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The Mellin Type Integral Transform (MTIT) in the range (a,∞)

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Abstract

In this paper Laplace operators are used to solve The Mellin Type integral transform in the interval a to ∞ ,A work which is put forwarded to understand how Laplace operators would leads to properties , theorems ,derivatives and applications to the Mellin Type Integral Transform in this range.The main view of my work is to give procedure from Laplace Transform that turns out to be valid for for the Mellin Type Integral Transform in the range a to ∞. ,where a is positive ,.this Integral Transform ,can be a technique for solving boundary and initial value problems. . The results have been modified by applying suitable functions which leads to the results in The Mellin Type Integral Transform for the interval a to ∞ To illustrate the advantages and use of this transformation, some important differential equations have been solved at the end .the Matlab programmes are given for the basic and other formulae.The graphical concept is represented by assigning different values to the parameter by using tools of Matlab which gives a brighter view of applications to the Mellin Type Integral Transforms.

Keywords:

Laplace transform, Mellin transform, Finite transform,

AMS Mathematical Subject Classification

44A10,44A35, 44A05, 92D25, 46F12, 44A15, 35G15 , 44A85

1 .Introduction

In the theory of Integral Transform , Mellin Integral Transform has presented a direct and systematic technique, for resolution of certain types of classical boundary and initial value problems .To be successful the transform must be adopted to the form of differential operators to be eliminated as well as to the range of interest and associated boundary conditions. This transform is the extension of the Mellin integral transform and have a similar inversion formulae. This transform is suited to regions bounded by the natural coordinate surfaces of a cylindrical or spherical coordinate system and apply to finite or infinite regions bounded internally.

Historically ,Riemann (1876) first recognized the Mellin transform in the famous memoir on prime numbers, Its explicit formulation was given by Cahen (1894) . Almost simultaneously Mellin (1896, 1902) gave an elaborate discussion of the Mellin transform and its inversion formula

In this paper Laplace Transform considered as the basic definition, ,by using respective substitution ,the Mellin Type Integral Transform is derived. All properties , theorems, derivatives and applications of Laplace Transform are satisfied by the "The Mellin Type integral transform". in the range a to ∞.

2.Preliminary Results:

Let f(x) be a given function of x which is defined for all x $\geq$ 0 and's' is a parameter $L[f(x)]=$

$$\int_0^\infty e^{-sx} f(x)dx \qquad (1)$$

Substitute $x =$ log (.t/a) , $dx= \dfrac{dt}{t}$

If x=0 then t=a and if $x=\infty$ then $t=\infty$,then from (1)

$L\,[f(x)] = \int_a^\infty a^s t^{-s-1}\ f(t)dt$, denoted by $M_3\,[f(t), s, a, \infty]$

$$M\,[f(t), -s, a, \infty] = \int_a^\infty a^s t^{-s-1}\ f(t)dt \qquad (2)$$

thus obtained a the Mellin Type integral transform in the range a to infinity, with the new kernel $a^s t^{-s-1}$ and a new integral transform .

3. Properties for MTIT in a to ∞

3.1.Linearity property:

This Mellin Type Integral Transformation is a Linear operation, that is for any function *f(t)* and *g(t)* whose the Mellin Type integral transform exists in the range a to infinity and α and β are constants, then

$$M[\alpha f(t)+\beta g(t),-s,a,\infty]=\alpha M[f(t),-s,a,\infty]+\beta M[g(t)-,s,a,\infty]$$

Proof :

$$M[\alpha f(t)+\beta g(t),-s,a,\infty]=\int_a^\infty a^{-s}\{\alpha f(t)+\beta g(t)\}t^{-s-1}dt$$

$$=\alpha\int_a^\infty a^{-s}f(t)t^{-s-1}dt+\beta\int_a^\infty a^{-s}g(t)\}t^{-s-1}dt$$

$$=\alpha M[f(t)-,s,a,\infty]+\beta M[g(t),-s,,a,\infty]$$

$$M[\alpha f(t)+\beta g(t)-,s,a,\infty]=\alpha M[f(t),-s,a,\infty]+\beta M[g(t),-s,,a,\infty] \qquad (3)$$

Scaling Property:

Consider The Mellin Type integral transform in a to ∞,

$$\text{M}[f(t),-s,a,\infty]=\int_a^\infty a^s t^{-s-1}f(t)dt \text{ then}$$

$$\text{M}[f(bt),-s,a,\infty]=\int_a^\infty a^s t^{-s-1}f(bt)dt$$

Substituting *bt=p then t=p/b .dt=dp/b ,if t=a then p=ab and if t=*∞ *then p=*∞ , we have

$$\text{M}_3[f(bt),-s,a,\infty]=\int_a^\infty a^s t^{-s-1}f(bt)dt$$

$$=\int_{ab}^\infty a^s\left(\frac{p}{b}\right)^{-s-1}f(p)\frac{dp}{b}$$

$$=b^s\int_{ab}^\infty a^s p^{-s-1}f(p)dp$$

$$=b^{s} M[f(p), -s, ab, \infty]$$

$$M[f(bt), -s, a, \infty] = b^{s} M[f(p), -s, ab, \infty] \quad (4)$$

3.3. Other Property

Consider the equation (2)

$$M f(t^{b}), -s, a, \infty] = \int_{a}^{\infty} a^{s} t^{-s-1} f(t^{b}) dt$$

Substituting $t^{b} = q$; $dt = \frac{1}{b} q^{\frac{1}{b}-1} dq$ and

if $t=a$ then $q=a^{b}$; and $t=\infty$ then $q=\infty$

$$M[f(t^{b}), -s/a, a, \infty] = \frac{1}{b} \int_{a^{b}}^{\infty^{a}} a^{s} q^{-\frac{s}{b}-1} f(q) dq$$

$$M[f(t^{b}), -s, a, \infty] = \frac{1}{b} M[f(q), -s/b, a^{b}, \infty^{b}] \quad (5)$$

4. Main Results

3. 4.1. Inversion Theorem: for MTIT in a to ∞

The MTIT is in a to ∞

$$M[f(t), -s, a, \infty] = \int_{a}^{\infty} a^{s} t^{-s-1} f(t) dt$$

then its inversion formula is

$$f(t) = \frac{1}{2\pi i} \int_{c-iN}^{c+iN} \frac{t^{-s}}{s} M[f(t), -s, a, \infty] ds$$

Proof:

Assume that $M[f(t), s, a, \infty]$ is a regular equation in the strip $|\mathrm{Re}(s)| < r$ ('r' to be real number) of the s-plane and that $0<c<\nu$, $c-i\infty \le s \le c+i\infty$, where c is constant then

$$M[f(t), -s, a, \infty] = \int_{a}^{\infty} a^{s} t^{-s-1} f(t) dt$$

$$= \int_{a}^{\infty} a^{s} t^{-s-1} \, dt \; [\frac{1}{2\pi i} \int_{c-iN}^{c+iN} t^{-s} M[f(t), -s, a, \infty] ds]$$

$$= \frac{1}{2\pi i} \int_{c-iN}^{c+iN} t^{-s} M[f(t), s, a, \infty] ds \int_{1/a}^{\infty} t^{-s-1} a^{s} dt$$

$$= \frac{1}{2\pi i} \int_{c-iN}^{c+iN} t^{-s} M[f(t), -s, a, \infty] \left| \frac{t^{-s}}{-s} a^{s} \right|_{a}^{\infty} dx$$

$$M[f(t), -s, a, \infty] = \frac{1}{2\pi i} \int_{c-iN}^{c+iN} \frac{t^{-s}}{s} \; M \; [f(t), -s, a, \infty] ds \qquad (6)$$

Let $N \to \infty$ and assume that $\left| M[f(t), s, a, \infty] \right|$ remains unbounded as $\left| \text{limit } x \right| \to \infty$, when $\left| Re(s) \right| \leq$ c then the integral on R.H.S of the equation (6) tends *f(x)*.Hence

$$f(t) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \frac{t^{-s}}{s} \; M \; [f(t), -s, a, \infty] ds \qquad (7)$$

4. 4.2. Convolution Theorem: for MTIT in a to ∞

The MTIT is in a to ∞

$$M[f(t), -s, a, \infty] = \int_{a}^{\infty} a^{s} t^{-s-1} \; f(t) dt \quad \text{,thn}$$

$$M[f(t)\, g(x-t), s, a, \infty] = \frac{1}{2\pi i} \int_{c-iN}^{c+iN} \frac{t^{-s}}{s} \; M \; [f(t), -s, a, \infty] \; M[g(x-u), -s, a-t, \infty] \, ds$$

Proof

Consider the equations

$$M[f(t), -s, a, \infty] = \int_{a}^{\infty} a^{s} t^{-s-1} \; f(t) dt$$

$$M[g(t), -s, a, \infty] = \int_{a}^{\infty} a^{s} t^{-s-1} \; g(t) dt$$

then its inverses are

$$M^{-1}[f(t), s, a, \infty] = f(x) = \frac{1}{2\pi i} \int_{c-iN}^{c+iN} \frac{t^{-s}}{s} \; M \; [f(t), -s, a, \infty] ds$$

$M^{-1}[g(t),s,a,\infty] = g(x) = \frac{1}{2\pi i}\int_{c-iN}^{c+iN}\frac{t^{-s}}{s}\, M\,[g(t),-s,a,\infty]ds$,then

$M[f(t)\, g(x\text{-}t),\, s,\, a,\, \infty] = \int_a^{\infty} a^s t^{-s-1}\, f(t)g(x-t)dt$

$$= \frac{1}{2\pi i}\int_{c-iN}^{c+iN}\frac{t^{-s}}{s}\, M\,[f(t),-s,a,\infty]ds \int_a^{\infty} a^s t^{-s-1}\, g(x-t)dt$$

$$= \frac{1}{2\pi i}\int_{c-iN}^{c+iN}\frac{t^{-s}}{s}\, M\,[f(t),-s,a,\infty]\; M[g(x\text{-}t),\text{-}\, s,a,\, \infty]\, ds$$

$$M[f(t)\, g(x\text{-}t),\, s,\, 0,\, a] = \frac{1}{2\pi i}\int_{c-iN}^{c+iN}\frac{t^{-s}}{s}\, M\,[f(t),-s,a,\infty]\; M[g(x\text{-}t),\text{-}\, s,a,\, \infty]\, ds \qquad (8)$$

3. 4.3. Parsevals Theorem: for MTIT in a to ∞a,

If $M_3[f(t),\text{-}\, s,\, a,\, \infty] = \int_a^{\infty} a^s t^{-s-1}\, f(t)dt$,thn

$$M_3[f(t)\, g(\text{-}t),\, s,\, a,\, \infty] = \frac{1}{2\pi i}\int_{c-iN}^{c+iN}\frac{t^{-s}}{s}\, M\,[f(t),-s,a,\infty]\; M_3[\mathrm{g}(\mathrm{t}),\text{-}\, s,a,\, \infty]\, ds$$

Proof

Consider the equations

$M[f(t),\text{-}\, s,\, a,\, \infty] = \int_a^{\infty} a^s t^{-s-1}\, f(t)dt$

$M[g(t),\text{-}\, s,\, a,\, \infty] = \int_a^{\infty} a^s t^{-s-1}\, g(t)dt$

then its inverses are

$M^{-1}[f(t)\text{-},s,a,\infty] = f(x) = \frac{1}{2\pi i}\int_{c-iN}^{c+iN}\frac{t^{-s}}{s}\, M\,[f(t),-s,a,\infty]ds$

$M^{-1}[g(t)\text{-},s,a,\infty] = g(x) = \frac{1}{2\pi i}\int_{c-iN}^{c+iN}\frac{t^{-s}}{s}\, M\,[g(t),-s,a,\infty]ds$,then

$$M[f(t)\ g(x-t),\ s,\ a,\ \infty] = \int_a^\infty a^s t^{-s-1}\ f(t)g(x-t)dt$$

$$= \frac{1}{2\pi i}\int_{c-iN}^{c+iN} \frac{t^{-s}}{s}\ M\ [f(t),-s,a,\infty]ds \int_a^\infty a^s t^{-s-1}\ g(t)dt$$

$$= \frac{1}{2\pi i}\int_{c-iN}^{c+iN} \frac{t^{-s}}{s}\ M\ [f(t),-s,a,\infty]ds\ M[g(t),-\ s,a,\ \infty]\ ds$$

$$M_3[f(t)\ g(x-t),\ s,\ 0,\ a] = \frac{1}{2\pi i}\int_{c-iN}^{c+iN} \frac{t^{-s}}{s}\ M\ [f(t),-s,a,\infty]\ M[\mathrm{g(t)},-s,\ a,\ \infty]\ ds \qquad (8)$$

4.4.Definitions

(a)Uhit Step Function

If U(t)=H(t)=1,when t>0

=0,when t<0, then U(t) or H(t) is known as the Unit Step Function

(b) Heviside Unit Step Function

If U(t-a)=H(t-a)=1, when t>a

=0,when t<a,then U(t-a(or H(t-a) is known as the Heviside Unit Step Function.

3. 4.5.First Shifting Theorem for MTIT in a to ∞

If $M[f(t),-\ s,\ a,\ \infty] = \int_a^\infty a^s t^{-s-1}\ f(t)dt$ then

$$\mathrm{M}[t^n f(t),-\ s,\ a,\ \infty] = \mathrm{M}[f(t),-\ s+,a,\infty]$$

Proof

If $M[f(t),-\ s,\ a,\ \infty] = \int_a^\infty a^s t^{-s-1}\ f(t)dt$ then

$$M[\mathrm{t}^n\ (t),-\ s,\ a,\ \infty] = \int_a^\infty a^s t^{-s-1}\ t^n f(t)dt$$

$$= \int_a^\infty a^s t^{-s+n-1} f(t)dt$$

$$= M[f(t), -s+n, a, \infty]$$

$$M[t^n f(t), -s, a, \infty] = M[f(t), -s,, a, \infty] \quad (9)$$

3. 4.6. Second Shifting Theorem for MTIT in a to ∞

If $M[f(t), -s, a, \infty] = \int_a^\infty a^s t^{-s-1} f(t)dt$

then $M[f(t-a), s, a, \infty] =$

Proof

If $M[f(t), -s, a, \infty] = \int_a^\infty a^s t^{-s-1} f(t)dt$, then

$$M[f(t-b)U(t-b), -s, a, \infty] = \int_a^\infty a^s t^{-s-1} f(t-b)U(t-b)dt$$

$$= \int_a^\infty a^s t^{-s-1} f(t-b)dt$$

$$M[f(t-b)U(t-b), -s, a, \infty] = M[f(t-b), -s, a, \infty] \quad (10)$$

5. MTIT for Derivatives

3. 5.1. MTIT of first order Derivatives of f(t) w.r.t. t in a to ∞

Theorem: Suppose that $f(t)$ is continuous for all $t \geq 0$ satisfying (2) for some value γ and m and has a derivative $f'(t)$ which is piecewise continuous on every finite interval in the range of $t \geq 0$. Then The Mellin Type integral transforms of the derivative $f(t)$ exists when $s > \gamma$ and $|f(t)| \leq m\, e^{\gamma t}$ for all $t \geq 0$ for some constants

Proof: Considering the case when $f'(t)$ is continuous for all $t \geq 0$. Then on integrating by parts, this follows

$$M[f(t), -s, a, \infty] = \int_a^\infty a^s t^{-s-1} f(t)dt$$

then

$$M[f'(t),-s,a,\infty] = \int_a^\infty a^s t^{-s-1} f'(t)dt$$

$$=[a^s t^{-s-1} f(t)]_a^\infty - \int_a^\infty a^s(-s-1)t^{-s-2} f(t)dt$$

$$=(s+1)\int_a^\infty a^s t^{-s-2} f(t)dt - a^s a^{-s-1} f(a)$$

$$= (s+1)\, M[f(t),-s-1,a,\infty] - \frac{1}{a} f(a)$$

$$M[f'(t),-s,a,\infty] = (s+1)\, M[f(t),-s-1,a,\infty] - \frac{1}{a} f(a) \qquad (11)$$

since $f(t)$ satisfies $|f(t)| \leq m\, e^{\gamma t}$ and thus The Mellin Type integral transforms for derivatives is obtained .

3. 5.2. MTIT of nth order Derivatives f(t) w.r.t. t in a to ∞

$$M[f(t),-s,a,\infty] = \int_a^\infty a^s t^{-s-1} f(t)dt, \text{ then}$$

$$M[f''(t),-s,a,\infty] = \int_a^\infty a^s t^{-s-1} f''(t)dt$$

$$=[a^s t^{-s-1} f'(t)]_a^\infty - \int_a^\infty a^s(-s-1)t^{-s-2} f'(t)dt$$

$$=(s+1)\int_a^\infty a^s t^{-s-2} f'(t)dt - a^s a^{-s-1} f'(a)$$

$$=(s+1)[[a^s t^{-s-2} f(t)_a^\infty - \int_a^\infty a^s(-s-2)t^{-s-3} f(t)dt] - \frac{1}{a} f'(a)$$

$$=(s+1)(s+2)\int_a^\infty a^s t^{-s-3} f(t)dt - (s+1)a^s a^{-s-2} f(a) - \frac{1}{a} f'(a)$$

$$=(s+1)(s+2)\, M[f(t),-s-2,a,\infty] - (s+1)\frac{1}{a^2} f(a) - \frac{1}{a} f'(a)$$

$$M[f''(t),-s,/a,\infty] = (s+1)(s+2)M[f(t),-s-2,a,\infty]$$

$$-(s+1)\frac{1}{a^2}f(a) - \frac{1}{a}f'(a) \quad (12)$$

similarly

$$M[f'''(t),-s,/a,\infty] = (s+1)(s+2)(s+3)[f(t),-s-3,a,\infty]$$

$$-(s+1)(s+2)\frac{1}{a^3}f(a) - (s+1)\frac{1}{a^2}f'(a) - \frac{1}{a}f''(a) \quad (13)$$

then

$$M[f^{(n)}(t),-s,/a,\infty] = (s+1)(s+2)(s+3)—(s+n)[f(r),-s-n,a,\infty]$$

$$-(s+1)(s+2)—(s+n-1)\frac{1}{a^{n-1}}f'(a) - , \quad (14)$$

This is the genreralixed n th order derivative of f(t) by using the Mellin Type Integral Transform in a to ∞.

3. 6. Applications: for MTIT in a to ∞

The Cauchy Linear differential e quation is $\Delta_2 f(t) = t^2 f''(t) + tf'(t) + f(t)$

6.1. If $M[f(t),-s,a,\infty] = \int_a^\infty a^s t^{-s-1} f(t)dt$ then

$$M[t f'(t),-s,a,\infty] = \int_a^\infty a^s t^{-s-1}\, t f'(t)dt$$

$$= [a^s s^{-s} f(t)]_a^\infty - \int_a^\infty a^s(-s)t^{-s-1} f(t)dt$$

$$= s\int_a^\infty a^s t^{-s-1} f(t)dt - f(a)$$

$$= s\, M[f(t),-s,a,\infty] - f(a)$$

$$M[t f'(t),-s,a,\infty] = s\, M[f(t),-s,a,\infty] - f(a) \quad (15)$$

and

$$M[t^2 f''(t),-s,a,\infty] = \int_a^\infty a^s t^{-s-1}\, t^2 f''(t)dt$$

$$= \int_a^\infty a^s t^{-s+1} \; f''(t)dt$$

$$=[a^s t^{-s+1} f'(t)]_a^\infty - \int_a^\infty a^s (-s+1)t^{-s} \; f'(t)dt$$

$$=-a^s a^{-s+1} f'(a) -(-s+1) \int_a^\infty a^s t^{-s} \; f'(t)dt$$

$$= -af'(a)+(s-1) \int_a^\infty a^s t^{-s} \; f'(t)dt$$

$$=-af'(a)+(s-1) \; [[a^s t^{-s} f(t)]_a^\infty - \int_a^\infty a^s (-s)t^{-s-1} \; f(t)dt]$$

$$= -af'(a)+ (s-1) \; [-^s a^{-s} f(a) - \int_a^\infty a^s (-s)t^{-s-1} \; f(t)dt]$$

$$= -af'(a)-s(s-1) \int_a^\infty a^s t^{-s-1} \; f(t)dt -(s-1)f(a) -af'(a)$$

$$=s(s-1)f(t),- s, a, \infty]-(s-1)f(a) -af'(a)$$

$$M[t^2 \; f''t),-s,a, \infty]=s \, M[f(t),- s, a, \infty]-(s-1)f(a) -af'(a) \qquad (16)$$

If $\Delta_2 f(t) = \quad t^2 f''t)+tf'(t)+f(t)$

$$M \; [f(t),-s,a,\infty] \; = \; \int_a^\infty a^s t^{-s-1} \; f(t)dt \;\; \text{,then}$$

$$M \; [\Delta_2 f(t),-s,a,\infty] \; = \; \int_a^\infty a^s t^{-s-1} \; \Delta_2 f(t)dt$$

$$= \int_a^\infty a^s t^{-s-1} \; [t^2 f''(t) + tf'(t) + f(t)]dt$$

$$= s(s-1)M \; [f(t),-s,a, \infty]-(s-1)f(a) -af'(a)$$

$$+ s \, M[f(t),- s, a, \infty] \;\; -f(a)+ M_3[f(t),- s, a, \infty]$$

$$=(s^2 +1) \;\; M_3[f(t),- s, a, \infty]-sfa)-af'(a)$$

$M\,[\Delta_2 f(t),-s,a,\infty] = (s^2+1)\ M[f(t),-s,a,\infty]$-sfa)-af'(a) (17)

If $\Delta_2 f(x)=0$ then $x^2 f_{xx}(x)+xf_x(x)+f_{yy}(x)=0$

R.H.S of (17) gives the value zero theb the rHS of (17) is zero

$(s^2+1)\ M[f(t),-s,a,\infty]$-sfa)-af'(a) =0

$(s^2+1)\ M[f(t),-s,a,\infty]$=sf(a)+af'(a)[sf(a)+af'(a)]

$$M[f(t),-s,a,\infty]=\frac{1}{(s^2+1)}[sf(a)+af'(a)] \quad (18)$$

5. 6. Functions and Result for MTIT in a to ∞

Sr.No. Fnctions $M\,[f(t),-s,a,\infty] = \int_a^\infty a^s t^{-s-1}\ f(t)dt$

1) t^n $-(\frac{a^n}{s+n})$

2) e^{at} $-(\frac{1}{s}+\frac{a^2}{s+1}+\frac{a^4}{2!(s+2)}+\frac{a^6}{3!(s+3)}+---)$

3) e^{iat} $-(\frac{1}{s}-\frac{a^4}{2!(s+2)}+\frac{a^8}{4!(s+4)}---) + i(\frac{a^2}{s+1}-\frac{a^6}{3!(s+3)}+\frac{a^{10}}{5!(s+5)}--)$

4) sin(at) $-(\frac{a^2}{s+1}-\frac{a^6}{3!(s+3)}+\frac{a^{10}}{5!(s+5)}----)$

5) sinh(at) $-(\frac{a^2}{s+1}+\frac{a^6}{3!(s+3)}+\frac{a^{10}}{5!(s+5)}----)$

6) cos(at) $-(\frac{1}{s}-\frac{a^4}{2!(s+2)}+\frac{a^8}{4!(s+4)}----)$

7) cosh(at) $-(\frac{1}{s}+\frac{a^4}{2!(s+2)}+\frac{a^8}{4!(s+4)}+----)$

8)	tan(at)	$-\left(\frac{a^2}{s+1}+\frac{a^6}{3(s+3)}+\frac{2a^{10}}{15(s+3)}+---\right)$
9)	tanh(at)	$-\left(\frac{a^2}{s+1}-\frac{a^6}{3(s+3)}+\frac{2a^{10}}{15(s+3)}+---\right)$
10)	$t^n \sin(at)$	$-a^n\left[\frac{a^2}{s+n+1}-\frac{a^8}{3!(s+n+3)}+\frac{a^{10}}{5!(s+n+5)}---\right]$
11)	$t^n \sinh(at)$	$-a^n\left[\frac{a^2}{s+n+1}+\frac{a^8}{3!(s+n+3)}+\frac{a^{10}}{5!(s+n+5)}---\right]$
12)	$t^n \cos(at)$	$-a^n\left[\frac{1}{s+n}-\frac{a^4}{2!(s+n+2)}+\frac{a^8}{4!(s+n+4)}----\right]$
		12
13)	$t^n \cosh(at)$	$-a^n\left[\frac{1}{s+n}+\frac{a^4}{2!(s+n+2)}+\frac{a^8}{4!(s+n+4)}----\right]$
14)	$t^n \tan(at)$	$-a^n\left[\frac{a^2}{s+n+1}+\frac{a^6}{3(s+n+3)}+\frac{2a^{10}}{15(s+n+3)}+---\right]$
15)	$t^n \tanh(at)$	$-a^n\left[\frac{a^2}{s+n+1}-\frac{a^6}{3(s+n+3)}+\frac{2a^{10}}{15(s+n+3)}+---\right.$
16)	$\sin^{-1}(t)$	$-\left(\frac{a}{s+1}+\frac{a^3}{6(s+3)}+\frac{a^5}{40(s+5)}+--\right)$
17)	$\cos^{-1}(t)$	$-\left(\frac{\pi}{2}-\sin^{-1}(t)\right)$
18)	$\tan^{-1}(t)$	$-\left(\frac{a}{s+1}-\frac{a^3}{3(s+3)}+\frac{a^5}{5(s+5)}---\right)$

19)	$\sinh^{-1}(t)$	$-\left(\frac{a}{s+1}+\frac{a^3}{6(s+3)}+\frac{a^5}{40(s+5)}+--\right)$
20)	$\cosh^{-1}(t)$	$-\left(\frac{\pi}{2}-\sinh^{-1}(t)\right)$
21)	$\tanh^{-1}(t)$	$-\left(\frac{a}{s+1}+\frac{a^3}{3(s+3)}+\frac{a^5}{5(s+5)}+--\right)$
22)	$t^n \sin^{-1}(t)$	$-a^n\left[\frac{a}{s+n+1}-\frac{a^3}{6(s+n+3)}+\frac{a^5}{40(s+n+5)}---\right]$
23)	$t^n \cos^{-1}(t)$	$-a^n\left[\frac{a}{s+n+1}-\frac{a^3}{3(s+n+3)}+\frac{a^5}{5(s+n+5)}---\right]$
24)	$t^n \tan^{-1}(t)$	$-a^n\left[\frac{a}{s+n+1}-\frac{a^3}{3(s+n+3)}+\frac{a^5}{5(s+n+5)}---\right]$
25)	$t^n \sinh^{-1}(t)$	$-a^n\left[\frac{a}{s+n+1}+\frac{a^3}{6(s+n+3)}+\frac{a^5}{40(s+n+5)}---\right]$
26)	$t^n \cosh^{-1}(t)$	$-\left(\frac{\pi}{2}-\sinh^{-1}(t)\right)$
27)	$t^n \tanh^{-1}(t)$	$-a^n\left[\frac{a}{s+n+1}-\frac{a^3}{3(s+n+3)}+\frac{a^5}{5(s+n+5)}---\right]$
28)	tan(t)	$-\left(\frac{a}{s+1}+\frac{a^3}{3(s+30}+\frac{2a^3}{15(s+3)}+---\right)$
29)	log(1+t)	$-\left(\frac{a}{s+1}-\frac{a^2}{2(s+2)}+\frac{a^3}{3(s+3)}----\right)$

30) log(1-t) $-\left(-\frac{a}{s+1}-\frac{a^2}{2(s+2)}-\frac{a^3}{3(s+3)}----\right)$

31) $(1-t)^{-1}$ $-\left(\frac{1}{s}+\frac{a}{s+1}+\frac{a^2}{2(s+2)}+\frac{a^3}{3(s+3)}+--\right)$

32) $(1+t)^{-1}$ $-\left(\frac{1}{s}-\frac{a}{s+1}+\frac{a^2}{2(s+2)}-\frac{a^3}{3(s+3)}+--\right)$

33) $\sec(t)$ $-\left(\frac{1}{s}+\frac{a^2}{2(s+2)}+\frac{5a^4}{24(s+4)}+---\right)$

34) a^x $-\left(\frac{1}{s}+\log a\frac{a}{s+1}+(\log a)^2\frac{a^2}{2!(s+2)}+--\right)$

35) $\log\cos(t)$ $-\left(-\frac{a}{s+1}-\frac{a^4}{12(s+4)}-\frac{a^6}{45(s+6)}---\right)$

36) $\log\sec(t)$ $-\left(\frac{a^2}{2(s+2)}+\frac{a^4}{12(s+4)}+\frac{a^6}{45(s+6)}---\right)$

37) $e^{\sin t}$ $-\left(\frac{1}{s}+\frac{a}{s+1}+\frac{a^2}{2(s+2)}+\frac{a^3}{3(s+3)}+--\right)$

38) $\log(1+\cos t)$ $-\left(\frac{\log a}{s}-\frac{a^2}{4(s+2)}-\frac{a^4}{48(s+4)}----\right)$

39) $\log(1+\tan t)$ $-\left(\frac{a}{s+1}-\frac{a^2}{2(s+2)}+\frac{2a^3}{3(s+3)}------\right)$

40) $e^t\sin t$ $-\left(\frac{a}{s+1}+\frac{a^2}{(s+2)}+\frac{2a^3}{6(s+3)}---\right)$

41) $\log\sec(\frac{\pi}{4}+t)$ $-\left(\frac{1}{2s}\log 2+\frac{a}{s+1}+\frac{a^2}{s+2}+\frac{2a^3}{3(s+3)}--\right)$

7. Graphical representation by using tools of MATLAB:

The Mellin Type Integral Transform graph plotted between x,y for various values of 's' parameter.

Here the program has been shown with one value of the parameter.

Consider the equation

$$M[f(t), -s, 1, \infty] = \left(\frac{1}{s^2+1}\right)(sf(1) + f'(1))$$

(d) % f(x)=x^2= $\frac{1}{s^2+1}$ (s $f(a)+f'(a)$)

% f'(x)=2*x

% if a=0 then f1=f(0)=0,f2=f'(0)=0 then

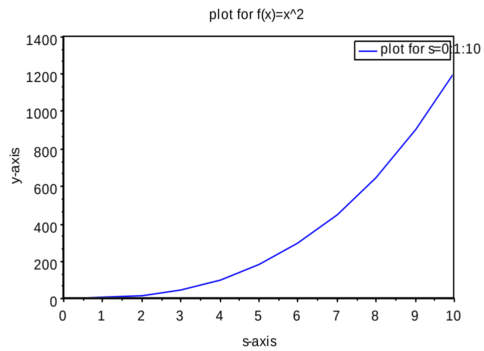
%y= $\frac{1}{s^2}$ (s $f1$ +$f2$) =0

% if a=1 then f1=f(1)=1 and f2=f'(1)=2

%y=1/(s^2+1)*(s*f1+f2))

Matlab Programme

```
f1=1,
f2=2,
a=1;
s=0:1:10;
y=(s*f1 +f2).*(s^2+1)
plot(s,y)
xlabel('s-axis'),
ylabel('y-axis'),
title('plot for f(x)=x^2'),
legend('plot for s=0:1:10'),
```



(*e*) % f(x)=e^x= $\frac{1}{s^2+1}$ ((s $f(a)+f'(a)$)

% f'(x)=e^x

% if a=0 ,s=0then f1=f(0)=1,f2=f'(0)=1 then

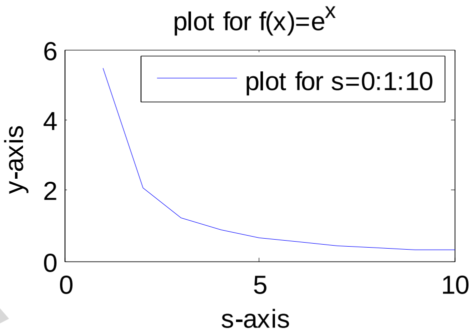
%y= $\frac{1}{s^2}$ (s $f1$ + $f2$) =0

% if a=1 then f1=f(1)=e, and f2=f'(1)=e

%y=1/(s^2+1)*(s*f1+f2))

Matlab Programme

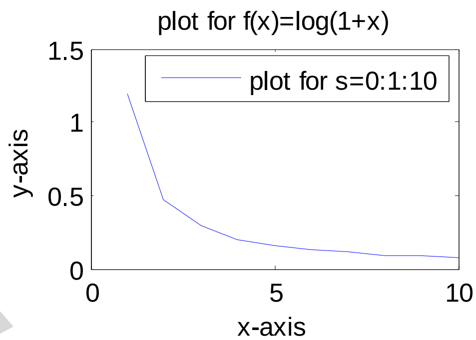
```
syms e
f1=2.71828,
f2=2.71828,
s=0:1:10;
y=(s*f1 +f2).*(s^2+1)
plot(s,y)
xlabel('s-axis'),
ylabel('y-axis'),
title('plot for f(x)=e^x '),
legend('plot for s=0:1:10'),
```




```
(f)% f(x)=log(1+x)
   %,f'(x)=1/(1+x)
   % if s=a=0 then f1=f(1)=0,f2=f'(1)=1,
   %then  y=1/(s^2+1)*(s*f1+f2)) =0
    %if a=1 then f1=f(1)=log(2) and f2=f'(1)=.5
    % then  y=1/(s^2+1)*(s*f1+f2))
```

Matlab Programme

```
f1=0.6931,
f2=0.5,
s=0:1:10;
 y=(s*f1 +f2).*(s^2+1)
plot(s,y)
xlabel('x-axis'),
ylabel('y-axis'),
title('plot for f(x)=log(1+x)'),
legend('plot for s=0:1:10'),
```



8. Conclusion

All the properties ,theorems,derivatives ,application ,examples of the MTIT are satiesfied. This Integral transform gives best results for technical problems.

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