) , a Predator –Ammensal($\mathcal{P}$) that endures on Ammensal –prey($\mathcal{A}$), enemy-Ammensal($\mathcal{E}$) and malice($\mathcal{M}$) for which $\mathcal{A}$, $\mathcal{P}$ are Ammensal respectively i.e., $\mathcal{E}$ and $\mathcal{M}$ adversely effect on $\mathcal{A}$ and $\mathcal{P}$ without themselves getting effected in any manner. Further $\mathcal{E}$ is Ammensal for $\mathcal{M}$ and $\mathcal{M}$ harms for $\mathcal{E}$..The three levels of Ammensalism are comprised in the pairs of $(\mathcal{A}, \mathcal{E})$, $(\mathcal{P}, \mathcal{H})$ and $(\mathcal{E},\mathcal{H})$. The model is constituted with a set of four first order non-linear ordinary differential coupled equations.In this model, sixteen equilibrium points are obtained. Global stabiliy of this model is ascertained in the normal steady state.

Keywords: Ammensal,Prey,Predator,Enemy,Malice,Eco-System,Equillibrium point, stable.

AMS Classification: 92D25, 92D40

1. INRTRODUCTION:

Computational Ecology is a scientific disciplined subject.It works as a central and main role in emphasizing the real life situations with the help of mathematical models.It is an advanced methodology in applied mathematics having prominent performance in the investigation of the complex physical models. It helps to develop efficient algorithms for studying ecological mathematical models.. Mathematical modeling of ecosystems was pioneered by Lotka [9] in 1925 and by Volterra [11] in 1931.], Kapur [7,8] and several authors. N.C. Srinivas [12] studied the competitive ecosystems of two, three or four species with limited and unlimited resources. The basic concepts of modeling have been presented in the treatises of Meyer [12.Recently Acharyulu et al. [1-6] obtained some noteworthy results "on the stability of many ecological Ammensal models.

In this article,an ecological Ammensal model with Four Species (A,P,E,H) Syn Eco-System of Ammensal-Prey(A),Predator-Ammensal (P),Enemy-Ammensal(E) and malice(M) species is discussed. The System contains Ammensal –prey(A) , a Predator –Ammensal(P), enemy-Ammensal(E) and malice(M).The three levels of Ammensalism are considered with First level Ammensal pair(A,E), the second level Ammensal pair (P, H) and (E,H) as third level Ammensalism. The model equations formed by a set of four first order non-linear ordinary differential coupled equations. Sixteen equilibrium points are identified. Global stability of this model is established.

A Schematic diagram of the present system under investigation can be shown as in Fig.1.

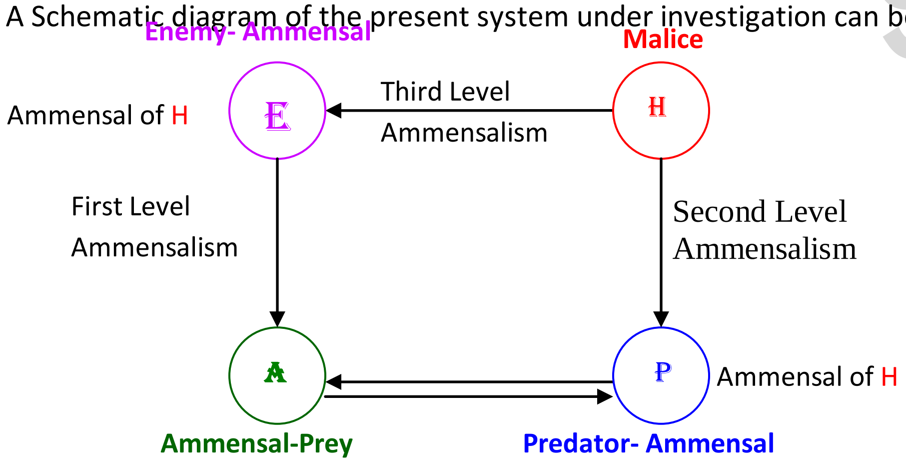

Fig. 1 Schematic diagram of the Eco- System

2. NOTATIONS :

$\mathbb{A}$: Prey-Ammensal:Prey for $\mathbb{P}$ and Ammensal for $\mathbb{E}$.

$\mathbb{P}$: Predator-Ammensal:Predator surviving upon $\mathbb{A}$ and Ammensal for $\mathbb{M}$.

$\mathbb{E}$: Enemy-Ammensal:Enemy for the Ammensal-Prey ($\mathbb{A}$) and Ammensal for $\mathbb{M}$.

$\mathbb{M}$: Malice of the Predator-Ammensal ($\mathbb{P}$) & Enemy-Ammensal ($\mathbb{E}$)

A,P,E,M : The Population growth rates of $\mathbb{A},\mathbb{P},\mathbb{E}\&\mathbb{M}$ at time t.

t : Time instant

n_i : Natural growth rates of $\mathbb{A},\mathbb{P},\mathbb{E}\&\mathbb{M}$,i=a,p,e & m respectively.

n_{ii} : Self inhibition coefficient of $\mathbb{A},\mathbb{P},\mathbb{E}\&\mathbb{M}$,i=a,p,e& m respectively.

n_{ap},n_{pa} : Interaction coefficients of $\mathbb{A}$ due to $\mathbb{P}$ and $\mathbb{P}$ due to $\mathbb{A}$.

n_{ae}, n_{pm} , n_{em}: Inhibition coefficients for $\mathbb{A}$ due to the enemy $\mathbb{E}$, $\mathbb{P}$ due to the malice $\mathbb{M}$, $\mathbb{E}$ due to malice $\mathbb{M}$ respectively

$K_i = \dfrac{n_i}{n_{ii}}$: Carrying capacities of $\mathbb{A},\mathbb{P},\mathbb{E}\&\mathbb{M}$,i=a,p,e & m respectively

$\alpha_{ij} = \dfrac{n_{ij}}{n_{ii}}$: Ammensal coeficient of $\mathbb{A},\mathbb{P},\mathbb{E}\&\mathbb{M}$, i,j=a,p,e & m(i≠j)

β_{ab}, β_{ba} : Interaction (A-P) coefficients of $\mathbb{A}$ due to $\mathbb{P}$ and $\mathbb{P}$ due to $\mathbb{A}$

Further the variables $\mathbb{A},\mathbb{P},\mathbb{E}\&\mathbb{M}$ are non-negative and the model parameters n_a, n_p, n_e, n_m; n_{aa}, n_{pp}, n_{ee}, n_{mm}; n_{ap}, n_{pa}, n_{ae}, n_{pm}, n_{em} are assumed to be non-negative constants.

3.BASIC EQUATIONS:

The model equations for an ecological Ammensal model with Four Species $(\mathbb{A},\mathbb{P},\mathbb{E},\mathbb{H})$ Syn Eco-System with three Ammensal levels are given by the following system of first order non-linear ordinary differential equations

The model equations for the growth rates of $\mathbb{A},\mathbb{P},\mathbb{E},\mathbb{M}$ are

$$(i) \quad \frac{dA}{dt} = n_{aa}(K_a A - A^2 - \beta_{ap} AP - \alpha_{ae} AE) \qquad (1)$$

$$(ii) \quad \frac{dP}{dt} = n_{pp}(K_p P - P^2 + \beta_{pa} PA - \alpha_{pm} PM) \qquad (2)$$

$$(iii) \quad \frac{dE}{dt} = n_{ee}(K_e E - E^2 - \alpha_{em} EM) \qquad (3)$$

$$(iv) \quad \frac{dM}{dt} = n_{mm}(K_m M - M^2) \qquad (4)$$

4. EQUILIBRIUM STATES:

The system under this investigation has sixteen equilibrium states defined by

$$\frac{dS}{dt} = 0, \ where\ S = \mathrm{A}, \mathrm{P}, \mathrm{E}, \mathrm{M}$$

A.Fully washed out state: $(i)\ \overline{A} = 0, \overline{P} = 0, \overline{E} = 0, \overline{M} = 0$

B.States in which three of the four species are washed out and the fourth is not.

(ii)Only the Ammensal-Prey (A) survives: $\overline{A} = \frac{n_a}{n_{aa}}, \overline{P} = 0, \overline{E} = 0, \overline{M} = 0$

(iii)Only the Predator-Ammensal (P) endures: $\overline{A} = 0, \overline{P} = \frac{n_p}{n_{pp}}, \overline{E} = 0, \overline{M} = 0$

(iv)Only the Enemy-Ammensal (E) of A exists: $\overline{A} = 0, \overline{P} = 0, \overline{E} = \frac{n_e}{n_{ee}}, \overline{M} = 0$

(v).Only the Malice (M) of P and E survives: $\overline{A} = 0, \overline{P} = 0, \overline{E} = 0, \overline{M} = \frac{n_m}{n_{mm}}$

C.States in which two of the four species are washed out and the other two are not:

(vi)Ammensal-Prey (A) and Predator-Ammensal (P) survive:

$$\overline{A} = \frac{n_a n_{pp} - n_p n_{ap}}{n_{aa} n_{pp} + n_{ap} n_{pa}}, \overline{P} = \frac{n_a n_{pa} + n_p n_{aa}}{n_{aa} n_{pp} + n_{ap} n_{pa}}, \overline{E} = 0, \overline{M} = 0$$

It will exist only when $n_a n_{pp} > n_p n_{ap}$

(vii)Ammensal-Prey (A) and Malice (M) are washed out:

$$\overline{A} = 0, \overline{P} = \frac{n_p}{n_{pp}}, \overline{E} = \frac{n_e}{n_{ee}}, \overline{M} = 0$$

(viii) Ammensal-Prey (S_1) and Predator-Ammensal (S_2) are washed out:

$$\overline{A} = 0, \overline{P} = 0, \overline{E} = \frac{n_e n_{mm} - n_m n_{em}}{n_{ee} n_{mm}}, \overline{M} = \frac{n_m}{n_{mm}}$$

It will exist only when $n_e n_{mm} - n_m n_{em} > 0$

(ix)Predator-Ammensal (P) and Enemy-Ammensal (E) are washed out:

$$\overline{A} = \frac{n_a}{n_{aa}}, \overline{P} = 0, \overline{E} = 0, \overline{M} = \frac{n_m}{n_{mm}}$$

(x)Ammensal-Prey (A) and Enemy-Ammensal (E) are washed out:

$$\overline{A} = 0, \overline{P} = \frac{n_p n_{mm} - n_m n_{pm}}{n_{pp} n_{mm}}, \overline{E} = 0, \overline{M} = \frac{n_m}{n_{mm}}$$

It will exist only when $n_p n_{mm} - n_m n_{pm} > 0$

(xi).Predator -Ammensal(P) and Malice (M) are washed out:

$$\overline{A} = \frac{n_a n_{ee} - n_e n_{ae}}{n_{aa} n_{ee}}, \overline{P} = 0, \overline{E} = \frac{n_e}{n_{ee}}, \overline{M} = 0,$$

It will exist only when $n_a n_{ee} - n_e n_{ae} > 0$

D.States in which one of the four species is washed out and the other three are not:

(xii)Only the Enemy-Ammensal (E) is washed out:

$$\overline{A} = \frac{n_a n_{pp} n_{mm} - n_{ap}(n_p n_{mm} - n_m n_{pm})}{n_{mm}(n_{aa} n_{pp} + n_{ap} n_{pa})}, \overline{P} = \frac{n_a n_{pa} n_{ee} + n_{aa}(n_p n_{mm} - n_m n_{pm})}{n_{44}(n_{aa} n_{pp} + n_{ap} n_{pa})},$$

$$\overline{E} = 0, \overline{M} = \frac{n_m}{n_{mm}}$$

$$\textit{only when } \; n_a n_{pp} n_{mm} > n_{ap}(n_p n_{mm} - n_m n_{pm})$$
$$\textit{and } n_a n_{pa} n_{ee} > n_{aa}(n_p n_{mm} - n_m n_{pm})$$

(xiii)Only the predator-Ammensal (P) is washed out

$$\overline{A} = \frac{n_a n_{ee} n_{mm} + n_{ae}(n_e n_{mm} - n_m n_{em})}{n_{aa} n_{ee} n_{mm}}, \overline{P} = 0,$$

$$\overline{E} = \frac{n_e n_{mm} - n_m n_{em,}}{n_{ee} n_{mm}} \textit{ and } \; \overline{M} = \frac{n_m}{n_{mm}}$$

$$\textit{It will exist only when } n_e n_{mm} > n_m n_{em.}$$

(xiv)Only the Malice (M) is washed out

$$\overline{A} = \frac{n_{pp} \;\; n_a n_{ee} - n_e n_{ae} \;\; - n_p n_{ap} n_{ee}}{n_{ee} \;\; n_{aa} n_{pp} + n_{ap} n_{pa}}, \overline{P} = \frac{n_{pa} \;\; n_a n_{ee} - n_e n_{ae} \;\; + n_p n_{aa} n_{ee}}{n_{ee} \;\; n_{aa} n_{pp} + n_{ap} n_{pa}},$$

$$\overline{E} = \frac{n_e}{n_{ee}}, \overline{M} = 0.$$

(xv).Only the Prey-Ammensal (A) is washed out

$$\overline{A} = 0, \overline{P} = \frac{n_p n_{mm} - n_{pm} n_m}{n_{pp} n_{mm}}, \overline{E} = \frac{n_e n_{mm} - n_m n_{em,}}{n_{ee} n_{mm}}, \overline{M} = \frac{n_m}{n_{mm}}$$

$$\textit{It will exist only when } \; n_p n_{mm} > n_{pm} n_m, n_e n_{mm} > n_m n_{em}.$$

E.States in which none of the four species is washed out.

(xvi)The co-existent state (or)Normal steady state

$$\overline{A} = n_{pp} \frac{n_a n_{ee} n_{mm} + n_{ae}(n_e n_{mm} - n_m n_{em,})}{n_{ee} n_{mm} \;\; n_{aa} n_{pp} + n_{ap} n_{pa}} - n_{ap} \frac{n_p n_{mm} + n_{pm} n_m}{n_{mm} \;\; n_{aa} n_{pp} + n_{ap} n_{pa}},$$

$$\overline{P} = n_{aa} \frac{n_p n_{mm} - n_{pm} n_m}{n_{mm} \;\; n_{aa} n_{pp} + n_{ap} n_{pa}} + n_{pa} \frac{(n_{ee} n_{mm} - n_{ae}(n_e n_{mm} - n_m n_{em}))}{n_{ee} n_{mm} \;\; n_{aa} n_{pp} + n_{ap} n_{pa}},$$

$$\overline{E} = \frac{n_e n_{mm} - n_m n_{em}}{n_{ee} n_{mm}} \; and \; \overline{M} = \frac{n_m}{n_{mm}}$$

$$It\ will\ exist\ only\ when\ \ n_e n_{mm} > n_m n_{em,} n_p n_{mm} > n_{pm} n_m \ ,$$
$$and\ n_{ee} n_{mm} > n_{ae}(n_e n_{mm} - n_m n_{em})$$

5. Liapunov's Function for Global Stability:

The Global stability can be investigated with the help of Liapunov's function. The examination of global stability is mainly based on the construction of Liapunov's fuction.It is discussed at normal state.It is observed that the system is stable at normal steady state.

$$i.e \quad \overline{A} = n_{pp} \frac{n_a n_{ee} n_{mm} + n_{ae}(n_e n_{mm} - n_m n_{em,})}{n_{ee} n_{mm} \;\; n_{aa} n_{pp} + n_{ap} n_{pa}} - n_{ap} \frac{n_p n_{mm} + n_{pm} n_m}{n_{mm} \;\; n_{aa} n_{pp} + n_{ap} n_{pa}},$$

$$\overline{P} = n_{aa} \frac{n_p n_{mm} - n_{pm} n_m}{n_{mm} \;\; n_{aa} n_{pp} + n_{ap} n_{pa}} + n_{pa} \frac{(n_{ee} n_{mm} - n_{ae}(n_e n_{mm} - n_m n_{em}))}{n_{ee} n_{mm} \;\; n_{aa} n_{pp} + n_{ap} n_{pa}},$$

$$\overline{E} = \frac{n_e n_{mm} - n_m n_{em}}{n_{ee} n_{mm}} \; and \; \overline{M} = \frac{n_m}{n_{mm}}$$

$$It\ will\ exist\ only\ when\ \ n_e n_{mm} > n_m n_{em,} n_p n_{mm} > n_{pm} n_m \ ,$$
$$and\ n_{ee} n_{mm} > n_{ae}(n_e n_{mm} - n_m n_{em})$$

Consider Liapunov function

$$L(A,P,E,M) = A - \overline{A} + \overline{A}[\log \overline{A} - \log A] + \lambda_1 \;\; P - \overline{P} + \overline{P}[\log \overline{P} - \log P]$$

$$+\lambda_2\ E-\overline{E}+\overline{E}[\log\overline{E}-\log E\ +\lambda_3\ M-\overline{M}+\overline{P}[\log\overline{M}-\log M$$

where $\lambda_1,\lambda_2\ and\ \lambda_3$ are scalars which protect the nature of Liapunov's function.

The change in Liapunov's function can be derived as below

$$\frac{dL}{dt}=\left(\frac{A-\overline{A}}{A}\right)\frac{dA}{dt}+\lambda_1\left(\frac{P-\overline{P}}{P}\right)\frac{dN_2}{dt}+\lambda_2\left(\frac{E-\overline{E}}{E}\right)\frac{dE}{dt}+\lambda_3\left(\frac{M-\overline{M}}{M}\right)\frac{dM}{dt}$$

$$\frac{dL}{dt}=\left(1-\frac{\overline{A}}{A}\right)\frac{dA}{dt}+\lambda_1\left(1-\frac{\overline{P}}{P}\right)\frac{dP}{dt}+\lambda_2\left(1-\frac{\overline{E}}{E}\right)\frac{dE}{dt}+\lambda_3\left(1-\frac{\overline{M}}{M}\right)\frac{dM}{dt}$$

$$\frac{dL}{dt}=\left(1-\frac{\overline{A}}{A}\right)A\ \ n_{aa}K_a-n_{aa}A-n_{aa}\beta_{ap}P-n_{aa}\alpha_{ae}E$$

$$+\lambda_1\left(1-\frac{\overline{P}}{P}\right)P(n_{pp}K_p-n_{pp}P+n_{pp}\beta_{pa}A-n_{pp}\alpha_{pm}M)$$

$$+\lambda_2\left(1-\frac{\overline{E}}{E}\right)E\ \ n_{ee}K_e-n_{ee}E-n_{ee}\alpha_{em}M$$

$$+\lambda_3\left(1-\frac{\overline{M}}{M}\right)M\ \ n_{mm}K_m-n_{mm}M$$

$$=\ A-\overline{A}\ \ n_{aa}K_a-n_{aa}A-n_{aa}\beta_{ap}P-n_{aa}\alpha_{ae}E$$

$$+\ \lambda_1\ P-\overline{P}\ \{n_{pp}K_p-n_{pp}P+n_{pp}\beta_{pa}A-n_{pp}\alpha_{pm}M\}$$

$$+\lambda_2\ E-\overline{E}\ \ n_{ee}K_e-n_{ee}E-n_{ee}\alpha_{em}M$$

$$+\lambda_3\ M-\overline{M}\ \ n_{mm}K_m-n_{mm}M$$

$$\frac{dL}{dt}=\ A-\overline{A}\ \ n_{aa}\overline{A}+n_{aa}\beta_{ap}\overline{P}+n_{aa}\alpha_{ae}\overline{E}-n_{aa}A-n_{aa}\beta_{ap}P-n_{aa}\alpha_{ae}E$$

$$+\lambda_1\ P-\overline{P}\ \ n_{pp}\overline{P}-n_{pp}\beta_{pa}\overline{A}+n_{pp}\alpha_{pm}\overline{M}-n_{pp}P+n_{aa}\beta_{ap}A-n_{pp}\alpha_{pm}M$$

$$+\lambda_2\ E-\overline{E}\ \ n_{ee}\overline{E}+n_{ee}\alpha_{em}\overline{M}-n_{ee}E-n_{ee}\alpha_{em}M$$

$$+\lambda_3\ M-\overline{M}\ \ n_{mm}K_m\overline{M}-n_{mm}M\qquad)$$

$$=\ A-\overline{A}\ \ -n_{aa}\ \ A-\overline{A}\ \ -n_{aa}\beta_{ap}\ \ P-\overline{P}\ \ -n_{aa}\alpha_{ae}\ \ E-\overline{E}$$

$$+\lambda_1\ P-\overline{P}\ \ -n_{pp}\ \ P-\overline{P}\ \ +n_{pp}\beta_{pa}\ \ A-\overline{A}\ \ -n_{pp}\alpha_{pm}\ \ M-\overline{M}$$

$$+\lambda_2 \;\; E-\bar{E} \;\; -n_{ee} \;\; E-\bar{E} \;\; -n_{ee}\alpha_{em} \;\; M-\bar{M}$$

$$+\;\lambda_3 \;\; M-\bar{M} \;\; -n_{mm} \;\; M-\bar{M} \qquad (5)$$

$$\frac{dL}{dt}=-n_{aa} \;\; A-\bar{A}^{\;2} -n_{aa}\beta_{ap} \;\; A-\bar{A} \;\; P-\bar{P} \;\; -n_{aa}\alpha_{ae} \;\; A-\bar{A} \;\; E-\bar{E}$$

$$+\lambda_1 \;\; (-n_{pp}) \;\; P-\bar{P}^{\;2} +n_{pp}\beta_{pa} \;\; A-\bar{A} \;\; P-\bar{P} \;\; -n_{pp}\alpha_{pm} \;\; M-\bar{M} \;\; P-\bar{P}$$

$$+\lambda_2 \;\; -n_{ee} \;\; E-\bar{E}^{\;2} -n_{ee}\alpha_{em} \;\; E-\bar{E} \;\; M-\bar{M}$$

$$+\;\lambda_3 \;\; -n_{mm} \;\; M-\bar{M}^{\;2} \qquad (6)$$

selecting $\lambda_1=\dfrac{n_{aa}\beta_{ap}}{n_{pp}\beta_{pa}}, \lambda_2$ *and* λ_3 are any positive constants, (6) becomes,

$$\frac{dL}{dt}=-n_{aa} \;\; A-\bar{A}^{\;2} -n_{aa}\alpha_{ae} \;\; A-\bar{A} \;\; E-\bar{E} \;\; -n_{aa}\beta_{ap} \;\; P-\bar{P}^{\;2} -\frac{n_{aa}\beta_{ap}\alpha_{pm}}{\beta_{pa}} \;\; P-\bar{P} \;\; M-\bar{M}$$

$$-\lambda_2 n_{ee} \;\; E-\bar{E}^{\;2} -\lambda_2 n_{ee}\alpha_{em} \;\; E-\bar{E} \;\; M-\bar{M} \;\; -\lambda_3 n_{mm} \;\; M-\bar{M}^{\;2}$$

Let $n_o=n_{aa}\alpha_{ae}\,, n_1=\dfrac{n_{aa}\beta_{ap}}{\beta_{pa}}, n_2=\dfrac{n_{aa}\beta_{ap}\alpha_{pm}}{\beta_{pa}}, n_3=\lambda_2 n_{ee}, n_4=\lambda_2 n_{ee}\alpha_{em}$ & $n_5=\lambda_3 n_{mm}$

$$\frac{dL}{dt}=-n_{aa} \;\; A-\bar{A}^{\;2} -n_0 \;\; A-\bar{A} \;\; E-\bar{E} \;\; -n_1 \;\; P-\bar{P}^{\;2} -n_2 \;\; P-\bar{P} \;\; M-\bar{M}$$

$$-n_3 \;\; E-\bar{E}^{\;2} -n_4 \;\; E-\bar{E} \;\; M-\bar{M} \;\; -n_5 \;\; M-\bar{M}^{\;2}$$

$$\frac{dL}{dt}= \;\; -n_{aa}K_a \;\; A-\bar{A}^{\;2} -n_1 \;\; P-\bar{P}^{\;2} -n_3 \;\; E-\bar{E}^{\;2} -n_5 \;\; M-\bar{M}^{\;2}$$

$$-E(n_0(\bar{A}-A)-n_4 M)-\bar{E}(n_0(A-\bar{A})-n_4\bar{M})$$

$$-M(n_2\bar{P}-n_4(E-\bar{E})-\bar{M}(n_2(P-\bar{P})+n_4 E_3)$$

$$\frac{dL}{dt}=-n_{aa} \;\; A-\bar{A}^{\;2} -n_1 \;\; P-\bar{P}^{\;2} -n_3 \;\; E-\bar{E}^{\;2} -n_5 \;\; M-\bar{M}^{\;2}$$

$$-E(n_0\overline{A}-n_oA-n_4M)-\overline{E}(n_0A-n_o\overline{A}-n_4\overline{M})\ -M(n_2\overline{P}-n_4E+n_4\overline{E})-\overline{N_4}(n_2P-n_2\overline{A}+n_4E)$$

$$\therefore \frac{dL}{dt}<0 \quad provided\ \ n_0\overline{A}-n_oA-n_4M\geq 0,\ n_0A-n_o\overline{A}-n_4\overline{M}\geq 0$$

$$n_2\overline{P}-n_4E+n_4\overline{E}\geq 0, n_2P-n_2\overline{A}+n_4E\geq 0$$

Hence the normal steady state is globally and asymptotically stable.

6. CONCLUSION: Global stability of this model is established at normal steady state in the mathematical model of a three level ecological Ammensalism with four species.

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