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Global Attractivity Results For Neutral Functional Differential Equations In Banac Algebras

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ABSTRACT

In this paper, we prove existence theorem and global attractivity solutions for neutral functional differential equations in Banac algebras by using the characterizations of measures of noncompactness and a fixed point theorem. The investigation are placed in the Banac space of real functions defined, continuous and bounded on interval.

Keywords: Global attractivity, Neutral functional differential equations, Fixed point theorem, Measures of noncompactness

1. Statement of problem

Let ℝ denote the real line and let $I_0 = [-r,0]$ and $I=[0,a]$ be closed and bounded intervals in ℝ. Let $J=I_0 \cup I$, then J is closed and bounded interval in ℝ. Let C denote the Banac space of all continuous real valued functions ϕ on I_0 with the supremum norm $\|\cdot\|_C$ defined by

$$\|\phi\|_C = \sup_{t\in I_0}|\phi(t)|$$

Clearly C is Banac algebra with this norm. Consider the first order functional differential equation (in short FDE)

$$\frac{d}{dt}\left[\frac{x(t)}{f(t,x(t),x_t)}\right] = (t,x(t),x_t)\,, \qquad a.e.\ t \in I \qquad (1.1)$$

$$x(t) = \phi(t)\,, \qquad t \in I_0$$

Where $f: I \times \mathbb{R} \times C \to \mathbb{R} - \{0\}$, $g: I \times \mathbb{R} \times C \to \mathbb{R}$ and for each $t \in I$, $x_t: I_0 \to C$ is continuous function defined by $x_t \coloneqq x(t+\theta)$ for all $\theta \in I_0$.

By a solution of FDE (1.1) we mean a function $x \in C(J,\mathbb{R}) \cap AC(I,\mathbb{R}) \cap C(I_0,\mathbb{R})$ that satisfies the equation in (1.1), where $AC(I,\mathbb{R})$ is the space ofall absolutely continuous real valued functions on J.

2. Auxiliary Results

We mention some concepts concerning the measures of noncompactness [1,2] in Banach spaces.

Denote by $\overline{\mathcal{B}_r}(x)$ the closed ball centered at x and with radius r. Thus $\overline{\mathcal{B}_r}(0)$ is the closed ball centered at origin of radius r. Assume that $(E, \|\cdot\|)$ is an infinite dimensional Banach space with zero elementsθ.If X is subset of E then the symbols $\bar{X}$, convX stand for the closure and closed convex hull of X, respectively. Denote by $\mathcal{P}_{bd}(E)$ the family of all nonempty and bounded subsets of E and by $\mathcal{P}_{rcp}(E)$ its subfamily consisting of all relatively compact subsets of E.

The measures of noncompactness defined as following which is appears in Banas and Goebel [1].

Definition (2.1): The mapping $\mu: \mathcal{P}_{bd}(E) \longrightarrow \mathbb{R}_+ = [0, \infty)$ is said to be a measures of noncompactness in E if it satisfies the following conditions.

a) The family $Ker\mu$=$\{X \in \mathcal{P}_{bd}(E)/\mu(X) = 0\} \neq \phi$ and $Ker\mu \subset \mathcal{P}_{bd}(E)$.
b) $X \subset Y \Longrightarrow \mu(X) \leq \mu(Y)$.
c) $\mu(\bar{X}) = \mu(X)$.
d) $\mu(convX) = \mu(X)$.
e) $\mu(\lambda X + (1-\lambda)y) \leq \lambda\mu(X) + (1-\lambda)\mu(Y)$ for $\lambda \in [o, 1]$.
f) If (X_n) is sequence of closed sets from $\mathcal{P}_{bd}(E)$ such that $X_{n+1} \subset X_n(n = 1,2,3, \dots \dots \dots)$. and if $\lim_{n\to\infty} \mu(X_n) = 0$, then the intersection set $X_\infty = \bigcap_{n=1}^{\infty} X_n$ is nonempty.

Also $Ker\mu$ given in 'a' is said to be the kernel of the measures of noncompactness μ. And from $'f'$ by observation set X_∞ is the member of family $Ker\mu$. In fact, for any n,

$\mu(X) \leq \mu(X_n)$.We infers that μ.

We will used following fixed point theorem of Darbo type in the sequel (Banas 2).

Theorem (2.2): Let Ω be a nonempty, bounded, and convex subset of the Banac space E and let $F: \Omega \to \Omega$ be continuous mapping. Assume that $\exists\, k \in [0,1)$ such that $\mu(FX) \leq k\mu(X)$ for any nonempty subset X of Ω. Then F has a fixed point in the set Ω.

Remark (2.3): Let the set of all fixed points of the operator F denoted by $F_{iX}F$ which belongs to Ω and also belongs to the family $Ker\mu$. (see .1).

We will be considered the Banac space $BC(\mathbb{R}, \mathbb{R})$ consisting of all real valued functions $x = x(t)$ defined continuous and bounded on $\mathbb{R}$ with the supremum norm

$$\|x\| = sup|x(t)|, \quad t \in \mathbb{R} \tag{2.4}$$

To define the measures of noncompactness, let us fix a nonempty and bounded subset X of the space $BC(\mathbb{R}, \mathbb{R})$ and number T> 0, for $x \in X$, $\epsilon \geq 0$ denote by $\omega^T(x, \epsilon)$ the modulus of continuity of the function x on the interval [0,T] i.e.

$$\omega^T(x, \epsilon) = \{|x(t) - x(s)|: t, s \in [0, T], |t - s| \leq \epsilon\}$$

Next, let us put

$$\omega^T(X,\epsilon) = sup\{\omega^T(x,\epsilon): x \in X\}$$

$$\omega^T(X) = \lim_{\epsilon\to 0}\omega^T(X,\epsilon)$$

$$\omega_0(X) = \lim_{T\to\infty}\omega_0^T(x)$$

Now, for a fixed number $t \in \mathbb{R}$, let us denote

$$X(t) = \{x(t): x \in X\}$$

And,

$$diam X(t) = sup|x(t) - y(t)| \quad x, y \in X$$

Finally, let us consider the function μ defined on the family $BC(\mathbb{R},\mathbb{R})$ by the formula

$$\mu(X) = \omega_0(X) + \lim_{t\to\infty}\sup diam\left(X(t)\right) \tag{2.5}$$

The function μ is a measures of noncompactness in the space $BC(\mathbb{R},\mathbb{R})$ (see Banas[2]).

The $Ker\mu$ of this mearsure consists nonempty and bounded subsets X of $BC(\mathbb{R},\mathbb{R})$ such that functions from X are locally equicontinuous on $\mathbb{R}$, by functions from $X \to 0$ at ∞. This characteristic of $Ker\mu$ has been utilized in establishing the global attractivity of solutions.

We will be used that let us assume that Ω is a nonempty subset of the space $BC(\mathbb{R},\mathbb{R})$, let Q be an operator defined on Ω with values in $BC(\mathbb{R},\mathbb{R})$. Consider the operator equation of the form

$$X(t) = Qx(t), \quad t \in \mathbb{R} \tag{2.6}$$

Definition (2.7); we say that solutions of the equation (2.5) are locally attractivity if there exists a ball $\overline{B}r(x_0)$ in $BC(\mathbb{R},\mathbb{R})$ such that for arbitrary solutions $x = x(t)$ and $y = y(t)$ of equation (2.5) belonging to $\overline{B}r(x_0) \cap \Omega$, we have

$$\lim_{t\to\infty}[x(t) - y(t)] = 0 \tag{2.7}$$

When for each $\epsilon > 0, \exists\, T > 0$ such that

$$|x(t) - y(t)| \le \epsilon \tag{2.8}$$

For all $x, y \in \overline{B}r(x_0) \cap \Omega$ being solution of (2.6) is said to globally attractive if (2.7) holds for each solution $y = y(t)$ of (2.6). If for every $\epsilon > 0, \exists\, T > 0$ such that the inequality (2.8) is satisfied for all $x, y \in \Omega$ being solution of (2.6) and for $t \ge T$, we will say that solutions of the equation (2.6) are uniformly globally attractive. The concept of attractivity of solutions was introduced in [8] while the concept of global attractivity of solution given in the paper [4] and [8].

3. Existence Theory

We will use the following hypotheses in the sequel

(H[1]) The functions $\alpha, \beta, \gamma : \mathbb{R} \to \mathbb{R}$ are continuous and $\lim_{t\to\infty} \alpha(t) = \infty$.

(H[2]) The function $f: I \times \mathbb{R} \times C \to \mathbb{R} - \{0\}$ is continuous and there exist bounded function $k: \mathbb{R} \to \mathbb{R}$ with bound L such that

$$\left|f(t, x_1, y_1) - f(t, x_2, y_2)\right| \le \ell(t) \max \ \{|x_1 - x_2|, \|y_1 - y_2\|_c\}, \qquad a.e.\ t \in \mathrm{I}$$

for all $x, y \in \mathbb{R}$, $y_1, y_2 \in C$.

(H[3]) The function $F: \mathbb{R} \to \mathbb{R}$ defined by $F(t) = |f(t, 0,0)|$ is bounded on $\mathbb{R}$ with

$F_0 = \sup_{t\ge 0} F(t)$

(H[4]) The function $g: I \times \mathbb{R} \times C \to \mathbb{R}$ is continuous and there exist functions $a, b: \mathbb{R} \to \mathbb{R}$

such that $|g(t, x, y)| \le a(t) b(t)$ for $t, s \in \mathbb{R}$.

Moreover, we assume that

$$\lim_{t\to\infty} a(t) \int_0^{\beta(t)} b(s)\, ds = 0$$

4. Main Result:

Theorem (4.1): Assume that the hypotheses (H[1]) through (H[4]) hold. Furthermore, if $k < 1$ where $k = \sup_{t\ge 0} a(t) \int_0^{\beta(t)} b(s)\, ds$, then the equation (1.1) has at least one uniformly globally attractive solution on J.

Proof: Now the FDE (1.1) is equivalent to the functional integral equation (in short FIE)

$$x(t) = [f(t, x(t), x_t)] \left(\phi(0) + \int_0^t g(t, x(s), x_s) ds\right) \qquad \text{if } \ t \in \mathrm{I} \qquad (4.2)$$

$$x(t) = \phi(t) \quad \text{if } \ t \in \mathrm{I}_o \qquad (4.3)$$

Let set X= $BC(\mathbb{R}, \mathbb{R})$. Consider the operator Q defined on the space $BC(\mathbb{R}, \mathbb{R})$ by the formula

$$Qx(t) = [f(t, x(t), x_t)] \left(\phi(0) + \int_0^t g(t, x(s), x_s) ds\right) \qquad (4.4)$$

By our hypothesis, for any function$x \in X$, the function Qx is continuous on $\mathbb{R}$. Since the function $\upsilon: \mathbb{R} \to \mathbb{R}$ defined by

$$\nu(t) = \lim_{t\to\infty} a(t) \int_0^{\beta(t)} b(s)\, ds \qquad (4.5)$$

Is continuous and by hypothesis (H4), the number $\nu_0 = \sup_{t\ge 0} \nu(t)$ exsits. Moreover, for arbitrarily fixed $\in \mathbb{R}$, we obtain,

$$|Qx(t)| \leq |f(t,x(t),x_t)|\left(\|\phi\|_c + \left|\int_0^t g(t,x(s),x_s)ds\right|\right)$$

$$\leq [|f(t,x(t),x_t) - f(t,0,0)| + |f(t,o,o)|] \times \left(\|\phi\|_c + \int_0^t |g(t,x(s),x_s)|ds\right)$$

$$|x(t)| \leq [\ell(t)max\{|x(t)|, \|x_t\|_c\} + F] \times \left(\|\phi\|_c + \int_0^t |g(t,x(s),x_s)|ds\right)$$

$$\leq \ell(t)max\{|x(t)|, \|x_t\|_c\} \times \left(\|\phi\|_c + \int_0^t |g(t,x(s),x_s)|ds\right)$$

$$+F \times \left(\|\phi\|_c + \int_0^t |g(t,x(s),x_s)|ds\right)$$

$$\leq Lmax\{|x(t)|, \|x_t\|_c\} \times \left(\|\phi\|_c + a(t)\int_0^{\beta(t)} b(s)\,ds\right)$$

$$+F \times \left(\|\phi\|_c + a(t)\int_0^{\beta(t)} b(s)\,ds\right)$$

$$\leq Lmax\{|x(t)|, \|x_t\|_c\}\big(\|\phi\|_c + v(t)\big) + F\|\phi\|_c + Fv(t)$$

$$|x(t)| \leq L|x(t)|\big(\|\phi\|_c + v(t)\big) + F\|\phi\|_c + Fv(t)$$

$$|x(t)|\left(1 - L\big(\|\phi\|_c + v(t)\big)\right) \leq F\|\phi\|_c + Fv(t)$$

$$|x(t)| \leq \frac{F\|\phi\|_c}{1-L\big(\|\phi\|_c+v(t)\big)} + \frac{Fv(t)}{1-L\big(\|\phi\|_c+v(t)\big)}$$

$$\leq \frac{F\big(\|\phi\|_c+v(t)\big)}{1-L\big(\|\phi\|_c+v(t)\big)} = \frac{F_0v_0}{1-Lv_0}$$

For all ∈ ℝ , taking the supremum over t ,where $v_0 = \|\phi\|_c + v(t)$ supremum over t. We have

$$\|Qx\| \leq \frac{F_0v_0}{1-Lv_0} \qquad \text{for all } x \in X \tag{4.5}$$

This implies that operator Q transforms the space X into itself. Also from (4.5)

We get that the operator Q transforms the space X into the ball $\overline{\mathcal{B}}r(0)$, where $r = \frac{F_0v_0}{1-Lv_0}, Lv_0 \leq 1$

as result Q defines a mapping $Q: \overline{\mathcal{B}}r(0) \to \overline{\mathcal{B}}r(0)$.

Now we have to show that the operator Q is continuous on the ball $\overline{\mathcal{B}}r(0)$, let us fixed arbitrary $\epsilon > 0$, and take $x, y \in \overline{\mathcal{B}}r(0)$ such that $\|x-y\| \leq \epsilon$. Then

$|Qx(t) - Qy(t)|$

$$\leq \left| f(t,x(t),x_t)\left(\phi(0)+\int_0^t g(t,x(s),x_s)ds\right) - f(t,y(t),y_t)\left(\phi(0)+\int_0^t g(t,y(s),y_s)ds\right)\right|$$

$$\leq \left| f(t,x(t),x_t)\left(\phi(0)+\int_0^t g(t,x(s),x_s)ds\right) - f(t,y(t),y_t)\left(\phi(0)+\int_0^t g(t,x(s),x_s)ds\right)\right|$$

$$+\left| f(t,y(t),y_t)\left(\phi(0)+\int_0^t g(t,x(s),x_s)ds\right) - f(t,y(t),y_t)\left(\phi(0)+\int_0^t g(t,y(s),y_s)ds\right)\right|$$

$$\leq [|f(t,x(t),x_t)-f(t,y(t),y_t)|]\left(\phi(0)+\int_0^t g(t,x(s),x_s)ds\right)+[|f(t,y(t),y_t)-f(t,0,0)|+ \quad |f(t,0,0)|]\left(\int_0^t[|g(t,x(s),x_s)|+|g(t,x(s),x_s)|]ds\right)$$

$$\leq \left[\ell(t)max\{|x-y|,\|x_t-y_t\|_c\}\times\left(\|\phi\|_c+a(t)\int_0^{\beta(t)} b(s)\,ds\right)\right]$$

$$+[\ell(t)max\{|y(t)|\}+F_0]2a(t)\int_0^{\beta(t)} b(s)\,ds$$

$$\leq LM(\|\phi\|_c+v(t))+[L\|y\|+F_0]2v(t)$$

$$\leq LK\epsilon+2[Lr+F_0]v(t) \qquad \text{where } K=\frac{M(\|\phi\|_c+v(t))}{\epsilon} \text{ and } M=max\{|x-y|,\|x_t-y_t\|_c\}.$$

Hence, in virtue of hypothesis (H_4), that $\exists T>0$ such that $v(t)=\frac{\epsilon}{2[Lr+F_0]}$ for $t\geq T$

Thus for $t\geq T$, from equation (4.5), we obtain that

$$|Qx(t)-Qy(t)|\leq(LK+1)\epsilon \tag{4.6}$$

Further, let us consider that $t\in[0,T.]$ Then evaluating similarly as above we get

$$|Qx(t)-Qy(t)|\leq LK\epsilon+[Lr+F_0]\left(\int_0^t|g(t,x(s),x_s)-g(t,y(s),y_s)|ds\right)$$

$$\leq LK\epsilon+[Lr+F_0]\left(\int_0^{\beta_T}|g(t,x(s),x_s)-g(t,y(s),y_s)|ds\right)$$

$$\leq LK\epsilon+\beta_T\omega_r^T(g,\epsilon) \tag{4.7}$$

Where $\beta_T = sup\{\beta(t): t \in [0,T]\}$

$$\omega_r^T(g,\epsilon) = sup\{|g(t,s,x) - g(t,s,y)|: t,s \in [0,T], x,y \in [-r,r], |x-y < \epsilon|\}.$$

In view of continuity of β, $\beta_T < \infty$, moreover from the uniform continuity of the function $g(t,s,x)$ on the set$[0,T] \times [o,T] \times [-r,r]$ we derive that $\omega_r^T(g,\epsilon) \to 0$ as $\epsilon \to 0$ and by equation (4.6),(4.7), and above factors , the operator Q maps continuously the ball $\overline{\mathcal{B}}r(0)$ into itself. Further on let us take a nonempty subset X of the ball $\overline{\mathcal{B}}r(0)$. And fixed arbitrary $T > 0, \epsilon > 0$. Let us choose $x \in X, t_1, t_2 \in [o,T]$ with $|t_1 - t_2| \le \epsilon$, with loss of generality we may assume that $t_1 < t_2$. Then taking into our assumptions, we get

$$|Qx(t_1) - Qx(t_2)| \le$$

$$\le \left| f(t_1, x(t_1), x_{t_1}) \left(\phi(0) + \int_0^{t`_1} g(t\ , x(s), x_s) ds \right) - f(t_2, x(t_2), x_{t_2}) \left(\phi(0) + \int_0^{t_2} g(t, x(s), x_s) ds \right) \right|$$

$$\le \left| f(t_1, x(t_1), x_{t_1}) \left(\phi(0) + \int_0^{t`_1} g(t\ , x(s), x_s) ds \right) - f(t_2, x(t_2), x_{t_2}) \left(\phi(0) + \int_0^{t_1} g(t, x(s), x_s) ds \right) \right|$$

$$+ \left| f\big(t_2, x(t_2), x_{t_2}\big) \left(\phi(0) + \int_0^{t_1} g(t, x(s), x_s) ds \right) - f(t_2, x(t_2), x_{t_2}) \left(\phi(0) + \int_0^{t_2} g(t, x(s), x_s) ds \right) \right|$$

$$\le \left| f\big(t_1, x(t_1), x_{t_1}\big) - f(t_2, x(t_2), x_{t_2}) \right| \left| \phi(0) + \int_0^{t_1} g(t, x(s), x_s) ds \right|$$

$$+ \left| f(t_2, x(t_2), x_{t_2}) \right| \left| \int_0^{t`_1} g(t\ , x(s), x_s) ds - \int_0^{t_2} g(t, x(s), x_s) ds \right|$$

$$\le \ell(t) max \left\{ |x(t_1) - x(t_2)|, \left\| x_{t_1} - x_{t_2} \right\|_c \right\} \big(\|\phi\|_c + v(t_1) \big)$$

$$+ (Lr + F_0) \left| \int_0^{\beta(t_1)} g(s, x(s), x_s) ds - \int_0^{\beta(t_2)} g(s, x(s), x_s) ds \right|$$

$$\le LM\big(\|\phi\|_c + v(t_1)\big) + \quad (Lr + F_0) \left| \int_0^{\beta(t_1)} g(s, x(s), x_s) ds - \int_0^{\beta(t_2)} g(s, x(s), x_s) ds \right| \tag{4.8}$$

$$\le LM\big(\|\phi\|_c + v(t_1)\big) + (Lr + F_0)|v(t_1) - v(t_2)|$$

$$\le LK\epsilon + [Lr + F_0]\omega_r^T(v,\epsilon) \tag{4.9}$$

Where we have denoted

$$\omega_r^T(v,\epsilon) = sup\{|v(t_1) - v(t_2)|: t_1, t_2 \in [0,T] |t_1 - t_2| < \in, v \in [-r,r]\}$$

From the above inequality we obtained

$$\omega^T(Q(t),\epsilon) \le LK\epsilon + [Lr + F_0]\omega_r^T(\upsilon,\epsilon) \tag{4.10}$$

Observe that
$\omega^T(g,\epsilon) \to 0 \; as \; \epsilon \to 0$ which is simple consequence of the uniform continuity of function g on $[0,T] \times [o,T] \times [-r,r]$ moreover from uniform continuity of $g, \alpha, \beta, \upsilon \; on[0,T]$, it follows that $\omega_r^T(\upsilon,\epsilon) \to 0 \; as \; \epsilon \to 0$ with the equation (4.9) we get

$$\omega_0^T\big(Q(X)\big) \le LK \in \omega_0^T(X)$$

$$i.e. \qquad \omega_0\big(Q(X)\big) \le LK \in \omega_0(X) \tag{4.11}$$

Taking into account our hypothesis, for fixed $t \in \mathbb{R} \;\; x, y \in X$, we deduce following (see 4.6)

$$|Qx(t) - Qy(t)| \le LK\epsilon + 2[Lr + F_0]\upsilon(t)$$

From the above inequality it follows that

$$diam\big(QX(t)\big) \le LK_1 diam\big(X(t)\big) + 2[Lr + F_0]\upsilon(t)$$

Where $K_1 = K\epsilon$, there fore

$$\lim_{t\to\infty} sup diam\big(QX(t)\big) \le LK_1 \lim_{t\to\infty} sup diam\big(X(t)\big) \tag{4.12}$$

Further using the measure of noncompactness μ defined by the formula (4.2),(4.3) and by estimates (4.11),(4.12), we obtain

$$\mu\big(Q(X)\big) \le LF_1\mu(X)$$

Since $LF_1 \le 1$ then theorem (2.1) yield to the operator equation $Qx = x$ and deduce that the operator Q has fixed point x in the ball $\overline{B}r(0)$. Obviously x is solution of the FIE (4.2). Moreover, taking into account that the image of the space X under the operator Q is contained in $\overline{B}r(0)$. We infer that the set $FixQ$ of all fixed points of Q is contained in $\overline{B}r(0)$. obviously the set $FixQ$ contains all solutions of the equation (4.1). Other hand we conclude that the set $FixQ$ belongs to the family$Ker\mu$ $(see \; 2.3)$

Now taking into account the description of sets belonging to $Ker\mu$ we deduce that all solutions of the FIE(4.2) are uniformly globally attractive on ℝ. This completes the proof.

5. An Example:

Given the closed and bounded intervals $I_0 = \left[-\frac{\pi}{2}, 0\right] \; and \; I = \left[0, \frac{\pi}{2}\right]$ in ℝ.

Consider FDE

$$\frac{d}{dt}\left[\frac{x(t)}{f(t,x(t),x_t)}\right] = \frac{e^{-t}x_t}{1+\|x_t\|_c} \qquad t \in I$$

$$x(t) = \sin t \qquad\qquad t \in I_0$$

(5.1)

Where $f: I \times \mathbb{R} \times C \to \mathbb{R} - \{0\}$ is defined by $f(t, x(t), x_t) = \frac{1}{2}[1 + \alpha(|x(t)| + \|x_t\|_c)]$

for $t \in I, \alpha > 0$ and define a function $g: I \times \mathbb{R} \times C \to \mathbb{R}$ by $g(t, x(t), x_t) = \frac{e^{-t} x_t}{1+\|x_t\|_c + |x(t)|}$ we get

$\alpha(t) = t$, $\beta(t) = \frac{\pi t}{2}$ for all $t \in \mathbb{R}, x \in \mathbb{R}$. We shall show that all the above functions satisfy the conditions of theorem (4.1).

Clearly the functions α, β are continuous and map $\mathbb{R}$ into itself with $\lim_{t\to\infty} \alpha(t) = \infty$, further on the function f is continuous on $I \times \mathbb{R} \times C$ and satisfied (H_2) with $= \frac{1}{2}$. Let $x, y \in \mathbb{R}$ then

$$|f(t, x(t), x\_t) - f(t, y(t), y\_t)| \leq \frac{1}{2}[\alpha(|x(t)| + \|x_t\|_c) - \alpha(|y(t)| + \|y_t\|_c)]$$

$$\leq |x - y| \quad \text{for all } t \in \mathbb{R}.$$

Finally the function g is continuous on $I \times \mathbb{R} \times C$ and

$$g(t, x(t), x_t) = \left|\frac{e^{-t} x_t}{1 + \|x_t\|_c + |x(t)|}\right|$$

$$\leq e^{-t}$$

$$\leq a(t)b(s) \quad \text{for all } s, t \in \mathbb{R}, \ x \in \mathbb{R}$$

Moreover

$$\lim_{t\to\infty} a(t) \int_0^{\beta(t)} b(s) ds = \lim_{t\to\infty} e^{-t} \int_0^{\frac{\pi}{2}t} ds = 0 \quad \text{and}$$

$$K = \sup_{t\geq 0} e^{-t} \int_0^{\frac{\pi}{2}t} ds \leq 1$$

As $= \frac{1}{2} < 1$, we apply theorem (4.1) to yield that the FDE i.e. FIE (4.2) has a solution and all solutions are uniformly globally attractive on $\mathbb{R}$.

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