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Performance Analysis of Delay and Optimality of Scheduling Policies for Multi-Hop wireless Networks

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Abstract— **In this paper, we analyze the delay performance of a multi-hop wireless network in which the routes between source-destination pairs are fixed. We develop a new queue grouping technique to handle the complex correlations of the service process resulting from the multi-hop nature of the flows and their mutual sharing of the wireless medium. A general set based interference model is assumed that imposes constraints on links that can be served simultaneously at any given time. These interference constraints are used to obtain a fundamental lower bound on the delay performance of any scheduling policy for the system. We present a systematic methodology to derive such lower bounds. For a special wireless system, namely the clique, we design a policy that is sample path delay optimal. For the tandem queue network, where the delay optimal policy is known, the expected delay of the optimal policy numerically coincides with the lower bound. The lower bound analysis provides useful insights into the design and analysis of optimal or nearly optimal scheduling policies. We conduct extensive numerical studies to demonstrate that one can design policies whose average delay performance is close to the lower bound computed by the techniques presented in this paper.**

Keywords— **Delay, Performance, Analysis, Optimality, Multi -Hop, wireless.**

I. INTRODUCTION

Multi-hop wireless networks have been devoted to System stability while maximizing metrics like Throughput or utility. These metrics measure the performance of a system over a long time-scale. For a large class of applications such as video or voice over IP, embedded network control and for system design; metrics like delay are of prime importance. The delay performance of wireless networks, however, has largely been an open problem. This problem is notoriously difficult even in the context of wire line networks, primarily because of the complex interactions in the network (e.g., superposition, routing, departure, etc.) that make its analysis amenable only in very special cases like the product form networks. The problem is further exacerbated by the mutual interference inherent in wireless networks

which, complicates both the scheduling mechanisms and their analysis. Some novel analytical techniques to compute useful lower bound and delay estimates for wireless networks with single hop traffic were developed in [12]. How ever. The analysis is not directly applicable to multi-hop wireless network with multi hop flows, due to the difficulty in characterizing the departure
process at intermediate links. The metric of interest in this paper is the system-wide average delay of a packet from the source to its corresponding destination. We present a new, systematic methodology to obtain a fundamental lower bound on the average packet delay in the system under any scheduling policy. Furthermore, we re-engineer well known scheduling policies to achieve good delay performance viz-a-viz the lower bound.

Here we analyze a multi-hop wireless network with multiple source-destination pairs, given routing and traffic information. Each source injects packets in the network, which traverses through the network until it reaches the destination.
We now summarize our main contributions in this paper:
• Development of a new queue grouping technique to handle the complex correlations of the service process resulting from the multi-hop nature of the flows. We also introduce a novel concept of (K, X)-bottlenecks in the network.
• Development of a new technique to reduce the analysis of queuing *upstream* of a bottleneck to studying simple single queue systems. We derive sample path bounds on a group of queues upstream of a bottleneck.
• Derivation of a fundamental lower bound on the system wide average queuing delay of a packet in multi-hop wireless network, regardless of the scheduling policy used, by analyzing the single queue systems obtained above.
• Extensive numerical studies and discussion on useful insights into the design of optimal or nearly optimal scheduling policies gained by the lower bound analysis.

II. SYSTEM MODEL

We consider a wireless network G = (V,L), where V is the set of nodes and L is the set of links. Each link has unit capacity. There are N flows, each distinguished by its source destination pair (si, di). There is a fixed route (set of links) between the source si and corresponding destination di. Each route is a simple path. Each flow has its own exogenous arrival stream {Ai(t)}1 t=1.Each packet has a deterministic service time equal to one unit. The exogenous arrivals at each source are assumed to be independent. Let A(t) = (A1(t), . . . ,AN(t)) represent the vector of exogenous arrivals, where Ai(t) is the number of packets injected into the system by the source si during time slot t (for i ∈ 1, . . . ,N). Let _ = (_1, . . . , _N) represent the corresponding long-term average arrival rate vector.
The path on which flow i is routed is specified as Pi :=(v0 i , v1i , . . . , vj i , . . . , v|Pi| i), where vj i is a node at a j-hop distance from the source node si. The source node si is denoted by v0 i and the destination node di by v|Pi| i , where |Pi| is the path length. The packets arriving at each node are queued. Each node maintains a separate queue for each flow that passes through the node. Let Qj i (t) denote the queue length at node vj i corresponding to flow i. After reaching the destination node, each packet leaves the system, i.e. Q|Pi| i = 0. The queue length vector is denoted by Q(t) = (Qj i (t) : i ∈ {1, 2, . . . ,N} and j ∈ {1, 2, . . . , Pi}). Multiple flows can share a link e. A link can be activated in a time slot t only if the corresponding queue is

non empty. We use the term activation (scheduling) of a link or a queue interchangeably. At most, one packet is served at a queue in a given time slot. The service structure is slotted.

III. DERIVING LOWER BOUNDS ON AVERAGE DELAY

In this section, we present our methodology to derive lower bounds on the system-wide average packet delay for a given multi-hop wireless network. At a high level, we partition the flows into several groups. Each group passes through a (K, X)-bottleneck and the queuing for each group is analyzed individually. The grouping is done so as to maximize the lower bound on the system wide expected delay. For flows passing through a given bottleneck (a group), we lower bound the sum of queues upstream and downstream of the bottleneck separately. We *reduce* the analysis of queuing *upstream* of a (K, X)-bottleneck to studying single queue systems fed by appropriate arrival processes. These arrival processes are simple functions of the exogenous arrival processes of the original network. For example, Figure 1 shows the reduction of two (1, X) bottlenecks in the network. A separate lower bound can be established for the queues downstream of the network.

The lower bound on the system-wide average delay of a packet is then computed using the statistics of the exogenous arrival processes. We derive analytical expressions of the lower bounds for a large class of arrival processes. Bottle necks, and returns a lower bound on the system-wide average packet delay.

IV. DESIGN OF DELAY EFFICIENT POLICIES

We now address the important question of designing a delay-efficient scheduler for general multi-hop wireless networks. We will see that although delay optimal policies can be derived for some simple networks like the clique and the tandem, deriving such policies in general is extremely complex. Intuitively, such a scheduler must satisfy the following properties.

• *Ensure high throughput:* This is important because if the scheduling policy does not guarantee high throughput then the delay may become infinite under heavy loading.

• *Allocate resources equitably:* The network resources must be shared among the flows so as not to starve some of the flows. Also, non-interfering links in the network have to be scheduled such that certain links are not starved for service. Starvation leads to an increase in the average delay in the system. The above properties are difficult to achieve; given the dynamics of the network and the lack of apriority information of the packet arrival process. In the light of the previous work [20], [30], we choose to investigate the *back-pressure policy*
with fixed routing . The back-pressure policy has been widely used to develop solutions for a variety of problems in the context of wireless networks [8], [20]; and the importance of studying the trade-offs in stability, delay, and complexity of these solutions is now being realized by the research community. This policy tries to maintain the queues corresponding to each flow in decreasing order of size from the source to the destination. This is achieved by using the value of differential backlog (difference of backlogs at the two ends of a link) as the weight for the link and scheduling

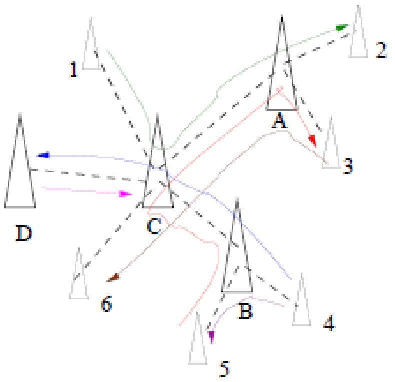

Fig1.An example network (clique) with interference constraints such that only one pair of nodes can communicate at any given time. Note that the packets may still need to traverse multiple hops.

The matching with the highest weight. As a result, the policy is throughput optimal. Henceforth, we shall refer to this policy as only the back-pressure policy.

We first study the delay optimal policy for a clique network. We then modify the back-pressure policy using the intuition gained from the nature of the delay optimal scheduling for the clique and tandem networks. A clique network is one in which the interference constraints allow only one link to be scheduled at any given time. Such a situation may arise, for example, in the down-link of a base station which employs relays to increase coverage and/or data rates (see for eg. [29]). Suppose there are N flows in the clique network. An example network with six flows is shown in Fig. 1. Every link lies in the interference range of the other and hence only one link can be scheduled at any given time.

V. DISCUSSION AND RELATED WORK

The general research on the delay analysis of scheduling policies has progressed in the following main directions:

• *Heavy traffic regime using fluid models:* Fluid models have typically been used to either establish the stability of the system or to study the workload process in the heavy traffic regime. It has been shown in [5] that the maximum-pressure policy (similar to the back-pressure policy) minimizes the workload process for a stochastic processing network in the heavy traffic regime when processor splitting is allowed.

• *Stochastic Bounds using Lyapunov drifts:* This method is developed in [8], [19], [23], [24] and is used to derive upper bounds on the average queue length for these systems. However, these results are order results and provide only a limited characterization of the delay of the system. For example, it has been shown in [24] that the maximal matching policies achieve O(1) delay for networks with single-hop traffic when the input load is in the reduced capacity region. This analysis however, has not been extended to the multi-hop traffic case, because of the lack of an analogous Lyapunov function for the back-pressure policy.

• *Large Deviations:* Large deviation results for cellular and multi-hop systems with single hop traffic have been obtained in [32], [35] to estimate the decay rate of the queue-overflow probability. Similar analysis is much more difficult for the multi-hop wireless network considered here, due to the complex interactions between the arrival, service, and backlog process. Traditional heavy traffic results have focused on a single bottleneck in the system [18], [22], [25] and proving a state space collapse. We have shown in [9] (Section

4.3.1) that it in general, it is impossible to avoid *idling* in these systems. Hence, it may not even be possible to prove a state-space collapse except for some special scenarios. We on the other hand, analyze queuing in multiple bottlenecks by relaxing the constraints in the system, which is crucial to obtain tight lower bounds. Although it may not be possible to achieve the lower bound, our relaxation approach is novel and leads to non-trivial lower bounds.

VI. CONCLUSION

The delay analysis of wireless networks is largely an open problem. In fact, even in the wire line setting, obtaining analytical results on the delay beyond the product form types of networks has posed great challenges. These are further exacerbated in the wireless setting due to complexity of scheduling needed to mitigate interference. Thus, new approaches are required to address the delay problem in multi-hop wireless systems. To this end, we develop a new approach to reduce the bottlenecks in a multi-hop wireless to single queue systems to carry out lower bound analysis. For a special class of wireless systems (cliques), we are able to apply known techniques to obtain a sample path delay-optimal scheduling policy. We also obtain policies that minimize a function of queue lengths at all times on a sample path basis. Further, for a tandem queuing system, we show numerically that the expected delay of a previously known delay-optimal policy coincides with the lower bound. The analysis is very general and admits a large class of arrival processes. Also, the analysis can be readily extended to handle channel variations. The main difficulty, however is
in identifying the bottlenecks in the system. The lower bound not only helps us identify near-optimal policies, but may also help in the design of a delay-efficient policy as indicated by the numerical studies.

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