ISSN 2249 5460

INTERNATIONAL JOURNAL OF MATHEMATICAL SCIENCES, TECHNOLOGY AND HUMANITIES

www.internationalejournals.com

Available online at www.internationaleJournals.com

International eJournals

International Journal of Mathematical Sciences, Technology and Humanities 57 (2012) 616 – 634

Run-Upflow of a Rivlin-Ericksen Conducting Fluid Through a Planar Channel Bounded By a Porous Bed Under a Transverse Magnetic Field Darcy Lapwood Model

D.Malleswari[1], D.Raju[2] and A.LeelaRatnam[3]

[1] Department of Mathematics, Government Degree College for Women, Begumpet, Hyderabad. A.P. E-mail: malleswaridammalapati@yahoo.com

[2] Vidya Jyothi Institute of Technology, AzizNagar Gate, Hyderabad-500 075, India. E-mail: 20122102india@gmail.com

[3]Department of Applied Mathematics, Sri Padmavathi MahilaVisvavidyalayam, Tirupathi, India. E-mail: leeleratnamappikatla@yahoo.com.

ABSTRACT

In this paper, the influence of a densely packed porous lining on the flow of a Rivlin-Ericksen conducting viscoelastic fluid through a parallel plate channel subjected to a traverse magnetic field is considered. The behavior of the velocity of the fluid in clean fluid region as well as the slip velocity discussed at the interface for small and large thickness of the porous bed. The velocity, Shear stress and mass flux are discussed for various values of the parameters and are represented graphically. The results are obtained here are more general.

Keywords: Run – up flow, Visco – elasticity, Start – up flow, Newtonian fluid, Viscosity.

Mathematical Subject classification: 58 D 30.

1. INTRODUCTION:

Non-Newtonian fluid flows are encountered in a wide range of engineering applications, chemical technology and petroleum industry as well as geophysical fluid dynamics [1,2,3]. The study of these non-Newtonian fluids has great importance in lubrication, extrusion of plastics, flow in journal bearings; flow is a shock absorber etc. [4,5]. The increase in applications urged Scientists and Engineers to provide mathematical models for non-Newtonian fluids. The non-linearity between stress and deformation rate for phase fluids makes it, in general, impossible to obtain a simple mathematical model as in the case of Newtonian fluids. Viscoelastic fluids which possess certain degree of elasticity in addition to viscosity are categorized as second order fluids.

Rivlin and Ericksen [6] have proposed a mathematical model for viscoelastic fluids which predicts normal stress affect although maintain Newtonian viscosity. The constitutive equation governing the Rivlin and Ericksen [6] fluid is given by

$$T = -PI + \phi_1 A + \phi_2 B + \phi_3 A^2,$$

Where I = || δ_{ij} ||, δ_{ij} is the kronecker delta.

A = || a_{ij} ||, $a_{ij} = ½[u_{i,j} + u_{j,i}]$ in the deformation tension

B = || b_{ij} ||, $b_{ij} = b_{ij}^m = a_{i,j} + a_{j,i} + 2V_{m,i}V_{m,j}$ in the viscoelastic tensor

$A_{i,j}$, $a_{j,i}$ are the acceleration gradients

$V_{m,i}$, $V_{m,j}$ are the velocity gradients.

ϕ_1, ϕ_2 and ϕ_3 are material constants called the coefficient of viscosity, visco-elasticity and cross-viscosity respectively, which are considered to be constant in this paper.

Initially the flow is due to a prescribed pressure gradient with boundaries at rest and at time t > 0, the pressure gradient is withdrawn the upper plate suddenly moves with a uniform velocity while the lower plate continues to be at rest. Researchers in this field are initiated for the first time by Kazakia and Rivlin[7], in which they investigated run-up flow in an incompressible isotropic viscoelastic fluid contained between two infinite rigid parallel plates. Rivlin [8] also discussed run-up and spin-up flow in a viscoelastic fluid between two infinite parallel plates containing Maxwell fluid initially at rest. They have studied the fluid motion resulting from sudden velocities given to the plates and subsequently held constant.

The fluid flow is through a composite system consisting of two zones. The unsteady governing equations are solved as initial value problem. Zone -1 consisting of Rivlin-Ericksen fluid in the non-porous region bounded above by an impermeable boundary plane. The flow in zone - 2 consists of flow through porous region bounded below by the rigid plane. The flow in the non-porous region is governed by Navier-stokes equation. The porous region although densly packed allows slip through the interface. Hence, we choose Darcy-Lapwood model to govern the flow through porous bed. At the interface the slip velocity satisfies the Beavers- Joseph Condition. Also at the interface the continuity of the velocity is imposed so that the velocity the fluid in the clean fluid region at the interface equal to the slip velocity. The velocity in both the clean fluid a porous Zone the shear stress and the mass flux have been evaluated and their behaviour is discussed computationally for variations in the governing parameters.

2. FORMULATION AND SOLUTION OF THE PROBLEM:

The equation governing initial flow in clean fluid region zone – 1 in non-dimensional form is

$$\frac{d^2u}{dy^2} - M^2Ru = PR \qquad (1)$$

The corresponding non-dimensional boundary condition is

u = 0 at y = 1 (2)

The initial axial velocity in the porous bed (Zone – 2) is given by

$$u_p = \frac{PD^2R\lambda^{-1}}{1 + M^2D^2R\lambda^{-1}} \qquad (3)$$

Where $M^2 = \dfrac{\sigma\mu_e^2H_0^2h}{\rho U}$ (the Hartmann number)

$R = \dfrac{\rho U h}{\mu}$ (the Reynolds number)

$D^{-2} = \dfrac{h^2}{K}$ (the inverse Darcy Parameter)

$\lambda = \dfrac{\mu_{eff}}{\mu}$ (the ratio of the viscosities)

At $t \geq 0$ the momentum equations governing the flow in non-dimensional form in zone – 1 is

$$\frac{\partial u}{\partial t} = \frac{1}{R}\frac{\partial^2 u}{\partial y^2} + S\frac{\partial^3 y}{\partial t \partial y^2} - M^2u \qquad (4)$$

In zone – 2

$$\frac{\partial u_p}{\partial t} = \delta\frac{\partial p}{\partial x} - \delta D^{-1}R^{-1}\lambda u_P - M^2\delta u_P \qquad (5)$$

where $S = \dfrac{\alpha_1}{\rho h^2}$ (is the viscoelastic parameter)

The boundary and the interfacial conditions in non-dimensional form are

u = 1 at y = 1 (6)

$u = u_B;\ \dfrac{\partial u}{\partial y} = D^{-1}\alpha(u_B - u_P)$ at $y = s_1$ (7)

Solving (1) subjected to the conditions (2) the initial flow in the non-porous region is given by

$$u = \frac{C_1}{Sinh(M\sqrt{R})}\left[Sinh(M\sqrt{R}(1-y))\right] + \frac{PR}{M\sqrt{R}}\left[\frac{Sinh(M\sqrt{R}y)}{Sinh(M\sqrt{R})} - 1\right] \qquad (8)$$

where C_1 be an arbitrary constant to be determined.

We now solve (4) and (5) subjected to the conditions (6) and (7) using Laplace transforms method.

Let $\bar{u}, \bar{u}_p$ and $\bar{u}_B$ be the transformed velocities of u, u_p and u_B respectively.

The equations governing the transformed velocities, making use of the initial velocity expression reduces to

$$\frac{d^2\bar{u}}{dy^2} - \beta^2\bar{u} = \frac{1}{1+SRs}\left[A_3 Sinh(M\sqrt{R}(1-y)) + A_3 Cosh(M\sqrt{R}y) + A_8 Sinh(M\sqrt{R}) + A_7\right] \qquad (9)$$

where $\beta^2 = \frac{s+M^2}{1+SRs}$

A_1, A_2 etc. are constants given in the appendix

$$\bar{u}_P = \frac{P\left(sD^2R\lambda^{-1} + \delta A_{11}\right)}{s\left(s+\delta A_{12}\right)A_{11}} \qquad (10)$$

The boundary and the interfacial conditions in the transformed form are

$$\bar{u} = \frac{1}{s} \text{ at } y = 1 \qquad (11)$$

$$\bar{u} = \bar{u}_B \text{ at } y = s_1 \qquad (12)$$

$$\bar{u}_B = \bar{u}_p + D\alpha^{-1}\left(\frac{d\bar{u}}{dy}\right)_{y=s_1} \qquad (13)$$

Solving (9) subjected to the condition (11) we obtain

$$\bar{u} = \frac{(2+sRS)C_1'}{Sinh(\beta)}.$$

$$\left[\begin{array}{l} Sinh(\beta(1\text{-}y)) + \dfrac{A_8}{(1+SRs)(M^2R+\beta^2)Sinh(\beta)} \\ \left[Sinh(\beta y)Sinh(M\sqrt{R}) - Sinh(M\sqrt{R}y)Sinh(\beta)\right] \end{array}\right] +$$

$$+\frac{1}{(1+SRs)(M^2R+\beta^2)Sinh(\beta)}\cdot\left[\begin{array}{l} \mathrm{A}_3\left(Sinh((\beta y))\mathrm{Cosh}(\mathrm{M}\sqrt{\mathrm{R}}) - \mathrm{Sinh}(\mathrm{M}\sqrt{\mathrm{R}}(1-\mathrm{y}))\mathrm{Sinh}(\beta)\right) \\ -\mathrm{Cosh}(\mathrm{M}\sqrt{\mathrm{R}}\,\mathrm{y})\mathrm{Sinh}(\beta) \end{array}\right] +$$

$$+\frac{A_7}{(1+SRs)\beta^2}\frac{(Sinh(\beta y) - Sinh(\beta))}{Sinh(\beta)} + \frac{Sinh(\beta y)}{s(Sinh(\beta))} \qquad (14)$$

Where C_1' is an arbitrary constant to be determined.

$$\left(\frac{d\bar{u}}{dy}\right)_{y=s_1} = \frac{C_1'}{Sinh(\beta i}\left[\beta Cosh(\beta(1-s_1))\right] +$$

$$+\left(\frac{(2+SRs)\,A_8}{(1+SRs)(M^2R+\beta^2)Sinh(\beta)}\right)$$

$$\left[\beta Cosh(\beta s_1)Sinh(M\sqrt{R}) - M\sqrt{R}Cosh(M\sqrt{R}s_1)Sinh(\beta)\right] +$$

$$+\left[\frac{\left[\begin{array}{l} A_3\left(\beta Cosh(\beta s_1)Cosh(M\sqrt{R}) - M\sqrt{R}Cosh(M\sqrt{R}(1-s_1))Sinh(\beta)\right) \\ -M\sqrt{R}Sinh(M\sqrt{R}s_1)Sinh(\beta) \end{array}\right]}{(1+SRs)(M^2R+\beta^2)Sinh(\beta)}\right] \qquad (15)$$

$\bar{u}_B$ is obtained from (12) and (13) using (10) and (15)

$$\bar{u}_B = D\alpha^{-1}\left\{\frac{\beta Cosh(\beta(1-s_1))}{[Sinh(\beta(1-s_1)) - D\alpha^{-1}\beta Cosh(\beta(1-s_1))]Sinh(\beta)}\right.$$

$$\left[\frac{A_8(2+SRs)}{(1+SRs)(M^2R+\beta^2)}\right.$$

$$\left\{D\alpha^{-1}\left(\beta Cosh(\beta s_1)Sinh(M\sqrt{R}) - M\sqrt{R}Sinh(M\sqrt{R}s_1)Sinh(\beta)\right) - \right.$$

$$\left(Sinh(\beta s_1)Sinh(M\sqrt{R})\right)-Sinh(M\sqrt{R}s_1)Sinh(\beta)\Big\}+\left[\frac{A_3}{(1+SRs)(M^2R+\beta^2)}\right].$$

$$\left[(D\alpha^{-1})\begin{pmatrix}\beta Cosh(\beta s_1).Cosh(M\sqrt{R})-M\sqrt{R}Cos(M\sqrt{R}(1-s_1))Sinh(\beta)\\ -M\sqrt{R}.Sinh(M\sqrt{R})Sinh(\beta)\end{pmatrix}\right.$$

$$\left.-\begin{pmatrix}Sinh(\beta s_1)Cosh(M\sqrt{R})-Sinh(M\sqrt{R}(1-s_1))Sinh(\beta)\\ -Cosh(M\sqrt{R}s_1)Sinh(\beta)\end{pmatrix}\right]+$$

$$+\frac{A_7}{(1+SRs)\beta^2}\left[D\alpha^{-1}Cosh(\beta s_1)-\left(Sinh(\beta s_1)-Sinh(\beta)\right)\right]+$$
$$\frac{1}{s}\left[(D\alpha^{-1})\beta Cosh(\beta s_1)-Sinh(\beta s_1)\right]+$$

$$+\frac{P\left(sD^2R^{-1}\lambda+\delta A_{11}\right)}{sA_{11}(s+\delta A_{12})}\Bigg]+\left[\frac{A_8(2+SRs)}{(1+SRs)(M^2R+\beta^2)Sinh(\beta)}\right]$$

$$\begin{bmatrix}\beta Cosh(\beta s_1)Sinh(M\sqrt{R})\\ -M\sqrt{R}Cosh(M\sqrt{R}s_1)Sinh(\beta)+\end{bmatrix}$$
$$\left[A_3\begin{pmatrix}\beta Cosh(\beta s_1)Cosh(M\sqrt{R})-M\sqrt{R}Cosh(M\sqrt{R}(1-s_1))Sinh(\beta)\\ -M\sqrt{R}Sinh(M\sqrt{R}s_1)Sinh(\beta)\end{pmatrix}\right]\Bigg]+$$

$$\frac{A_7}{(1+SRs)\beta}\frac{Cosh(\beta s_1)}{Sinh(\beta)}+\frac{\beta Cosh(\beta s_1)}{sSinh(\beta s)}\Bigg\}+\frac{P\left[sDR^{-1}\lambda+\delta A_{11}\right]}{sA_{11}(s+A_{12}\delta)} \tag{16}$$

Taking inverse Laplace Transformations of (10), (14) and (16) we obtain

$$u_p=p\left[\frac{\delta}{\gamma_2}+\frac{\exp(-\gamma_2 t)(\gamma_1\gamma_3-\gamma_2)}{\gamma_1\gamma_2\gamma_3}\right]$$

$$u=A_8\left\{\frac{1}{Sinh(\beta_1)}\left(Sinh(\beta_1 y)Sinh(M\sqrt{R})-Sinh(M\sqrt{R}y)Sinh(\beta_1)\right)A_{15}+\right.$$

$$+A_3\begin{pmatrix} Sinh(\beta_1 y)Cosh(M\sqrt{R}) - Sinh(M\sqrt{R}(1-y))Sinh(\beta_1) \\ -Cosh(M\sqrt{R}y)Sinh(\beta_1) \end{pmatrix}+$$

$$+\sum_{n=1}^{\infty} e^{-s_n t}(-1)^n\Bigg[A_{14}\left(Sinh(\beta_n y)Sinh(M\sqrt{R}) - Sinh(M\sqrt{R}y)Sinh(\beta_n)\right)(2+SRs_n)+$$

$$+A_3\begin{pmatrix} Sinh(\beta_n y)Cosh(M\sqrt{R}) - Sinh(M\sqrt{R}(1-y))Sinh(\beta_n) \\ -Cosh(M\sqrt{R}y)Sinh(\beta_n) \end{pmatrix}\Bigg]\Bigg\}+$$

$$+A_{16}Sinh(\beta_1(1-y))\Big\{A_{15}A_8\left(D\alpha^{-1}A_{17} - A_{18}\right) + A_3A_{19} - A_{20} +$$

$$D\alpha^{-1}MCosh(Ms_1) - Sinh(Ms_1) + \frac{PSinh(M)}{A_{12}}\Big\}+$$

$$+\sum_{n=1}^{\infty}(-1)^n e^{-s_n t}\left(\frac{A_7}{(s_n + M^2)Sin(\beta_n)}\left[Sinh(\beta_n y) - Sinh(\beta_n)\right]\right) + \frac{Sinh(My)}{Sinh(M)}+$$

$$+\frac{Sinh(\beta_2(1-y))Sinh(\beta_2)}{A_{22}}A_{23}$$

$$u_B = D\alpha^{-1}\left\{\frac{A_8A_{15}}{1+SRA_9}\left\{\frac{\beta_1 Cosh(\beta_1(1-s_1))}{A_{16}}\left(D\alpha^{-1}\right)A_{21} - A_{18} + \frac{1}{Sinh(\beta_1)}A_{17}\right\}+\right. +$$

$$\frac{A_3}{(1+SRA_9)}\left(\begin{matrix}\beta_1 Cosh(\beta_1(1-s_1)). \\ \left[D\alpha^{-1}\begin{pmatrix}\beta_1 Cosh(\beta_1 s_1)Cosh(M\sqrt{R}) - M\sqrt{R}Cosh(M\sqrt{R}(1-s_1)) \\ Sinh(\beta_1) - M\sqrt{R}Sinh(M\sqrt{R})Sinh(\beta_1)\end{pmatrix}-\right.\end{matrix}\right.$$

$$\left.\begin{pmatrix} Sinh(\beta_1 s_1)Cosh(M\sqrt{R}) - Sinh(M\sqrt{R}(1-s_1))Sinh(\beta_1) \\ -Cosh(M\sqrt{R}s_1)Sinh(\beta_1) \end{pmatrix}\right] + \frac{1}{Sinh(\beta_1)}A_{19}\Bigg)+$$

$$\sum_{n=1}^{\infty} e^{-s_n t}(-1)^n\left\{A_{14}A_8(2+SRs_n)\begin{pmatrix}\beta_n Cosh(\beta_n s_1).M\sqrt{R} - \\ M\sqrt{R}Sinh(M\sqrt{R}s_1.)Sinh(\beta_n)\end{pmatrix}+\right.$$

$$+A_3A_{14}\begin{bmatrix}\beta_n Cosh(\beta_n s_1)Cosh(M\sqrt{R}) - M\sqrt{R}Cosh(M\sqrt{R}(1-s_1))Sinh(\beta_n) - \\ -M\sqrt{R}Sinh(M\sqrt{R}s_1)Sinh(\beta_n)\end{bmatrix}+$$

$$+\frac{A_7}{(1+SRs_n)\beta_n^2}\left(\beta_n Cosh(\beta_n s_1)+\frac{\beta_n}{s_n}Cosh(\beta_n s_1)\right)\Bigg\}+$$

$$+\frac{A_7}{1+SRA_{10}}\left[\frac{\beta_2 Cosh(\beta_2(1-s_1))}{A_{22}}\begin{Bmatrix}D\alpha^{-1}\beta_2 Cosh(\beta_2 s_1)\\ -Sinh(\beta_2 s_1)+Sinh(\beta_2)\end{Bmatrix}+\frac{\beta_2 Cosh(\beta_2 s_1)}{Sinh(\beta_2)}\right]+ \qquad +$$

$$\frac{P}{A_{12}}\left[\frac{MCosh(M(1-s_1).)D\alpha^{-1}Sinh(M)}{Sinh(M(1-s_1))-D\alpha^{-1}MCosh(M(1-s_1))}+1\right]+$$

$$\left[\begin{array}{l}\dfrac{MCosh(M(1-s_1))}{SinhM((1-s_1))-D\alpha^{-1}MCosh(M(1-s_1))}\begin{pmatrix}D\alpha^{-1}MCosh(Ms_1)\\ -Sinh(Ms_1)\end{pmatrix}\\ +\dfrac{MCosh(Ms_1)}{Sinh(M)}\end{array}\right]\Bigg\}$$

$$+\ A_{23}\left(\frac{\beta_2 Cosh(\beta_2(1-s_1))D\alpha^{-1}Sinh(\beta_2)}{A_{22}}+1\right)$$

Where β_1, β_2 etc. and A_9, A_{10} etc. A_{23} are constants.

The shear stress are calculated using the formula $\tau=\frac{du}{dy}$

$$\tau_{y=1}=A_8\left\{\frac{1}{Sinh(\beta_1)}\begin{pmatrix}\beta_1 Cosh(\beta_1)Sinh(M\sqrt{R})\\ -M\sqrt{R}Cosh(M\sqrt{R})Sinh(\beta_1)\end{pmatrix}A_{15}+\right.$$

$$A_3\left[\begin{array}{l}\beta_1 Cosh(\beta_1)Cosh(M\sqrt{R})-M\sqrt{R}Cosh(M\sqrt{R})Sinh(\beta_1)\\ -M\sqrt{R}-Sinh(M\sqrt{R})Sinh(\beta_1)\end{array}\right]+$$

$$\sum_{n=1}^{\infty}(-1)^n e^{-s_n t}\left[A_{14}\left(\beta_n Cosh(\beta_n)Sinh(M\sqrt{R})-M\sqrt{R}Cosh(M\sqrt{R})Sinh(\beta_n)\right)(2+SRs_n)+\right.$$

$$\left.\left.A_3\begin{pmatrix}\beta_n Cosh(\beta_n)Cosh(M\sqrt{R})-M\sqrt{R}Cosh(M\sqrt{R})Sinh(\beta_n)\\ -M\sqrt{R}Cosh(M\sqrt{R})Sinh(\beta_n)\end{pmatrix}\right]\right\}+$$

$$\sum_{n=1}^{\infty}(-1)^n e^{-s_n t}\frac{A_7}{(s_n+M^2)Sinh(\beta_n)}\left[\beta_n Cosh(\beta_n)-Sinh(\beta_n)\right]+\frac{MCosh(M)}{Sinh(M)}+$$

$$\frac{\beta_1 Cosh\beta_1}{A_{16}}\Big\{A_{15}A_8\left(D\lambda^{-1}A_{17}-A_{18}\right)+\ A_3\left[A_{19}\right]-\ A_{20}\ +$$

$$D\alpha^{-1}MCosh(M\,s_1)-Sinh(M\,s_1)+\frac{PSinh(M)}{A_{12}}\Big\}+\frac{A_{23}}{A_{22}}$$

We also determine the mass flux by the formula

$$\int_{s_1}^{1}udy=A_8\Bigg\{\frac{1}{Sinh\beta_1}\Bigg[\left(\frac{Cosh(\beta_1)}{\beta_1}Sinh(M\sqrt{R})-\frac{Cosh(M\sqrt{R})}{M\sqrt{R}}Sinh(\beta_1)\right)$$

$$-\left(\frac{Cosh(\beta_1 s_1)}{\beta_1}Sinh(M\sqrt{R})-\frac{Cosh(M\sqrt{R}s_1)}{M\sqrt{R}}Sinh(\beta_1)\right)\Bigg]A_{15}+$$

$$A_3\Bigg[\left(\frac{Cosh(\beta_1)}{\beta_1}Cosh(M\sqrt{R})-\frac{Sinh(\beta_1)}{M\sqrt{R}}-\frac{Sinh(M\sqrt{R})}{M\sqrt{R}}Sinh(\beta_1)\right)-$$

$$\left(\frac{Cosh(\beta_1 s_1)}{\beta_1}Cosh(M\sqrt{R})-\frac{Cosh(M\sqrt{R}(1-s_1))}{M\sqrt{R}}Sinh(\beta_1)-\frac{Sinh(M\sqrt{R})}{M\sqrt{R}}Sinh(\beta_1)\right)\Bigg]+$$

$$\sum_{n=1}^{\infty}e^{-s_n t}(-1)^n\Bigg[\frac{1}{\left(M^2R+\beta_n^2\right)\left(1+SRs_n\right)}\Bigg\{\left(\frac{Cosh(\beta_n)}{\beta_n}Sinh(M\sqrt{R})-\frac{Cosh(M\sqrt{R})}{M\sqrt{R}}Sinh(\beta_n)\right)$$

$$-\left(\frac{Cosh(\beta_n s_1)}{\beta_n}Cosh(M\sqrt{R})-\frac{Cosh(M\sqrt{R}(1-s_1))}{M\sqrt{R}}Sinh(\beta_n)\right)\Bigg\}(2+SRs_n)\ +$$

$$A_3\Bigg[\left(\frac{Cosh(\beta_n)}{\beta_n}Cosh(M\sqrt{R})-\frac{Sinh(\beta_n)}{M\sqrt{R}}-\frac{Sinh(M\sqrt{R})}{M\sqrt{R}}Sinh(\beta_n)\right)-$$

$$\left(\frac{Cosh(\beta_n s_1)}{\beta_n}Cosh(M\sqrt{R})-\frac{Cosh(M\sqrt{R}(1-s_1))}{M\sqrt{R}}Sinh(\beta_n)-\frac{Sinh(M\sqrt{R}s_1)}{M\sqrt{R}}Sinh(\beta_n)\right)\Bigg]\Bigg\}+$$

$$\sum_{n=1}^{\infty}(-1)^n e^{-s_n t}\frac{A_7}{(s_n+M^2)Sinh(\beta_n)}\left[\left(\frac{Cosh(\beta_n)}{\beta_n}-Sinh(\beta_n)\right)-\left(\frac{Cosh(\beta_n s_1)}{\beta_n}-Sinh(\beta_n)\right)\right]$$

$$+\left(\frac{Cosh(M)-Cosh(Ms_1)}{MSinh(M)}\right)+\left[\frac{(1-Cosh(\beta_1(1-s_1)))}{A_{16})}\left\{A_{15}A_8\left(D\alpha^{-1}A_{17}-A_{18}\right)\right]\right. +$$

$$A_3A_{19}-A_{20}+\ D\alpha^{-1}Cosh(Ms_1)-Sinh(Ms_1)+\frac{PSinh(M)}{M^2A_{12}}\right\}+\left(\frac{(1-Cosh(\beta_2(1-s_1)))}{A_{22}}\right)A_{23}$$

3. DISCUSSIONS OF THE RESULTS:

The main aim of this investigation is to discuss the influence of a densely packed porous lining on the flow of a Rivlin–Ericksen conducting viscoelastic fluid through a parallel plate channel subjected to a traverse magnetic field. The behaviour of the velocity of the fluid in clean fluid region as well as the slip velocity at the interface for small and large thickness of the porous bed has been computationally analyzed for variations in the governing parameters. Fig. [1-4] corresponds to the fluid velocity in the clean fluid region when the thickness of the bed is small and figs [5-8] corresponds to its behavior when the thickness is fairly large. We notice from fig (1) in general when M > 5, u reduces the lower half till y ≤ 0.6 and later gradually rises to attain the prescribed value on the upper plate. When $M < 5$, the similar behavior is noticed till y ≤ 0.6 although a reversal flow is observed is the upper half with u steeply raising to its prescribed value on the upper boundary. An increasing M enhances the fluid velocity in the lower region while reducing in the upper half fig (1). From fig. (2) we observe that an increase in through smaller values R (<10) accelerates the fluid flow while for R>10 the velocity reduces in the flow field. A reversal flow is observed in the flow field except in the vicinity in the upper plate for higher value of R (=25). Lower the permeability of the porous bed greater the velocity of the fluid in the non-porous region, although for sufficiently high inverse Darcy parameter D^{-1} order 3 x 10^5 a slight retardation is observed in the fluid. Fig. (4) corresponds to the variation of u with S the viscoelastic parameter we observe that the magnitude of fluid enhances every where with increase in S except in the vicinity of the upper plate.

When the thickness of the bed is sufficiently large a reversal movement is observed in general with flow taking place in the direction of the imposed pressure gradient. We also notice that the magnitude of u enhances with increase in M and R, except perhaps near the upper boundary fig [5-8]. A similar behaviour is noticed with increase in D^{-1} (<2 x 10^5) and S (<2). When D^{-1} is 2 x 10^5 or S (=2) a slight depreciation is noticed in the axial velocity fig. [7,8]. The slip velocity u_B has been evaluated and tabulated in table-1 for different variations in the governing parameters in both the cases of small and large thickness of the bed. We notice that u_B enhances with any one of M S, R, or D^{-1}, fixing the remaining parameters.

The shear stress on the upper plate and mass flux have been evaluated for variations in the parameters and tabulated in tables 2 and 4. In either case of small and large thickness of porous bed we find that the shear stress enhance on the plates with M, R, S for fixed values of the other parameters while reduce with increase in D^{-1}. Hence lesser the permeability of the bed higher the stresses on the plate. The mass flux reduces with increase in M, R, S or D^{-1} irrespective of the thickness of the porous bed.

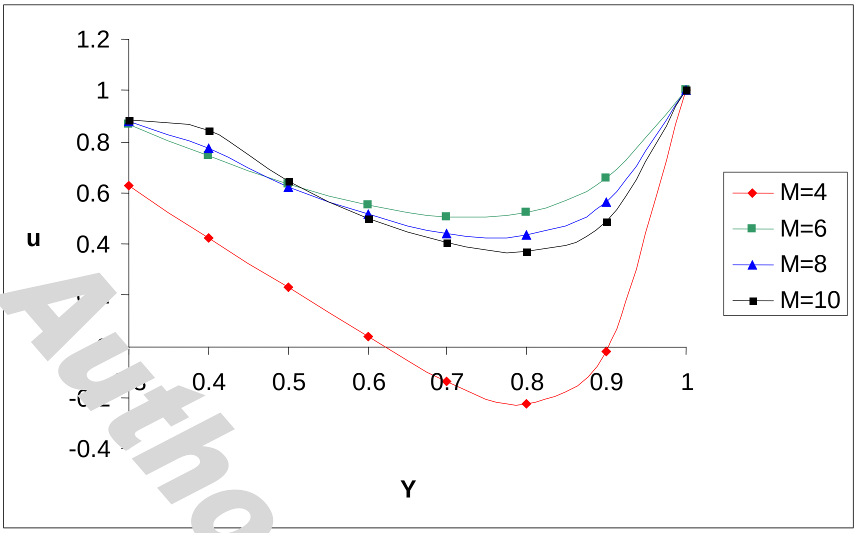

Fig. 1. Variation of u with M in the clean fluid region (0.3≤y≤1)

P = 1, t =1, D-1 = 104, R = 10, S = 2.5, α = 0.5, λ = 1.2, δ = 0.3, s_1 = 0.3

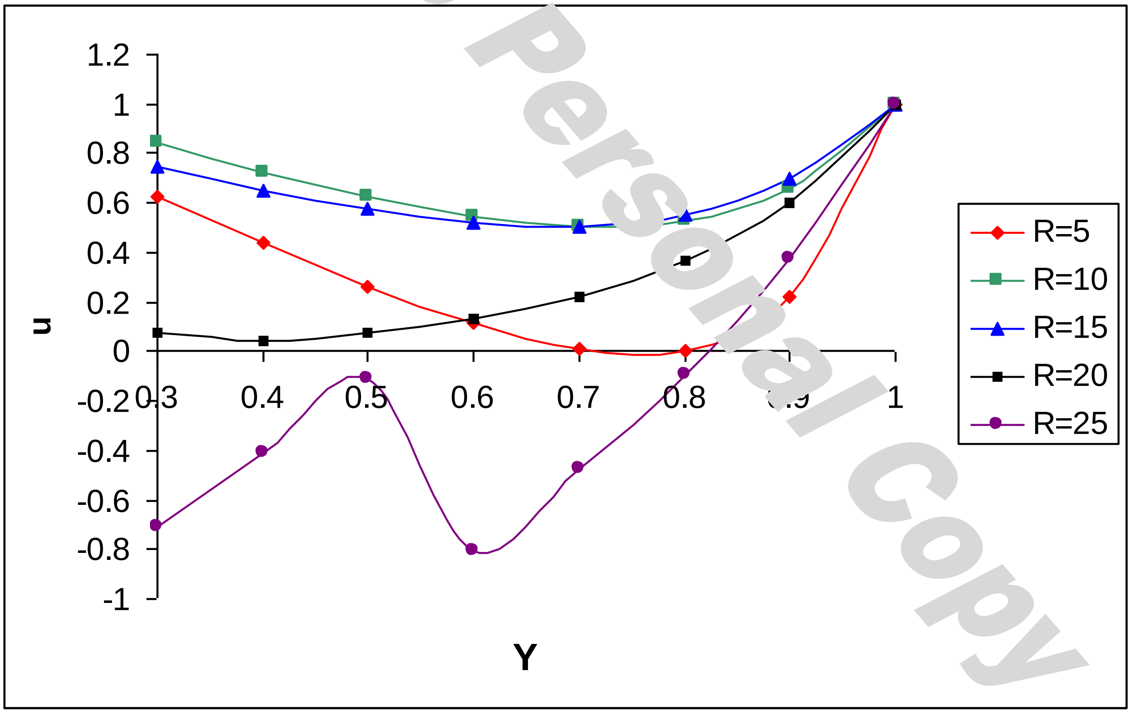

Fig.2. Variation of u with R

P = 1, t =1, $D^{-1} = 10^4$, M = 5, S = 2.5, α = 0.5, λ = 1.2, δ = 0.3, s_1 = 0.3

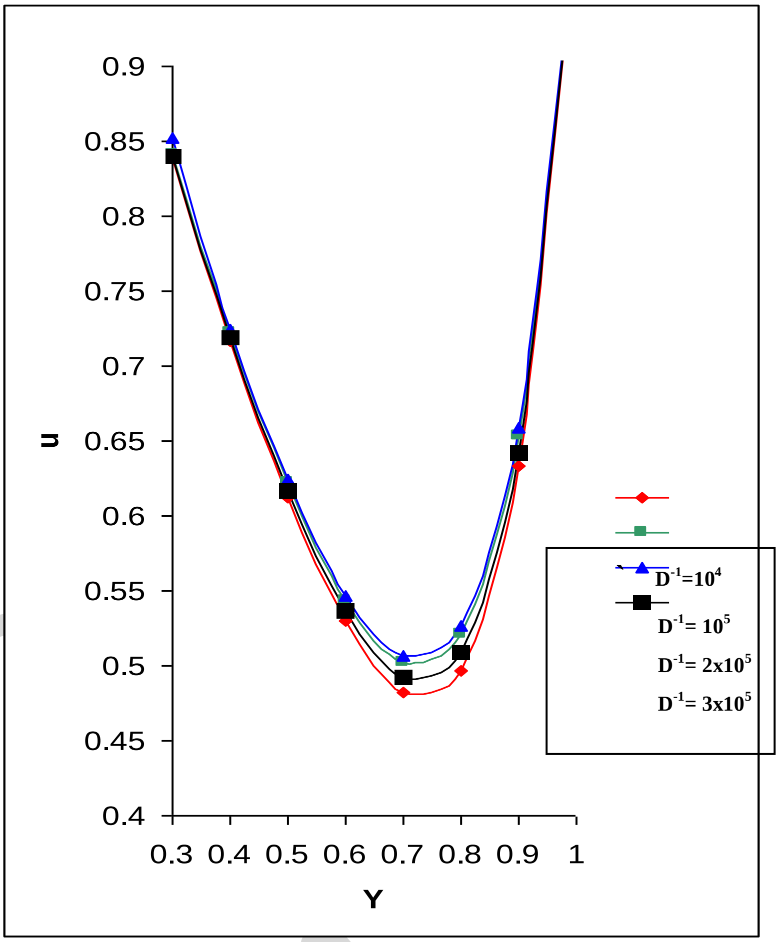

Fig.3. Variation of u with D-1

P = 1, t =1, R = 10, M = 5, S = 2.5, α = 0.5, λ = 1.2, δ = 0.3, s_1 = 0.3

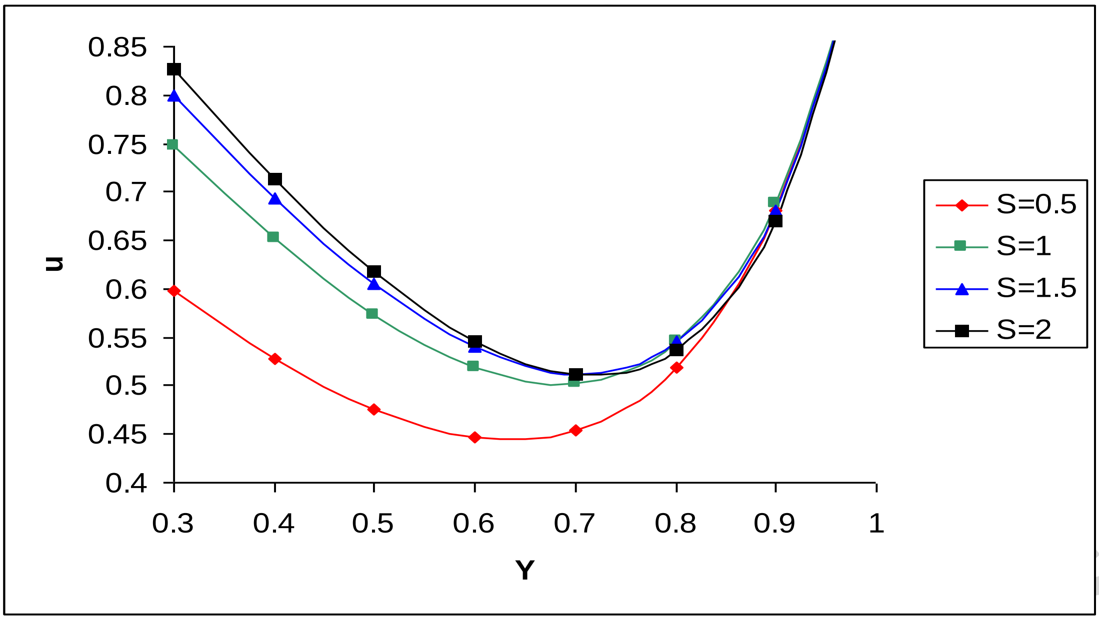

Fig.4. Variation of u with S

P = 1, t =1, R = 10, M = 5, D-1 = 10^4, α = 0.5, λ = 1.2, δ = 0.3, s_1 = 0.3

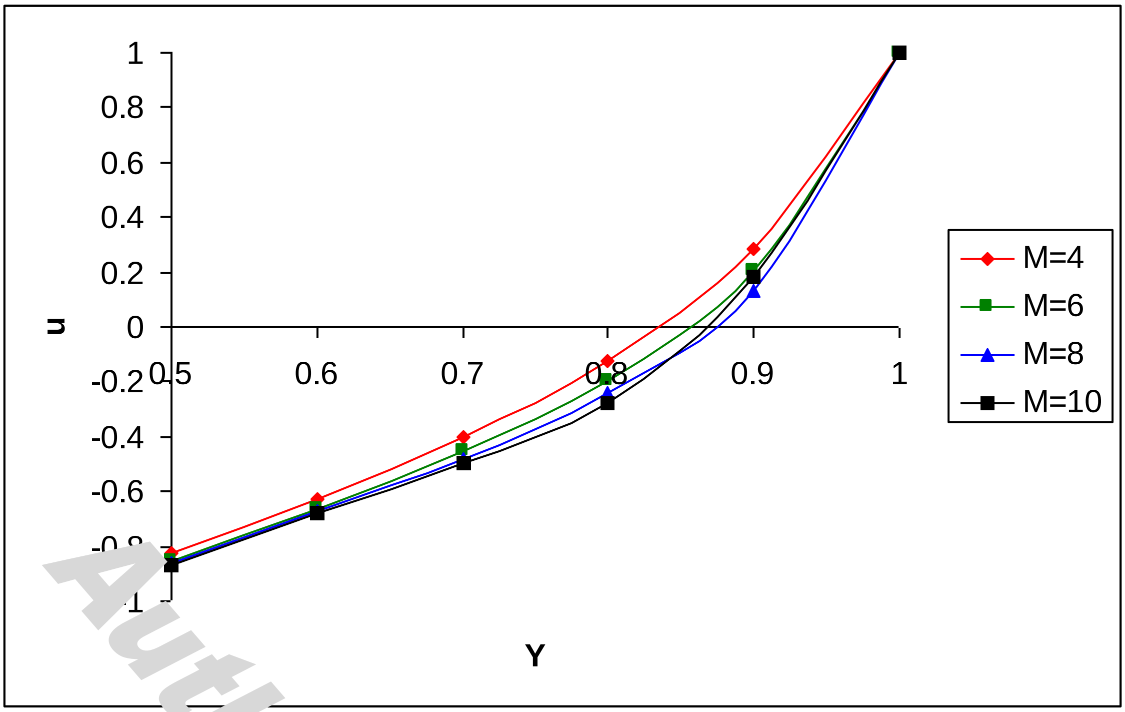

Fig. 5 Variation of u with M (0.5≤y≤1)

P = 1, t =1, D = 4, R = 10, S = 2.5, α = 0.5, λ = 1.2, δ = 0.3, s_1 = 0.5

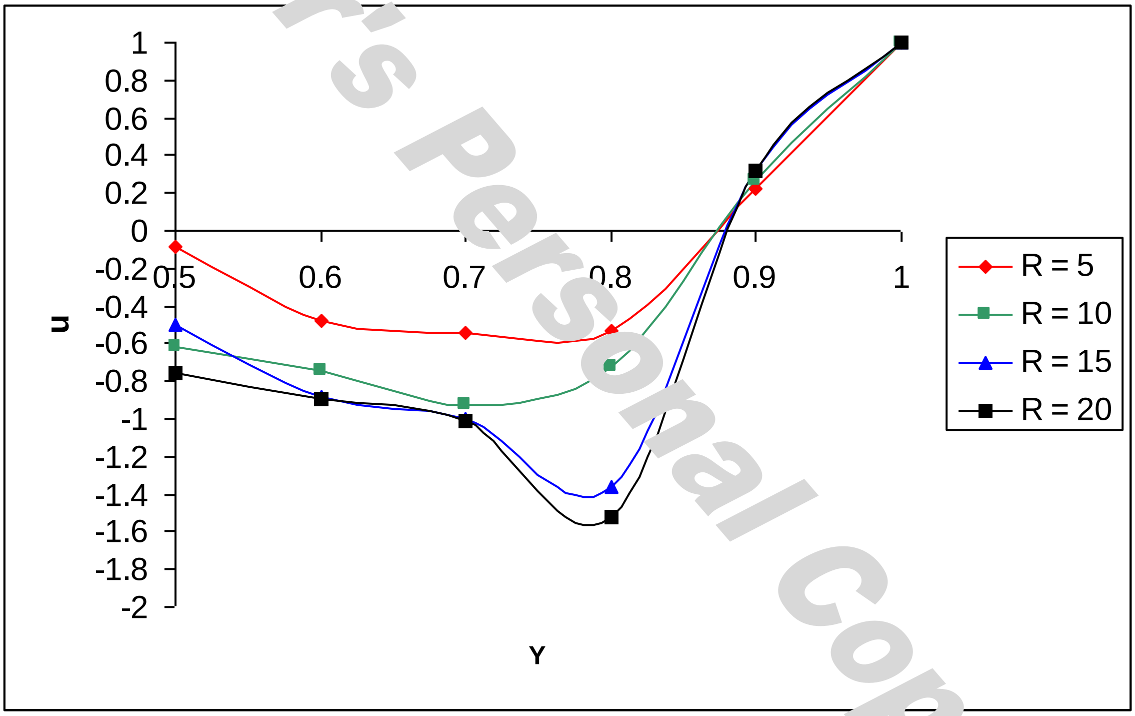

Fig. 6 Variation of u with R

P = 1, t =1, $D^{-1} = 10^4$, M = 5, S = 2.5, α = 0.5, λ = 1.2, δ = 0.3, s_1 = 0.5

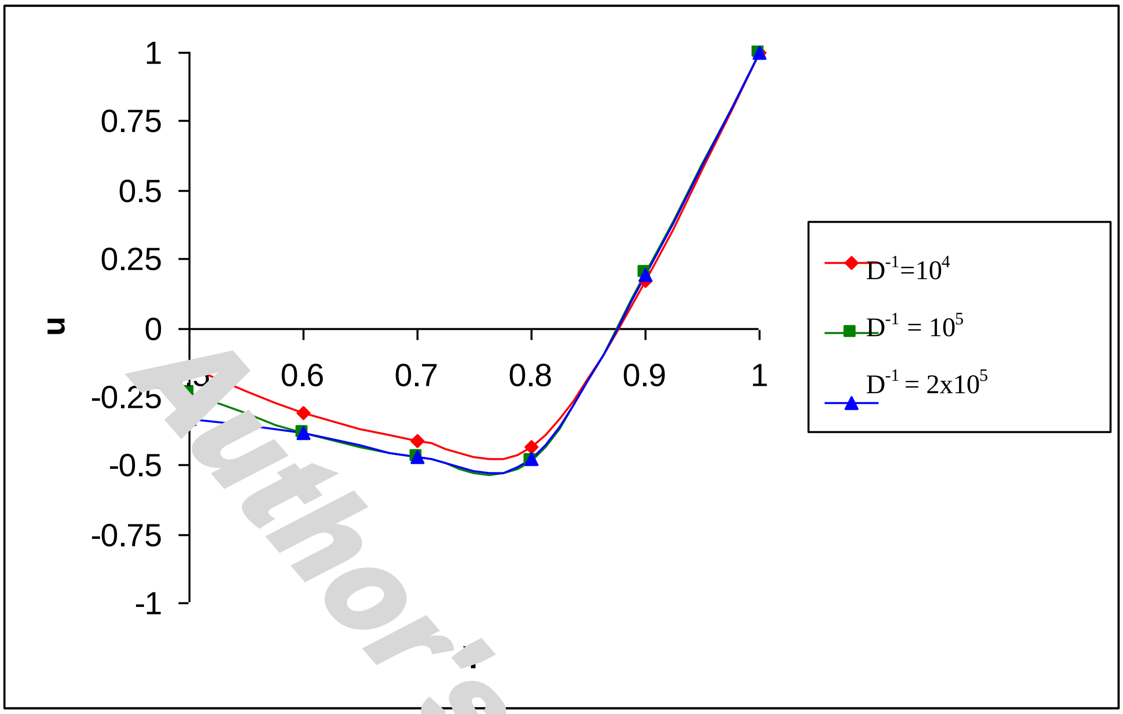

Fig. 7 Variation of u with D-1

P = 1, t =1, R =10, M = 5, S = 2.5, α = 0.5, λ = 1.2, δ = 0.3, s1 = 0.5

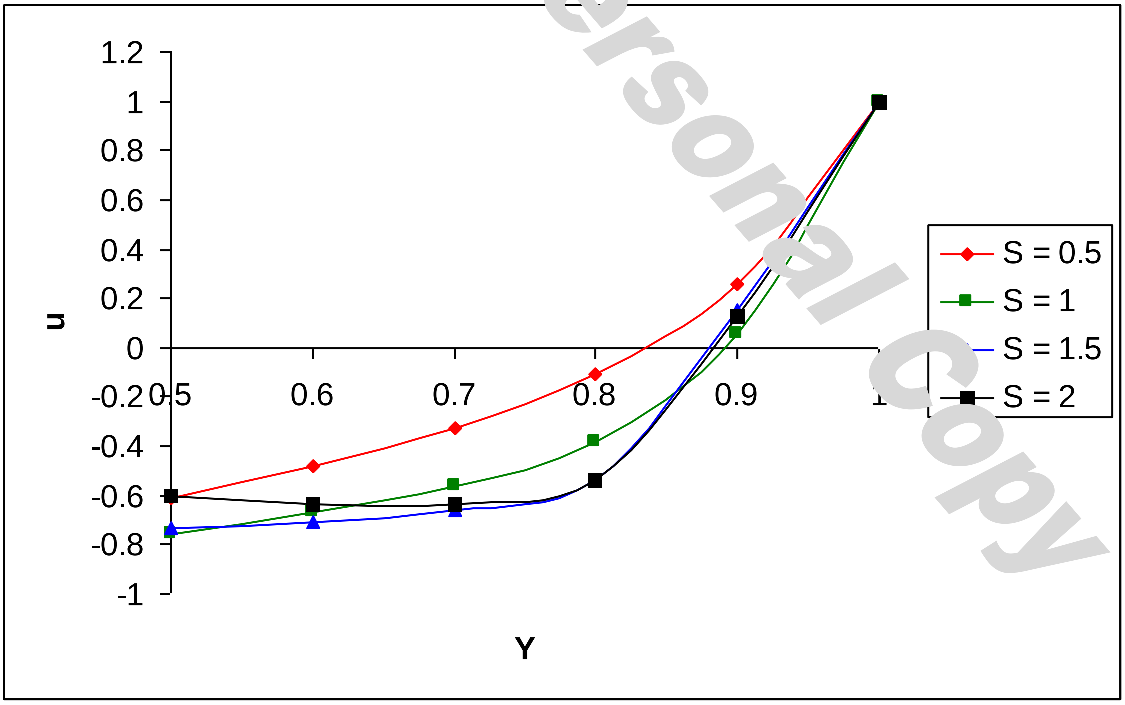

Fig. 8 Variation of u with S

P = 1, t =1, R =10, M = 5, $D^{-1} = 10^4$, α = 0.5, λ = 1.2, δ = 0.3, s_1 = 0.5

TABLE – 1

SLIP VELOCITY u_B

S_1	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
0.3	0.83835	0.876998	0.884333	0.627566	0.746105	0.0783026	0.71921	0.84119	0.8519	0.597547	0.74751	0.8004540
0.5	-0.64152	-0.863553	-0.867589	-0.089508	-0.50423	-0.752	-0.234986	-0.33257	-0.41592	-0.611466	-0.759402	-0.735836

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
M	5	8	10	5	5	5	5	5	5	5	5	5
R	10	10	10	5	15	20	10	10	10	10	10	10
S	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	0.5	1	1.5
D^{-1}	10^4	10^4	10^4	10^4	10^4	10^4	10^3	10^5	2×10^5	10^4	10^4	10^4

TABLE – 2

SHEAR STRESS AT Y = 1

S_1	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
0.3	3.58194	5.99344	6.03499	-7.36501	5.48447	3.86074	2.36915	3.36461	2.86777	4.73466	4.86434	4.5383
0.5	-120.425	-4.13109	-2.98803	-94.8123	-52.8313	-58.6806	-262.808	-133.91	-166.868	-15.9204	-35.2955	-59.1816

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
M	5	8	10	5	5	5	5	5	5	5	5	5
R	10	10	10	5	15	20	10	10	10	10	10	10
S	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	0.5	1	1.5
D^{-1}	10^4	10^4	10^4	10^4	10^4	10^4	10^3	10^5	2×10^5	10^4	10^4	10^4

TABLE – 3

MASS FLUX

S_1	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
0.3	0.52320	0.51627	0.45123	0.43628	0.40412	0.3216	0.31427	0.28612	0.26602	0.24321	0.2262	0.19630
0.5	0.26667	0.26430	0.25281	0.24084	0.24132	0.22513	0.16727	0.26325	0.15227	0.15165	0.15240	0.15302

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
M	5	8	10	5	5	5	5	5	5	5	5	5
R	10	10	10	5	15	20	10	10	10	10	10	10
S	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	0.5	1	1.5
D^{-1}	10^4	10^4	10^4	10^4	10^4	10^4	10^3	10^5	2×10^5	10^4	10^4	10^4

APPENDIX

$$A_1 = \frac{SR^2M^2}{Sinh(M\sqrt{R})}\left[D^{1/2}\alpha^{-1}PRCosh(M\sqrt{R}s_1) - \frac{PR}{M\sqrt{R}}\left(Sinh(M\sqrt{R}s_1) - Sinh(M\sqrt{R})\right)\right]$$

$$A_2 = Sinh(M\sqrt{R}(1-s_1)) - D^{1/2}\alpha^{-1}M\sqrt{R}Cosh(M\sqrt{R}(1-s_1))$$

$$+ D\delta R\lambda^{-1}Sinh(M\sqrt{R}$$

$$A_3 = \frac{A_1}{A_2}$$

$$A_4 = \frac{MPR^{1/2}}{Sinh(M\sqrt{R})}$$

$$A_5 = \frac{-Cosh(M\sqrt{R})}{Sinh(M\sqrt{R})}$$

$$A_6 = \frac{PR}{M\sqrt{R}Sinh(M\sqrt{R})}$$

$$A_7 = \frac{-PR}{M\sqrt{R}}$$

$$A_8 = A_4 + A_3 A_5 + A_6$$

$$A_9 = \frac{M^2(1+R)}{(1+SRs)}$$

$$A_{10} = \frac{-M^2}{(1+D^{-1}R^{-1}\lambda)}$$

$$s_n = \frac{-(n^2\pi^2 + M^2)}{(1+n^2\pi^2SR)}$$

$$\beta_1 = \sqrt{\frac{A_9 + M^2}{1+SRA_9}}$$

$$\beta_2 = \sqrt{\frac{A_{10} + M^2}{1+SRA_{10}}}$$

$$\beta_n = \sqrt{\frac{s_n + M^2}{1+SRs_n}}$$

$$A_{11} = \left(1 + M^2 DR\lambda^{-1}\right)$$

$$A_{12} = D^{-1}R^{-1}\lambda + M^2$$

$$A_{13} = A_{11}A_{12}$$

$$A_{14} = \frac{1}{\left(M^2R + \beta_n^2\right)\left(1 + SRs_n\right)}$$

$$A_{15} = \left(2 + SRA_9\right)$$

$$A_{16} = \frac{1}{Sinh(\beta_1)(1 - s_1) - D\alpha^{-1}\beta_1 Cosh\left(\beta_1(1 - s_1)\right)}$$

$$A_{17} = \left(\beta_1 Cosh(\beta_1 s_1.) Sinh(M\sqrt{R}) - M\sqrt{R} Cosh(M\sqrt{R}s_1).Sinh(\beta.\right)$$

$$A_{18} = \left(Sinh(\beta_1 s_1).Sinh(M\sqrt{R}) - Sinh(M\sqrt{R}s_1) Sinh(\beta_1)\right)$$

$$A_{19} = [\beta_1 D\alpha^{-1} Cosh(M\sqrt{R}) Cosh(\beta_1 s_1) - M\sqrt{R} Cosh(M\sqrt{R}(1 - s_1)) Sinh(\beta_1)$$
$$- M\sqrt{R} Sinh(M\sqrt{R}) Sinh(\beta_1)]$$

$$A_{20} = \begin{pmatrix} Sinh(\beta_1 s_1) Cosh(M\sqrt{R}) - Sinh(M\sqrt{R}(1 - s_1)) Sinh(\beta_1) \\ - Cosh(M\sqrt{R}s_1).Sinh(\beta_1) \end{pmatrix}$$

$$A_{21} = \left(\beta_1 Cosh(\beta_1 s_1.) Sinh(M\sqrt{R}) - M\sqrt{R} Sinh(M\sqrt{R}s_1) Sinh(\beta_1)\right)$$

$$A_{22} = Sinh(\beta_2(1 - s_1)) - D\alpha^{-1}\beta_2 Cosh(\beta_2(1 - s_1))$$

$$A_{23} = \left(\frac{P\left(A_{10}DR\lambda^{-1} + \delta A_{11}\right)}{A_{10}A_{11}\left(A_{10} + \delta A_{12}\right)}\right)$$

REFERENCES

1) Larson RG. Instabilities in viscoelastic flows. Rheol Acta 1992;31: 213-63(1992).
2) Khomami N, Su KC. An experimental/theoretical investigation of interfacial instabilities in superposed pressure-driven channel flow of Newtonian and well characterized viscoelastic fluids, Pt.I: Linear stability and encapsulation. J Non-Newtonian Fluid Mech: 91:59-84(2000).
3) Chin WC.Propagation in petroleum engineering. Houston, TX, USA: Gulf Publishing Company; (1993).
4) G.Bohme, Non-Newtonian Fluid Mechanics, North Holland, Amsterdam, (1981).
5) R.R. Huilgol, N.Phan-Thien, Fluid mechanics of viscoelasticity, Elsevier, Amsterdam, (1997).
6) Rivlin RS, Ericksen JL. Stress-deformation relaxations for isotropic materials, J.Rat.Mech.Anal; 4:323-425, (1955).
7) Kazakia J.Y. and Rivilin R.S,Run-up and Spin- up flows in a viscoelastic fluid –I-Rheologia- Acta,20.(1981).
8) Rivlin, Run-up and Spin-up in a viscoelastic fluids III, Rheologia- Acta, 21, pp.213-222, (1982).