INTERNATIONAL JOURNAL OF MATHEMATICAL SCIENCES, TECHNOLOGY AND HUMANITIES

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International eJournals

International Journal of Mathematical Sciences, Technology and Humanities 52 (2012) 541 – 560

α-Homeomorphisms in Topological Ordered Spaces

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ABSTRACT

In this paper we introduce I-α-homeomorphisms, D-α-homeomorphisms and B-α-homeomorphisms for topological ordered spaces after introducing I-α-continuous maps, D-α-continuous maps and B-α-continuous maps, I-α-open maps, D-α-open maps, B-α-open maps, I-α-closed maps, D-α-closed maps and B-α-closed maps for topological ordered spaces together with their characterizations.

KEYWORDS AND PHRASES. Topological ordered spaces, semi-open sets, semi-closed sets, pre-open sets, pre-closed sets, α-open sets, α-closed sets, increasing sets, decreasing sets, balanced sets, semi-continuous map, pre-continuous map, α-continuous map, semi-open map, pre-open map, α-open map, semi-closed map, pre-closed map and α-closed map.

INTRODUCTION. Leopoldo Nachbin [8] initiated the study of topological ordered spaces. A topological ordered space is a triple $(X, \tau, \leq)$, where τ is a topology on X and $\leq$ is a partial order on X. Let $(X, \tau, \leq)$ be a topological ordered space. For any $x \in X$, $[x, \rightarrow] = \{y \in X / x \leq y\}$ and $[\leftarrow, x] = \{y \in X / y \leq x\}$. A subset A of a topological ordered space $(X, \tau, \leq)$ is said to be increasing if $A = i(A)$ and is called decreasing if $A = d(A)$, where $i(A) = \bigcup_{a \in A} [a, \rightarrow]$ and $d(A) = \bigcup_{a \in A} [\leftarrow, a]$. Observe that the complement of an increasing set is a decreasing set and the complement of a decreasing set is an increasing set. A subset of a topological ordered space $(X, \tau, \leq)$ is said to be balanced if it is both increasing and decreasing. M.K.R.S. Veera Kumar [10] studied different types of maps between topological ordered spaces. O.Njastad [9] introduced α-open sets and A.S. Mashhour et al [5] introduced α-closed sets. A subset A of a topological space (X, τ) is called a semi-open set [3] if $A \subseteq cl(int(A))$ and a semi-closed set if $int(cl(A)) \subseteq A$. A subset A of a topological space (X, τ) is called a pre-open set [4] if $A \subseteq int(cl(A))$ and pre-closed if $cl(int(A)) \subseteq A$. A subset A of a topological space (X, τ) is called an α-open set [9] if $A \subseteq int(cl(int(A)))$ and α-closed set if $cl(int(cl(A))) \subseteq A$.

LEMMA 1.1: A subset A of a topological space (X, τ) is an α-closed set iff it is semi-closed and pre-closed.

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Note that the complement of an α-open set is an α-closed and vice versa. We denote the complement of A by C (A).

For a subset A of a topological ordered space (X, τ, ≤), we define

iαcl(A) = ∩{F/F is an increasing α-closed subset of X containing A },

dαcl(A) = ∩{F/F is a decreasing α-closed subset of X containing A},

bαcl(A) = ∩{F/F is a balanced α-closed subset of X containing A},

$A^{i\alpha o}$ = ∪{G/G is an increasing α-open subset of X contained in A},

$A^{d\alpha o}$ = ∪{G/G is a decreasing α-open subset of X contained in A} and

$A^{b\alpha o}$ = ∪{G/G is a balanced α-open subset of X contained in A}.

Clearly iαcl(A) (resp. dαcl (A), bαcl (A)) is the smallest increasing (resp. decreasing, balanced) α-closed set containing A. IαO(X) (resp. DαO(X), BαO(X)) denotes the collection of all increasing (resp. decreasing, balanced) α-open subsets of a topological ordered space (X, τ, ≤). IαC(X) (resp. DαC(X), BαC(X)) denotes the collection of all increasing (resp. decreasing, balanced) α-closed subsets of a topological ordered space (X, τ, ≤). A function f : (X, τ)→ (Y, σ) is called α-continuous [5] if f^{-1}(V) is an α-closed set of (X, τ) for every closed set V of (Y, σ). A function f : (X, τ) → (Y, σ) is called α-open [5] if f(G) is an α-open set in Y, for every open set G of X. A function f : (X, τ) → (Y, σ) is called α-closed [5] if f(F) is an α-closed set in Y for every closed set F of X. A function f : (X, τ) → (Y, σ) is called semi-continuous if f^{-1}(V) is a semi-open set of (X, τ) for every open set V of (Y, σ). A function f : (X, τ) → (Y, σ) is called pre-continuous if f^{-1}(V) is a pre-closed set of (X, τ) for every closed set V of (Y, σ). A function f : (X, τ) → (Y,σ) is called semi-open map if f(G) is a semi-open is set in (Y, σ) for every open set G of (X, τ). A function f : (X, τ) → (Y,σ) is called pre-open map if f(G) is a pre-open set in (Y, σ) for every open set G of (X, τ). A function f : (X, τ) → (Y,σ) is called semi-closed map if f(G) is a semi-closed set in (Y, σ) for every closed set G of (X, τ). A function f : (X, τ) → (Y,σ) is called pre-closed map if f(G) is a pre-closed set in (Y, σ) for every closed set G of (X, τ). A function f : (X, τ) → (Y,σ) is called an α-closed map if f(G) is an α-closed set in (Y, σ) , for every closed set G of (X, τ). A function f : (X, τ, ≤) → (X*, τ^*, ≤*) is called I-semi-continuous[1] (resp. D-semi-continuous, B-semi-continuous) map if f^{-1}(G)∈IPO(X) (resp f^{-1}(G)∈DPO(X), f^{-1}(G)∈BPO (X)) whenever G is an open set of (X*, τ^*). A function f : (X, τ, ≤) → (X*, τ^*, ≤*) is called I-pre-continuous[2] (resp. D-pre-continuous, B-pre-continuous) map if f^{-1}(G)∈IPC(X) (resp f^{-1}(G)∈DPC(X), f^{-1}(G)∈BPC (X)) whenever G is an open set of (X*, τ^*). A function f : (X, τ, ≤)→(X*, τ*, ≤***)** is called an I-semi-open (resp.a D-semi-open, B-semi-open) map [2] if f(G)∈ISO(X*) (resp f(G)∈DSO(X*), f(G)∈BSO(X*) whenever G is an open subset of (X, τ). A function f : (X, τ, ≤)→(X*, τ*, ≤***)** is called an I-pre-open (resp. a D-pre-open, B-pre-open) map [2] if f(G)∈IPO(X*) (resp f(G)∈DPO(X*), f(G)∈BPO(X*) whenever G is an open subset of (X, τ). A function f : (X, τ, ≤) → (X*, τ*, ≤*) is called an I-semi-closed (resp a D-semi-closed, B-semi-closed) map [1] if f(G)∈ISC(X*) (resp f(G)∈DSC(X*), f(G)∈BSC(X*)) whenever G is a closed subset of X. A function f : (X, τ, ≤) → (X*, τ*, ≤*) is called an I-pre-closed (resp a D-pre-closed, a B-pre-closed) map [2] if f(G) ∈ IPC(X*) (resp f(G) ∈ DPC(X*), f(G) ∈ BPC(X*)) whenever G is a closed subset of X, A bijection f : (X, τ, ≤) → (X*, τ*, ≤*) is called an I-semi-homeomorphism [1] (resp a D-semi-homeomorphism,a B-semi-homeomorphism) if both f and f^{-1} are I-semi-continuous (resp. D-semi-continuous and B-semi-continuous). A bijection f : (X, τ, ≤) → (X*,τ*, ≤*) is called an I-pre-homeomorphism [2] (resp a D-pre-homeomorphism, a B-pre-homeomorphism) if both f and f^{-1} are I-pre-continuous (resp. D-pre-continuous and B-pre-continuous).

Authors studied Semi-homeomorphisms in Topological Ordered Spaces [1] and Pre-homeomorphisms in Topological Ordered Spaces [2].

1. I-α-CONTINUOUS, D-α-CONTINUOUS AND B-α-CONTINUOUS MAPS

We introduce the following definition.

DEFINITION 2.01. A function f : (X, τ, ≤) → (X*, τ*, ≤*) is called an I-α-continuous (resp. D-α continuous, B-α-continuous) map if f^{-1}(V) ∈ IαC(X), (resp. f^{-1}(V) ∈ DαC(X), f^{-1}(V) ∈ BαC(X)) whenever V is closed in X.

It is evident that every x-α-continuous map is an α-continuous for x = I, D, B and that every B-α-continuous map is both I-α-continuous and D-α-continuous.

The following example shows that an α-continuous map need not be a x-α-continuous for x=I, D, B.

EXAMPLE 2.01. Let X = {a, b, c} = X*, τ = {ϕ, X, {a}, {b}, {a, b}} = τ* and ≤ = {(a, a), (b, b), (c, c), (a, b), (b, c), (a, c)}. Clearly (X, τ, ≤) is a topological ordered space. Let f be the identity map from (X, τ, ≤) onto itself. {a, c} is closed set in X, but f^{-1} ({a, c}) = {a, c} is neither an increasing nor a decreasing α-closed set. Thus f is not x-α-continuous for x = I, D, B. Clearly f is an α-continuous.

The following example shows that a D-α-continuous map need not be a B-α-continuous.

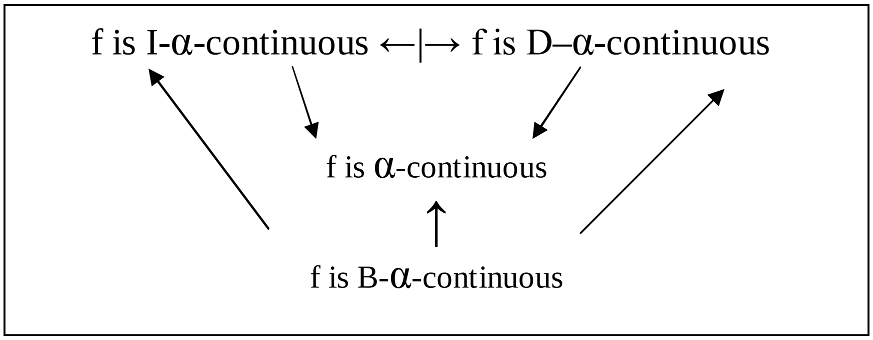
EXAMPLE 2.02. Let X = {a, b, c} = X*, τ = {ϕ, X, {a}, {a, b}} = τ* and ≤ = {(a, a), (b, b), (c,c), (b, a)}, and ≤* = {(a, b), (b, b), (c, c)}. Let g be the identity map from (X, τ, ≤) onto (X*,τ*,≤*). Then g is not a B-α-continuous map, however g is a D-α-continuous map.

The following example supports that an I-α-continuous map need not be a B-α-continuous map.

EXAMPLE 2.03. Let X={a, b, c} = X*,τ = {ϕ, X, {a}, {b}, {a, b}} = τ* and ≤ = {(a, a), (b, b), (c, c), (a, c), (b, c)} = ≤*. Let f : (X, τ, ≤) → (X*, τ*, ≤*) be the identity map. Then f is an I-α-continuous but not a B-α-continuous map.

2.01 Thus we have the following diagram.

For a function f : (X, τ, ≤) →(X*, τ*, ≤*)



,where p → q (resp. p ←|→ q) represents p implies q but q need not imply p (resp. p and q are independent of each other)
The following theorem characterizes I-α-continuous maps.

THEOREM 2.01. For a function f : (X, τ, ≤) → (X*, τ*, ≤*), the following statements are equivalent.

1) f is I-α-continuous.
2) f (iαcl(A)) ⊆ cl(f (A)) for any A ⊆ X.
3) iαcl(f^{-1} (B)) ⊆ f^{-1}(cl(B)) for any B ⊆ X*.
4) For any closed subset K of (X*, τ*, ≤*), f^{-1}(K) is an increasing α-closed subset of (X, τ, ≤).

Proof. (1) => (2) Since cl(f(A)) is closed in X* and f is I-α-continuous we have that f^{-1}(cl(f(A))) is an increasing α-closed set in X. f (A) ⊆ cl(f(A)) => A ⊆ f^{-1}(cl(f(A))) and iαcl(A) is the smallest increasing α-closed set containing A. Therefore iαcl(A) ⊆ f^{-1}(cl(f(A))) and hence f(iαcl(A)) ⊆ cl(f(A))).
(2) =>(3) Put A = f^{-1}(B). Then f(A) ⊆ B and cl(f(A)) ⊆ cl(B). Therefore iαcl (f^{-1}(B)) ⊆ f^{-1}(cl(B)).
(3) => (4) Let K be any closed set in X*. From (3) iαcl(f^{-1}(K)) ⊆ f^{-1}(cl(K)) = f^{-1}(K). But f^{-1}(K) ⊆ iαcl(f^{-1}(K)). Thus f^{-1}(K) is an increasing α-closed set in (X, τ, ≤) whenever K is closed subset in (X*, τ*, ≤*).
(4) => (1) follows from definition.

THEOREM 2.02. For a function f : (X, τ, ≤) →(X*, τ*, ≤*), the following statements are equivalent.

1) f is D-α-continuous.
2) f (dαcl(A)) ⊆ cl(f(A)) for any A ⊆ X.
3) dαcl(f^{-1}(B)) ⊆ f^{-1}(cl(B)) for any B ⊆ X*.
4) For every closed subset K of (X*, τ* , ≤*), f^{-1}(K) is a decreasing α-closed subset of (X, τ, ≤).

THEOREM 2.03. For a function f : (X, τ, ≤) →(X*, τ,* ≤*), the following statements are equivalent.

1) f is B-α-continuous.
2) f(bαcl(A)) ⊆ cl (f(A)) for any A ⊆ X.

3) bαcl(f^{-1}(B)) ⊆ f^{-1}(cl(B)) for any B ⊆ X*.

4) For every closed subset K of (X*, τ*, ≤*), f^{-1}(K) is a balanced α-closed subset of (X, τ, ≤).

THEOREM 2.04. Let (X, τ, ≤) and (X*, τ*, ≤*) be two topological ordered spaces. Let f : (X, τ, ≤) → (X*, τ*, ≤*) be a map. Then f is I-α-continuous iff it is I-semi-continuous and I-pre-continuous.

Proof. Follows from Lemma 1.1.

The following example shows that an I-semi-continuous map need not be an I-**α**-continuous.

EXAMPLE 2.04. Let X ={a, b, c}= X*, τ = {ϕ, X, {a}, {b}, {a, b}} and τ* = {ϕ, X*, {a}}. Let ≤ = {(a, a), (b, b), (c, c), (a, b), (c, b)} = ≤*. Define f : (X, τ, ≤) → (X*, τ*, ≤*) by f(a) = b, f(b) = a and f(c) = c. Then f is an I-semi-continuous but not an I-α-continuous.

The following example shows that an I-pre-continuous map need not be an I-α-continuous.

EXAMPLE 2.05. Let X = {a, b, c} = X*, τ = {ϕ, X, {a, b}}, τ* = {ϕ, X, {a}, {b}, {a, b}, {b, c}} and ≤ = {(a, a), (b, b), (c, c)} = ≤*. Define f : (X, τ, ≤) → (X*, τ*, ≤*) by f(a) = b, f(b) = a and f(c) = c. Then f is an I-pre-continuous map but not an I-α-continuous map.

THEOREM 2.05. Let (X, τ, ≤) and (X*, τ*, ≤*) be two topological ordered spaces. Let f : (X, τ, ≤) → (X*, τ*, ≤*) be a map. Then f is D-α-continuous iff it is D-semi-continuous and D-pre-continuous.

Proof. Follows from Lemma 1.1.

The following example shows a D-semi-continuous map need not be a D-α-continuous map.

EXAMPLE 2.06 : Let X = {a, b, c} = X*, τ = {ϕ, X, {a}, {b}, {a, b}}, τ* = {ϕ, X, {a}} and ≤ = {(a, a), (b, b), (c, c), (a, b), (b, c), (a, c)} = ≤* . Let f be the identity map from (X, τ, ≤) onto (X*, τ*, ≤*). Then f is a D-semi-continuous map, but not a D-α-continuous map.

The following example shows that a D-pre-continuous map need not be a D-α-continuous.

EXAMPLE 2.07. Let X = {a, b, c}=X*, τ = {ϕ, X, {a, b}}, τ* =. {ϕ, X, {a}, {b}, {a, b}, {b, c}}, and ≤ = {(a, a), (b, b), (c, c)} = ≤*. Define a map f : (X, τ, ≤) → (X*, τ*, ≤*) by f(a) = b, f(b) = a and f(c) = c. Then f is a D-pre-continuous map but not a D-α-continuous map..

THEOREM 2.06. Let (X, τ, ≤) and (X*, τ*, ≤*) be two topological ordered spaces. Let f : (X, τ, ≤) → (X*, τ*, ≤*) be a map. Then f is B-α-continuous iff it is B-semi-continuous and B-pre-continuous.

Proof. Follows from Lemma 1.1.

The following example shows that a B-semi-continuous map need not be a B-α-continuous map.

EXAMPLE 2.08. Let $X = \{a, b, c\} = X^*$, $\tau = \{\phi, X, \{a\}, \{a, b\}\}$ and $\tau^* = \{\phi, X, \{a, b\}\}$ and $\leq = \{(a, a), (b, b), (c, c)\} = \leq^*$. Define $f : (X, \tau, \leq) \to (X^*, \tau^*, \leq^*)$ by $f(a) = c$, $f(b) = a$ and $f(c) = b$. Then f is a B-semi-continuous but not a B-α-continuous.

The following example shows that a B-pre-continuous map need not be a B-α-continuous.

EXAMPLE 2.09. Let $X = \{a, b, c\} = X^*$, $\tau = \{\phi, X, \{a, b\}\}$ and $\tau^* = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$ and $\leq = \{(a, a), (b, b), (c, c)\} = \leq^*$. Define a map $f : (X, \tau, \leq) \to (X^*, \tau^*, \leq^*)$ by $f(a) = b$, $f(b) = a$ and $f(c) = c$. Then f is a B-pre-continuous map but not a B-α-continuous map.

3. I-α-OPEN, D-α-OPEN AND B-α-OPEN MAPS.

DEFINITION 3.01. A function $f : (X, \tau, \leq) \to (X^*, \tau^*, \leq^*)$ is called an I-α-open map (resp. D-α-open, B-α-open map if $f(G) \in I\alpha O(X^*)$ (resp. $f(G) \in D\alpha O(X^*)$, $f(G) \in B\alpha O(X^*)$) whenever G is an open subset of $(X, \tau, \leq)$.

It is evident that every x-α-open map is an α-open map for x= I, D, B and that every B-α-open map is both I-α-open and D-α-open.

The following example shows that an α-open map need not be x-α-open for x= I, D, B.

EXAMPLE 3.01. Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\} = \tau^*$ and $\leq = \{(a, a), (b, b), (c, c), (a, b), (b, c), (a, c)\}$. Clearly $(X, \tau, \leq)$ is a topological ordered space. Let f be the identity map from $(X, \tau, \leq)$ onto itself. {b}is an open set in X but $f(\{b\}) = \{b\}$ is not an I-α-open set, not a D-α-open set, not a B-α-open set. Thus f is not x-α-open map for x = I,D,B. Clearly f is an α-open map.

The following example shows that a D-α-open map need not be a B-α-open map.

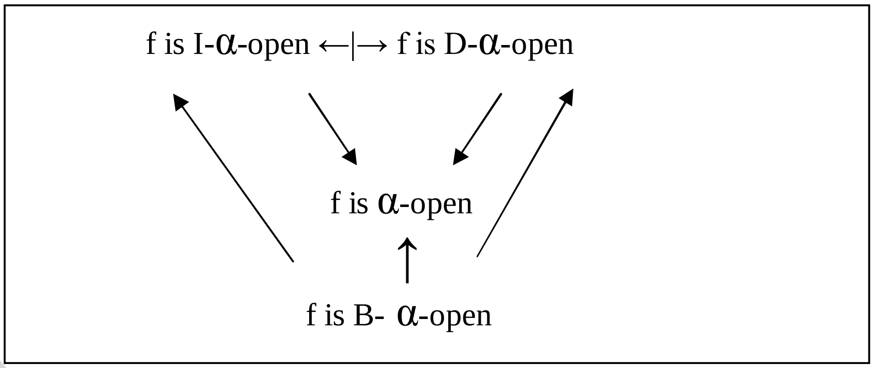
EXAMPLE 3.02. Let $X = \{a, b, c\} = X^*$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\} = \tau^*$, $\leq = \{(a, a), (b, b), (c, c), (a, c)\}$ and $\leq^* = \{(a, a), (b, b), (c, c), (a, c), (b, c)\}$. Let θ be the identity map from $(X, \tau, \leq)$ onto $(X^*, \tau^*, \leq^*)$. θ is a D-α-open map but not a B-α-open map.

The following example shows that an I-α-open map need not be a B-α-open map.

EXAMPLE 3.03. Let $X = \{a, b, c\} = X^*$, $\tau = \{\phi, X, \{a\}, \{a, c\}\} = \tau^*$ and $\leq = \{(a, a), (b, b), (c, c), (c, a), (b, c), (b, a)\} = \leq^*$. Let $f : (X, \tau, \leq) \to (X^*, \tau^*, \leq^*)$ be the identity map. Then f is an I-α-open map but not a B-α-open map.

3.01 Thus we have the following diagram

For a function $f : (X, \tau, \leq) \to (X^*, \tau^*, \leq^*)$



,where p→ q (resp.p ←|→ q) represents p implies q but q need not imply p (resp. p and q are independent of each other).

LEMMA 3.01. Let A be any subset of a topological ordered space $(X, \tau, \leq)$. Then

1) $C(d\alpha cl(A)) = (C(A))^{i\alpha o}$.
2) $C(i\alpha cl(A)) = (C(A))^{d\alpha o}$.
3) $C(b\alpha cl(A)) = (C(A))^{b\alpha o}$.

Proof. $C(d\alpha cl(A)) = C\cap\{F/F$ is a decreasing α-closed subset of X containing A$\}$
$= \cup\{C(F)/F$ is a decreasing α-closed subset of X containing A$\}$
$= \cup\{G/G$ is an increasing α-open subset of X contained in C(A)$\}$
$= (C(A))^{i\alpha o}$.

Proofs of (2) and (3) are analogous to as that of (1) and hence omitted.

The following theorem characterizes I-α-open functions.

THEOREM 3.01. For any function $f : (X, \tau, \leq) \to (X^*, \tau^*, \leq^*)$, the following statements are equivalent.

1) f is an I-α-open map.
2) $f(A^0) \subseteq [f(A)]^{i\alpha o}$ for any $A \subseteq X$.
3) $[f^{-1}(B)]^0 = f^{-1}(B^{i\alpha o})$ for any $B \subseteq X^*$.

Proof. (1) => (3). Let $B \subseteq X^*$. Since $[f^{-1}(B)]^0$ is open in X, f is an I-α-open, $f((f^{-1}(B))^0) \subseteq f(f^{-1}(B)) \subseteq B$ and $f([f^{-1}(^1(B)]^0)$ is I-α-open in X*.Then $f((f^{-1}(B))^0) \subseteq B^{i\alpha o}$ since $B^{i\alpha o}$ is the largest increasing α-open set contained in B. Therefore $[f^{-1}(B)]^0 \subseteq f^{-1}(B^{i\alpha o})$.

(3) => (2). Replacing B by f(A) in (3), we have $[f^{-1}(f(A))]^0 \subseteq f^{-1}([f(A)]^{i\alpha o})$.Since $A^0 \subseteq [f^{-1}(f(A))]^0$, we have $A^0 \subseteq f^{-1}([f(A)]^{i\alpha o})$. $f(A^0) \subseteq f(f^{-1}([f(A)]^{i\alpha o})) \subseteq [f(A)]^{i\alpha o}$. Hence $f(A^0) \subseteq [f(A)]^{i\alpha o}$.

(2) => (1). Let G be any open set in X. Then $f(G) = f(G^0) \subseteq [f(G)]^{i\alpha o} \subseteq f(G)$. Therefore f(G) is an increasing α-open set in X*. So f is an I-α-open map.

The following two theorems give characterizations for D-α-open map and B-α-open maps, whose proofs are similar to as that of the above theorem.

THEOREM 3.02. For any function $f : (X, \tau, \leq) \rightarrow (X^*, \tau^*, \leq^*)$, the following statements are equivalent.

1) f is a D-α-open map.
2) $f(A^0) \subseteq [f(A)]^{d\alpha o}$ for any $A \subseteq X$.
3) $[f^{-1}(B)]^0 \subseteq f^{-1}(B^{d\alpha o})$ for any $B \subseteq X^*$.

THEOREM 3.03. For any function $f : (X, \tau, \leq) \rightarrow (X^*, \tau^*, \leq^*)$, the following statements are equivalent.

1) f is a B-α-open map.
2) $[f(A^0] \subseteq [f(A)]^{b\alpha o}$ for any $A \subseteq X$.
3) $[f^{-1}(B)]^0 \subseteq f^{-1}(B^{b\alpha o})$ for any $B \subseteq X^*$.

THEOREM 3.04. Let $f : (X, \tau, \leq_1) \rightarrow (Y, \sigma, \leq_2)$ and $g: (Y, \sigma, \leq_2) \rightarrow (Z, \eta, \leq_3)$ be any two mappings. Then $gof : (X, \tau, \leq_1) \rightarrow (Z, \eta, \leq_3)$ is x-α-open if f is open and g is x-α-open for x = I,D,B.

Proof. Omitted.

THEOREM 3.05 : Let $(X, \tau, \leq)$ and $(X^*, \tau^*, \leq^*)$ be two topological ordered spaces. Let $f : (X, \tau, \leq) \rightarrow (X^*, \tau^*, \leq^*)$ be a map. Then f is I-α-open map iff it is I-semi-open map and I-pre-open map.

Proof. Follows from Lemma 1.1.

The following example shows that an I-semi-open map need not be an I-α-open map.

EXAMPLE 3.04. Let X = {a, b, c} = X*, τ = {ϕ, X, {a}}, τ*= {ϕ, X, {a}, {b}, {a, b}} and ≤ = {(a, a), (b, b), (c, c), (a, b), (c, b)} = ≤* Define $f : (X, \tau, \leq) \rightarrow (X^*, \tau^*, \leq^*)$ by f(a) = b , f(b) = c and f(c) = c. Then f is an I-semi-open map but not an I-α-open map.

The following example shows that an I-pre-open map need not be an I-α-open map.

EXAMPLE 3.05. Let X = {a, b, c} = X*, τ = {ϕ, X, {a}, {b}, {a, b}}, τ*= {ϕ, X, {a}, {b, c}} and ≤ = {(a, a), (b, b), (c, c)} = ≤*. Let f be the identity map from (X, τ, ≤) onto (X*, τ*, ≤*). Then f is an I- pre-open map but not an I-α-open map.

THEOREM 3.06 . Let $(X, \tau, \leq)$ and $(X^*, \tau^*, \leq^*)$ be two topological ordered spaces.

Let f : (X, τ, ≤) → (X*, τ^*, $\leq^*$) be a map. Then f is a D-α-open map iff it is a D-semi-open map and a D-pre-open map.

Proof. Follows from Lemma 1.1.

The following example shows that a D-semi-open map need not be a D-α-open map.

EXAMPLE 3.06. Let X = {a, b, c} = X*, τ = {ϕ, X, {a}}, τ*= {ϕ, X, {a}, {b}, {a, b}} and ≤ = {(a, a), (b, b), (c, c), (b, c), (c, a), (b, a)} = ≤*. Define f : (X, τ, ≤) → (X*, τ*, ≤*) by f(a) = b, f(b) = c and f(c) = c. Then f is a D-semi-open map but not a D-α-open map.

The following example shows that a D-pre-open map need not be a D-α-open map.

EXAMPLE 3.07. Let X = {a, b, c} = X*, τ = {ϕ, X, {a}, {b}, {a, b}}, τ*= {ϕ, X, {a}, {b, c}} and ≤ = {(a, a), (b, b), (c, c)} = ≤*. Let f be the identity map from (X, τ, ≤) onto **(**X*, τ***, ≤*).** Then f is a D-pre-open map but not a D-α-open map.

THEOREM 3.07. Let (X, τ, ≤) and (X*, τ^*, $\leq^*$) be two topological ordered spaces.
Let f : (X, τ, ≤) → (X*, τ^*, $\leq^*$) be a map. Then f is a B-α-open map iff it is a B-semi-open map and a B-pre-open map.

Proof. Follows from Lemma 1.1.

The following example shows that a B-semi-open map need not be a B-α-open map.

EXAMPLE 3.08. Let X = {a, b, c} = X*, τ = {ϕ, X, {a, b}}, τ*= {ϕ, X, {a}} and ≤ = {(a, a), (b, b), (c, c)} = ≤*. Let f : (X, τ, ≤) → (X*, τ*, ≤*) be the identity map. Then f is a B-semi-open map but not a B-α-open map.

The following example shows that a B-pre-open map need not be a B-α-open map.

EXAMPLE 3.09. Let X = {a, b, c} = X*, τ = {ϕ, X, {a}, {b}, {a, b}}, τ*= {ϕ, X, {a},{b, c}} and ≤ = {(a, a), (b, b), (c, c)} = ≤*. Let f be the identity map from (X, τ, ≤) onto (X*, τ*, ≤*). Then f is a B-pre-open map but not an B-α-open map.

4. α-CLOSED, D-α-CLOSED AND B-α-CLOSED MAPS

DEFINITION 4.01. A function f : (X, τ, ≤) → (X*, τ*, ≤*) is called an I-α-closed (resp. D-α-closed, B-α-closed) map if f(G) ∈ IαC(X*) (resp. f(G) ∈ DαC(X*), f(G) ∈ BαC(X*)) whenever G is a closed subset of X. Clearly every x-α-closed map is a α-closed map for x = I,D,B and every B-α-closed map is both I-α-closed and D-α-closed map.

The following example shows that an α-closed map need not be a x-α -closed map, for x = I,D,B

EXAMPLE 4.01. Let X = {a, b, c} = X*, τ ={ϕ, X, {a}, {b}, {a, b}} = τ^* and ≤ = {(a, a), (b, b), (c, c), (a, b), (b, c), (a, c) } = ≤* . Clearly (X, τ, ≤) is a topological ordered space. Let f be the identity map from (X, τ, ≤) onto itself. {a,c} is closed set, but f({a, c}) is neither an increasing nor a decreasing α-closed set. Thus f is not x-α-closed map for x = I, D, B. Clearly f is α-closed map.

The following example shows that an I-α-closed map need not be a B-α-closed map.

EXAMPLE 4.02. Let X = {a, b, c} = X*, τ = {ϕ, X, {a}, {b}, {a, b}} = τ *, ≤ = {(a, a), (b, b), (c, c), (a, c)} and ≤* = {(a, a), (b, b), (c, c), (a, c), (b, c)}. Let θ be the identity map from (X, τ, ≤) onto (X*, τ*, ≤*). θ is I-α-closed map but not a B-α-closed map.

The following example shows that an D-α-closed map need not be a B-α-closed map.

EXAMPLE 4.03. Let X = {a, b, c} = X*, τ = {ϕ, X, {c}, {b, c}} = τ*, ≤ = {(a, a), (b, b), (c, c), (a, b), (b, c), (a, c) } = ≤ * and f : (X, τ, ≤) → (X*, τ*, ≤*) be the identity map. Then f is D-α-closed map but not a B-α-closed map.

4.01 Thus we have the following diagram

For a function f : (X, τ, ≤) → (X*, τ*, ≤*)

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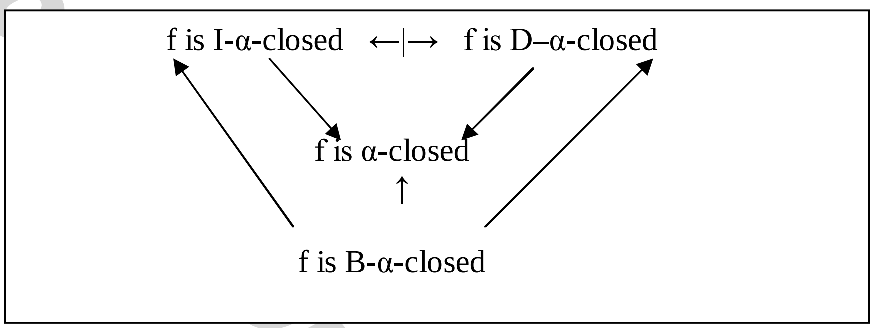


,where p → q (resp. p ←|→ q) represents p => q, but q need not imply p (p and q are independent of each other)

The following theorem characterizes I-α-closed maps.

THEOREM 4.01. Let f : (X, τ, ≤) → (X*, τ*, ≤*) be any map. Then f is I-α-closed iff iαcl (f(A)) ⊆ f(cl(A)) for any A ⊆ X.

Proof. Necessity: Since f is an I-α-closed, f(cl(A)) is an increasing α-closed subset of X. Clearly f(A) ⊆ f(cl(A)). Therefore iαcl(f(A)) ⊆ f (cl(A)) since iαcl(f(A)) is the smallest increasing α-closed set in X* containing f(A).

Sufficiency: Let F be any α-closed subset of X. Then f(F) ⊆ iαcl(f(F)) ⊆ f(cl(F)) = f(F). Thus f(F) = iαcl(F). So f(F) is an increasing α-closed subset of X*.Therefore f is an I-α-closed map.

The following two theorems characterize D-α-closed maps and B-α-closed maps.

THEOREM 4.02. Let f : (X, τ, $\leq$) → (X*, τ*, $\leq$*) be any map. Then f is D-α-closed iff dαcl (A) $\subseteq$ f(cl(A)) for every A $\subseteq$ X.

Proof. Omitted.

THEOREM 4.03. Let f : (X, τ, $\leq$) → (X*, τ*, $\leq$*) be any map. Then f is B-α-closed iff bαcl (A) $\subseteq$ f(cl(A)) for every A $\subseteq$ X.

Proof. Omitted.

THEOREM 4.04. Let f : (X, τ, $\leq$) → (X*, τ*, $\leq$*) be a bijection. Then

1) f is an I-α-open map iff f is a D-α -closed map.
2) f is an I-α-closed map iff f is a D-α -open map.
3) f is a B-α-open map iff f is a B-α-closed map.

Proof.(1) Necessity. Let F be any closed subset of X. Then f(C(F)) is an increasing α-open subset of X*. Since f(C(F)) = C(f(F)) and C(f(F)) is an increasing α-open subset of X*, f(F) is a decreasing α-closed subset of X*. Therefore f is a D-α-closed map
Sufficiency: Let G be any open subset of X. Then f (C(G)) is a decreasing α-closed subset of X*. Since f is a bijection, we have f(C(G)) = C(f(G)). So f(G) is an increasing α-open subset of X*. Therefore f is an I-α-open map.
Proofs of (2) and (3) are similar to that of (1).

THEOREM 4.05. Let f : (X, τ, $\leq_1$) →(Y, σ, $\leq_2$) and g:(Y,σ, $\leq_2$) → (Z, η, $\leq_3$) be any two mappings then gof : (X, τ, $\leq_1$) →(Z, η, $\leq_3$) is x-α -closed map if f is closed and g is x-α -closed map for x=I,D,B.

Proof. Omitted

THEOREM 4.06. Let f : (X, τ, $\leq$) → (X*, τ*, $\leq$*) be a bijection. Then the following statements are equivalent.

1) f is an I-α-open map.
2) f is a D-α-closed map.
3) f^{-1} is a D-α-continuous map.

Proof. (1) =>(2). Let f be an I-α-open map. Let F be a closed set of X, then C(F) is an open set. f(C(F)) is an increasing α-open set of X*. =>C(f(F)) is an increasing α-open set of X*. =>f(F) is a decreasing α-closed set of X*. =>f is D-α-closed map.
(2)=>(3). Let f be a D-α-closed map. Let F be a closed set in X, then f(F) is a decreasing α-closed set of X*.

=>[f^{-1}]$^{-1}$ (F) is a decreasing α-closed set of X*. => f^{-1} : X*→ X is D-α -continuous map.

(3)=>(1) Let F be an open set in X. Then C(F) is a closed set in X. => [f^{-1}]$^{-1}$(C(F)) is a decreasing closed subset of X* => C(f(F)) is a decreasing closed set in X*. =>f(F) is an increasing open set in X*. =>f is an I-α-open map.

THEOREM 4.07. Let f : (X, τ, ≤) → (X*, τ*, ≤*) be a bijection. Then the following are equivalent.

1) f is a D-α-open map.
2) f is an I-α-closed map.
3) f^{-1} is a D-α-continuous map.

Proof. Omitted.

THEOREM 4.08. Let f : (X, τ, ≤) → (X*, τ*, ≤*) be a bijection. Then the following statements are equivalent.

1) f is a B-α-open map.
2) f is a B-α-closed map.
3) f^{-1} is a B-α-continuous map.

Proof. Omitted.

THEOREM 4.09. Let f : (X, τ, ≤) → (X*, τ*, ≤*) be an I-α-closed map and B, C ⊆ X*. Then.

1) If U is an open neighborhood of f^{-1}(B), then there exists a decreasing α-open neighborhood V of B such that f^{-1}(B) ⊆ f^{-1}(V) ⊆ U.
2) If f^{-1}(B) and f^{-1}(C) have disjoint neighborhoods, then B and C have disjoint α-open neighborhoods.

Proof.1: Let U be an open neighborhood of f^{-1}(B). Take C(V) = f(C(U)). Since f is an I-α-closed map and C(V) is closed, then C(V) = f(C(U)) is an increasing α-closed subset of X. Since f^{-1} (B) ⊆ U , then C(V) = f(C(U)) ⊆ f(f^{-1}(C(U))) ⊆ C(B). Therefore B ⊆ V. Thus V is a decreasing α-open neighborhood B. => f^{-1}(B) ⊆ f^{-1}(V). Further C(U) ⊆ f^{-1}(f(C(U))) = f^{-1}(C(V)) = C(f^{-1}(V)) => f^{-1}(V) ⊆ U. Thus f^{-1}(B) ⊆ f^{-1}(V) ⊆ U.

Proof.2: Let U_B, U_C be disjoint open neighborhoods of f^{-1}(B), f^{-1}(C), where B, C ⊆ X* From (1) there exists V_B, V_C such that B ⊆ V_B, C ⊆ V_C. Also f^{-1}(B) ⊆ f^{-1}(V_B) ⊆ U_B, f^{-1}(C) ⊆ f^{-1}(V_C) ⊆U_C where V_B, V_C are decreasing closed neighborhoods of B and C respectively. Since U_B∩U_C = ϕ;
f^{-1}(V_B) ∩ f^{-1}(V_C) = ϕ. => V_B ∩ V_C = ϕ.

Similarly we have the following two theorems (proofs are omitted) regarding D-α-closed maps and B-α-closed maps.

THEOREM 4.10. Let f : (X, τ, ≤) → (X*, τ*, ≤*) be a D-α-closed map and B, C, ⊆, X*. Then

1. If U is an open neighborhood of f^{-1} (B), then there exists an increasing α-open neighborhood V of B such that f^{-1} (B) ⊆ f^{-1}(V) ⊆ U
2. If f^{-1}(B) and f^{-1}(C) have disjoint neighborhoods, then f^{-1}(B) and f^{-1}(C) have disjoint increasing α-open neighborhoods.

THEOREM 4.11. Let f : (X, τ, ≤) → (X*, τ*, ≤*) be a B-α-closed map and B, C, ⊆ X*. Then

1. If U is an open neighborhood of f^{-1}(B), then there exists an α-open neighborhood V of B, which is balanced such that f^{-1}(B) ⊆ f^{-1}(V) ⊆ U
2. If f^{-1}(B) and f^{-1}(C) have disjoint neighborhoods, then f^{-1}(B) and f^{-1}(C) have disjoint α-open neighborhoods, which are balanced.

THEOREM 4.12. Let (X, τ, ≤) and (X*, τ^*, $\leq^*$) be two topological ordered spaces. Let f : (X, τ, ≤) → (X*, τ^*, $\leq^*$) be a map. Then f is an I-α-closed map iff it is an I-semi-closed map and I-pre-closed map.

Proof. Follows from Lemma 1.1.

The following example shows that an I-semi-closed map need not be an I-α-closed map.

EXAMPLE 4.04. Let X = {a, b, c} = X*, τ = {ϕ, X, {a}}, τ*= {ϕ, X, {a}, {b}, {a, b}} and ≤ = {(a, a), (b, b), (c, c), (c, a), (b, c), (b, a) } = ≤*. Define a map f : (X, τ, ≤) → (X*, τ*, ≤*) by f(a) = c, f(b) = a and f(c) = a. Then f is an I-semi-closed map, but not an I-α-closed map.

The following example shows that an I-pre-closed map need not be an I-α-closed map.

EXAMPLE 4.05. Let X ={a, b, c} = X*, τ = {ϕ, X, {a}, {b}, {a, b}}, τ*= {ϕ, X, {a}, {b, c}} and ≤ = {(a, a), (b, b), (c, c)} = ≤*. Define a map f : (X, τ, ≤) → (X*, τ*, ≤*) by f(a) = c, f(b) = b and f(c) = a. Then f is an I-pre-closed map but not an I-α-closed map.

THEOREM 4.13. Let (X, τ, ≤) and (X*, τ^*, $\leq^*$) be two topological ordered spaces. Let f : (X, τ, ≤) → (X*, τ^*, $\leq^*$) be a map. Then f is a D-α-closed map iff it is a D-semi-closed map and a D-pre-closed map.

Proof. Follows from Lemma 1.1.

The following example shows that a D-semi-closed map need not be a D-α-closed map.

EXAMPLE 4.06. Let X ={a, b, c} = X*, τ = {ϕ , X, {a}}, τ*= {ϕ, X, {a}, {b}, {a, b}} and ≤ = {(a, a), (b, b), (c, c), (a, b), (c, b)} = ≤*. Define f : (X, τ, ≤) → **(**X*, **τ*, ≤*)** by f(a) = c, f(b) = a and f(c) = a. Then f is a D-semi-closed map, but not a D-α-closed map.

The following example shows that a D-pre-closed map need not be a D-α-closed map.

EXAMPLE 4.07. Let X ={a, b, c} = X*, τ = {ϕ, X, {a}, {b}, {a, b}}, τ*= {ϕ, X, {a}, {b, c}} and ≤ = {(a, a), (b, b), (c, c)} = ≤*. Define a map f : (X, τ, ≤) → (X*, τ*, ≤*) by f(a) = c, f(b) = b and f(c) = a. Then f is a D-pre-closed map but not a D-α-closed map.

THEOREM 4.14. Let (X, τ, ≤) and (X*, τ^***,** $\leq^*$) be two topological ordered spaces. Let f : (X, τ, ≤) → (X*, τ^***,** $\leq^*$) be a map. Then f is a B-α-closed map iff it is a B-semi-closed map and a B-pre-closed map.

Proof. Follows from Lemma 1.1.

The following example shows that a B-semi-closed map need not be a B-α-closed map.

EXAMPLE 4.08. Let X = {a, b, c} = X*, τ = {ϕ, X, {a, b}}, τ*= {ϕ, X, {a}, {b}, {a, b}} and ≤ = {(a, a), (b, b), (c, c)} = ≤*. Define f : (X, τ, ≤) → **(**X*, τ***, ≤*)** by f(a) = c, f(b) = b and f(c) = a. Then f is a B-semi-closed map but not a B-α-closed map.

The following example shows that a B-pre-closed map need not be a B-α-closed map.

EXAMPLE 4.09. Let X ={a, b, c} = X*, τ = {ϕ,X,{a},{b},{a, b}}, τ*= {ϕ,X,{a}{b, c}} and ≤ = {(a, a),(b, b),(c, c)} = ≤*. Define a map f : (X, τ, ≤) → (X*, τ*, ≤*) by f(a)=c, f(b)=b and f(c)=a. Then f is a B-pre-closed map but not a B-α-closed map.

5. I-α-HOMEOMORPHISMS, D-α-HOMEOMORPHISMS AND B-α-HOMEOMORPHISMS

We introduce the following definition

DEFINITION 5.01. A bijection f : (X, τ, ≤) → (X*, τ*, ≤*) is called an I-α-homeomorphism (resp. a D-α-homeomorphism, B-α-homeomorphism) if both f and f^{-1} are I-α-continuous (resp. D-α-continuous, B-α-continuous).

Clearly every x-α-homeomorphism is an α-homeomorphism for x = I, D, B and every B-α-homeomorphism is both an I-α-homeomorphism and a D-α-homeomorphism.

The following example shows that an α-homeomorphism need not be a x-α-homeomorphism for x= I, D, B.

EXAMPLE 5.01. Let X ={a, b, c} = X*, τ ={ϕ,X,{a},{b},{a, b}}= τ^* and ≤ = {(a, a),(b, b),(c, c), (a, b), (b, c),(a, c)}. Clearly (X, τ, ≤) is a topological ordered space. Let f be the identity map from (X, τ, ≤) onto itself. {a, c} is closed set but f^{-1}({a, c}) = {a, c} is neither an increasing nor a decreasing α-closed set. Thus f is not x-α-continuous for x= I, D, B. f is an α-homeomorphism but not a x-α-homeomorphism for x = I,D,B.

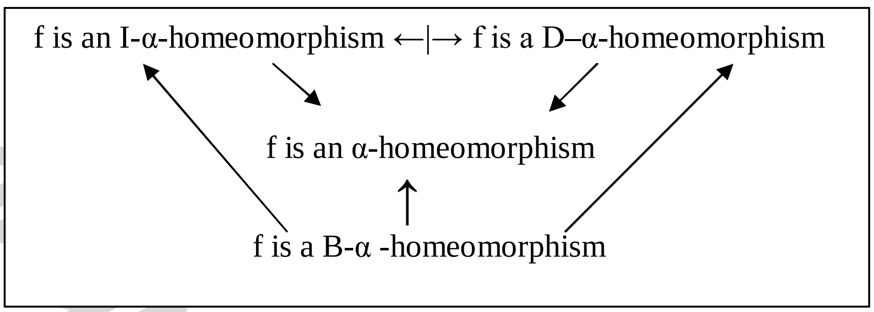
The following example shows that a D-α-homeomorphism need not be a B-α-homeomorphism.

EXAMPLE 5.02. Let X = {a, b, c} = X*, τ = {ϕ, X, {a}, {a, b}} = τ* and ≤ = {(a, a), (b, b), (c, c), (b, a)} = $\leq^*$. Let g be the identity map from (X, τ, ≤) onto (X*, τ*, ≤*). g is a D-α-homeomorphism but not a B-α-homeomorphism.

EXAMPLE 5.03. Let X ={a, b, c} = X*, τ = {ϕ, X, {a}, {b}, {a, b}}= τ* and ≤ = {(a, a), (b, b), (c, c), (a, c), (b, c)} = ≤*. Let f : (X, τ, ≤) → (X*, τ*, ≤*) be an identity map. Then f is an I-α-homeomorphism but not a B-α-homeomorphism.

5.01 Thus we have the following diagram

For a function f : (X, τ, ≤) → (X*, τ*, ≤*)



,where p→ q (resp. p ←|→ q) represents p implies q but q does not imply p (resp. p and q are independent of each other)

THEOREM 5.01. Let f : (X, τ, ≤) → (X*, τ*, ≤*) be a bijective I-α-continuous map. Then the following are equivalent.

1) f is an I-α -homeomorphism.
2) f is a D-α -open map.
3) f is an I-α -closed map.

Proof. Follows from the theorem 4.07.

THEOREM 5.02. Let f : (X, τ, ≤) → (X*, τ*, ≤*) be bijective D-α-continuous map. Then the following are equivalent.

1) f is a D-α-homeomorphism.
2) f is an I-α -open map.
3) f is a D-α -closed map.

Proof. Follows from the theorem 4.06.

THEOREM 5.03. Let f : $(X, \tau, \leq) \rightarrow (X^*, \tau^*, \leq^*)$ be a bijection and B-α-continuous map. Then the following are equivalent.

1) f is a B-α-homeomorphism.
2) f is a B-α-open map.
3) f is a B-α-closed map.

THEOREM 5.04. Let $(X, \tau, \leq)$ and $(X^*, \tau^*, \leq^*)$ be two topological ordered spaces. Let f : $(X, \tau, \leq) \rightarrow (X^*, \tau^*, \leq^*)$ be a map. Then f is an I-α-homeomorphism iff it is I-homeomorphism and I-pre-homeomorphism.

Proof. Follows from Lemma 1.1.

The following example shows that an I-semi-homeomorphism need not be an I-α-homeomorphism.

EXAMPLE 5.04. Let X = {a, b, c} = X*, τ = {ϕ, X, {a}, {a, b}}= τ* and ≤ = {(a, a), (b, b), (c, c), (c, a), (b, c), (b, a)} = ≤*. Define a map f : $(X, \tau, \leq) \rightarrow (X^*, \tau^*, \leq^*)$ by f(a) = a, f(b) = c and f(c) = b. Then f is an I-semi-homeomorphism but not an I-α-homeomorphism.

The following example shows that an I-pre-homeomorphism need not be an I-α-homeomorphism.

EXAMPLE 5.05. Let X = {a, b, c} = X*, τ = {ϕ, X, {a}, {b}, {a, b}, {a, c}} and τ*={{ϕ, X, {a}, {b, c}} and ≤ = {(a, a), (b, b), (c, c)} = ≤*. Define a map f : $(X, \tau, \leq) \rightarrow (X^*, \tau^*, \leq^*)$ by f(a)=b, f(b) =a and f(c) =c. Then f is an I-pre-homeomorphism but not an I-α-homeomorphism.

THEOREM 5.05. Let $(X, \tau, \leq)$ and $(X^*, \tau^*, \leq^*)$ be two topological ordered spaces.
Let f : $(X, \tau, \leq) \rightarrow (X^*, \tau^*, \leq^*)$ be a map. Then f is a D-α-homeomorphism iff it is a D-semi-homeomorphism and a D-pre-homeomorphism.

Proof. Follows from Lemma 1.1.

The following example shows that a D-semi- homeomorphism need not be a D-α-homeomorphism.

EXAMPLE 5.06. Let X ={a, b, c} = X*, τ = {ϕ, X, {a}, {a, c}}= τ* and ≤ = {(a, a), (b, b), (c, c), (a, b), (a, c)}= ≤*. Define a map f : $(X, \tau, \leq) \rightarrow (X^*, \tau^*, \leq^*)$ by f(a) = a, f(b) = c and f(c) = b. Then f is D-semi- homeomorphism but not a D-α-homeomorphism.

The following example shows that a D-pre-homeomorphism need not be a D-α-homeomorphism.

EXAMPLE 5.07. Let X ={a, b, c} = X*, τ = {ϕ, X, {a}, {b}, {a, b}, {a, c}} and τ*={{ϕ, X, {a}, {b, c}} and ≤ = {(a, a), (b, b), (c, c)} = ≤*. Define a map f : $(X, \tau, \leq) \rightarrow (X^*, \tau^*, \leq^*)$ by f(a) = b, f(b) = a and f(c) = c. Then f is a D-pre-homeomorphism but not a D-α-homeomorphism.

THEOREM 5.06. Let $(X, \tau, \leq)$ and $(X^*, \tau^*, \leq^*)$ be two topological ordered spaces.

Let f : (X, τ, ≤) → (X*, τ^*, $\leq^*$) be a map. Then f is a B-α-homeomorphism iff it is a B-semi-homeomorphism and a B-pre-homeomorphism.

Proof. Follows from Lemma 1.1.

The following example shows that a B-semi- homeomorphism need not be a B-α-homeomorphism.

EXAMPLE 5.08. Let X ={a, b, c} = X*, τ = {ϕ, X, {a}, {b}, {a, b}} and τ^* = {ϕ, X, {a}, {b}, {a, b}, {a, c}}, ≤ = {(a, a), (b, b), (c, c) }= ≤*. Define a map f : (X, τ, ≤) → **(**X*, **τ*, ≤*)** by f(a) = b, f(b) = a and f(c) = c. Then f is a B-semi- homeomorphism but not a B-α-homeomorphism.

The following example shows that a B-pre-homeomorphism need not be a B-α-homeomorphism.

EXAMPLE 5.09. Let X ={a, b, c} = X*, τ = {ϕ, X, {a}, {b}, {a, b}, {a, c}} and τ*={{ϕ, X, {a}, {b, c}} and ≤ = {(a, a), (b, b), (c, c)} = ≤*. Define a map f : (X, τ, ≤) → (X*, τ*, ≤*) by f(a) = b, f(b) = a and f(c) = c. Then f is a B-pre-homeomorphism but not a B-α-homeomorphism.

Standard separation axioms for topological ordered spaces have been studied systematically by S.D.McCartan [6, 7] Now we examine the separation properties of range spaces under some of these mappings.

DEFINITION 5.02. A topological ordered space (X, τ, ≤) is said to be upper strongly T_1-ordered iff for each pair of elements a ≤ b in X, there exists a decreasing τ-open neighborhood W of b such that a ∉W.

DEFINITION 5.03. A topological ordered space (X, τ, ≤) is said to be lower strongly T_1-ordered iff for each pair of elements a ≤ b in X, there exists an increasing τ-open neighborhood W of a such that b ∉ W. (X, τ, ≤) is said to be strongly T_1 ordered iff it is both lower and upper strongly T_1-ordered.

DEFINITION 5.04. A topological ordered space (X, τ, ≤) is said to be upper strongly α-T_1-ordered iff for each pair of elements a ≤ b in X, there exists a decreasing τ-α-open neighborhood W of b such that a ∉ W.

DEFINITION 5.05. A topological order space (X, τ, ≤) is said to be lower strongly α-T_1-ordered iff for each pair of elements a ≤ b in X there exists an increasing τ-α-open neighborhood W of a such that b ∉ W. (X, τ, ≤) is said to be strongly T_1-ordered iff it is both lower and upper strongly α-T_1-ordered

THEOREM 5.07. Let f : (X, τ, ≤) → (X*, τ*, ≤*) be a bijective I-α-open map as well as a poset isomorphism (i.e. x ≤ y iff f(x) ≤* f(y):∀x, y ∈ X). If (X, τ, ≤) is a lower strongly T_1-ordered space, then (X*, τ*, ≤*) is a lower strongly α-T_1-ordered space.

Proof. Let a,b∈ X* such that a ⊴* b. Then $f^{-1}(a) \trianglelefteq f^{-1}(b)$. Since (X, τ, ≤) is a lower strongly T_1-ordered space, there exists an increasing open neighborhood U of $f^{-1}(a)$ such that $f^{-1}(b) \notin U$. Thus f(U) is an increasing α-open neighborhood of f($f^{-1}(a)$) = a such that b= f ($f^{-1}(b)$) ∈ f (U). Therefore (X*, τ*, ≤*) is a lower strongly α-T_1-ordered space.

THEOREM 5.08. Let f : (X, τ, ≤) → (X*, τ*, ≤*) be a bijective D-α-open map as well as a poset isomorphism. If (X, τ, ≤) is an upper strongly α-T_1-ordered space then (X*, τ*, ≤*) is upper strongly α-T_1-ordered space.

Proof. Similar to as that of the theorem 5.07.

THEOREM 5.09. Let f : (X, τ, ≤) → (X*, τ*, ≤*) be a bijective B-α-open map. If f is a poset isomorphism and (X, τ, ≤) is strongly T_1-ordered space, then (X*,τ*, ≤*) is a strongly α-T_1-ordered space.

Proof. Follows from the theorems 5.07 and 5.08.

DEFINITION 5.06. A topological ordered space (X,τ, ≤) is called strongly α-T_2-ordered (or strongly α-Hausdorff ordered or strongly α-Hausdorff closed) iff for each pair of elements a ≤ b in X, there exists α-open neighborhoods U and V of a and b respectively such that U is an increasing set and V is a decreasing set.

THEOREM 5.10. Let f : (X, τ, ≤) → (X*, τ*, ≤*) be bijective B-α-open map. If (X,τ) is a Hausdorff space, then (X*,τ*, ≤*) is strongly α-Hausdorff ordered space.

Proof. Let a, b∈X* such that $a \not\leq^* b$. Then $f^{-1}(a) \neq f^{-1}(b)$. Since H is Hausdorff, there exists disjoint τ-open neighborhoods U and V of $f^{-1}(a)$ and $f^{-1}(b)$ respectively. Since f is B-α-open then f(U) and f(V) are two disjoint τ*, α-open neighborhoods of a and b respectively such that f(U) is an increasing set and f(V) is a decreasing set. Therefore (X*,τ*, ≤*) is a strongly Hausdorff ordered space.

DEFINITION 5.07. A topological ordered space (X,τ, ≤) is said to be a lower (an upper) strongly regular ordered space iff for each element a ∉ F there exists τ-open neighborhoods U of a and V of F such that U is an increasing (a decreasing) and V is a decreasing (an increasing) set in X and U ∩ V = ϕ.

DEFINITION 5.08. A topological ordered space (X,τ, ≤) is said to be a lower (upper) strongly α-regular ordered space iff for each decreasing (increasing) τ-closed set F and each element a ∈ F, there exist τ α-open neighborhoods U of a and V of F such that U is an increasing (a decreasing) and V is a decreasing (an increasing) set in X and U ∩V = ϕ. X is said to be strongly α-regular if it is upper and lower strongly α-regular ordered space.

THEOREM 5.11. Let f : (X, τ, ≤) → (X*, τ*, ≤*) be a bijective D-continuous map and a B-α-open map. If (X, τ) is regular space, then (X*, τ*, ≤*) is a lower strongly α-regular ordered space.

Proof. Let F be a decreasing closed subset of X* and a∈ X* such that a ∉ F. Since f is D-continuous f^{-1}(F) is a decreasing closed set in X. Since $f^{-1}(a) \notin f^{-1}(F)$ and X is regular, there exists two disjoint open neighborhoods U of $f^{-1}(a)$, V of $f^{-1}(F)$ in X. Since f is B-α-open, clearly $f^{-1}(U)$ is an increasing α-open set and f(V) is a decreasing α-open in X*. Also a ∈ f(U), F ⊆ f(V). (X*, τ*, ≤*) is lower strongly α-regular ordered space.

THEOREM 5.12. Let f : (X, τ, ≤) → (X*, τ*, ≤*) be a bijective D-continuous, B-open map. If (X, τ) is a regular space, then (X*, τ*, ≤*) is an upper strongly α-regular ordered space.

Proof. Analogous to as that of the theorem 5.11.

THEOREM 5.13. Let f : (X, τ, ≤) → (X*, τ*, ≤*) be a B-α-homomorphism. If (X, τ) is a regular space, then (X*, τ*, ≤*) is a strongly α-regular ordered space.

DEFINITION 5.09. A topological ordered space (X, τ, ≤) is said to be a strongly α-normally ordered space iff for each pair of disjoint τ-closed sets F_1 and F_2 in X, where F_1 is increasing F_2 is decreasing; there exists two disjoint τ-α-open neighborhoods U_1 of F_1 and U_2 of F_2 such that U_1 is increasing and U_2 is a decreasing in X.

DEFINITION 5.10. A topological ordered space (X, τ, ≤) is said to be a strongly α-T_3 -ordered iff it is both strongly α-T_1-ordered and strongly α-regular ordered
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DEFINITION 5.11. A topological ordered space (X, τ, ≤) is said to be a strongly α-T_4-ordered space iff it is both strongly α-T_1-ordered and strongly α-normally ordered.

THEOREM 5.14. Let f : (X, τ, ≤) → (X*, τ*, ≤*) be a bijective B-continuous and B-α-open map. Then:

1. If (X, τ,) is normal then (X*, τ*, ≤*) is a strongly α-normally ordered space.
3. If f is a poset isomorphism and (X, τ) is T_3, then (X*, τ*, ≤*) is strongly α-T_3-ordered space (Follows from the 5.09, 5.11).
2. If f is a poset isomorphism and (X, τ) is T_4, then (X*, τ*, ≤*) is strongly α-T_4-ordered space. (Follows from 5.09, 5.14(1)).

ACKNOWLEDGE: The first author is thankful to his guide Professor Y.V.Reddy, Department of Science and Humanities, University College of Engineering & Technology, Acharya Nagarjuna University, Nagarjuna Nagar, Guntur (Dt.), Andhra Pradesh, India, PIN-522 510. We have been thankful to M.K.R.S.Veera Kumar for proposing this topic and giving the guidelines to prepare this paper.

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