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DUO Chained -Semigroups

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ABSTRACT

In this paper we introduce the terms left α-cancelltive, right α-cancellative, α-cancellative, left Γ-cancellative, right Γ-cancellative, Γ-cancellative, strongly left cancellative, strongly right cancellative, strongly cancellative elements, α-inverse, Γ-inverse, complete inverse of an element, unit in a Γ-semigroup and a duo chained Γ-semigroup. It is proved that if P is a prime Γ-ideal of a duo chained Γ-semigroup S and $x \notin P$ then $P = \bigcap_{n=1}^{\infty}(x\Gamma)^n P$. It is also proved that every duo chained Γ-semigroup is a semiprimary Γ-semigroup. It is proved that (1) if $a \in S$ is a semisimple elements of a duo chained Γ-semigroup S , then $< a >^w \neq \emptyset$. (2) if a duo chained Γ-semigroup S has no -idempotent elements, then for any $a \in$ S, $< a >^w = \emptyset$ or $< a >^w$ is a prime -ideal. In a duo chained Γ-semigroup S if S $\neq$ SΓS then S\SΓS = { x } for some $x \in$ S. Further it is proved that in a duo chained Γ-semigroup S, if S $\neq$ SΓS such that S\SΓS = { x } for some $x \in$ S, then (1) S = x ΓS^1 = S^1 Γx and S ΓS = x ΓS = S Γx is the unique maximal Γ-ideal of S. (2) If $a \in$S and $a \notin < x >^w$ then $a \in (x\Gamma)^{n-1}x$ for some natural number $n > 1$. (3) If S contains strongly cancelable elements then x is a strongly cancelable element and $< x >^w$ is either empty or a prime Γ-ideal of S. It is proved that, if S is a duo chained Γ-semigroup, then S is an archemedian Γ-semigroup without Γ-idempotents if and only if $< a >^w = \emptyset$ for every $a \in S$. It is proved that if S be a strongly cancellative archemedian duo chained Γ-semigroup with $< a >^w \neq \emptyset$ for some $a \in S$, then S is a Γ-group. Also it is proved that, let S be a duo chained Γ-semigroup containing strongly cancellative elements and $< a >^w = \emptyset$ for every $a \in S$, then S is a strongly cancellative Γ-semigroup.

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KEY WORDS : chained Γ-semigroup, duo chained Γ-semigroup, α-cancellative element, Γ-cancellative element, strongly cancellative Γ-semigroup, α-inverse, Γ-inverse, complete inverse of an element, unit in a Γ-semigroup,

1. INTRODUCTION :

Γ- semigroup was introduced by SEN and SAHA [12] as a generalization of semigroup. ANJANEYULU. A [1], [2] and [3] initiated the study of pseudo symmetric ideals, radicals, semipseudo symmetric ideals in semigroups and N(A)-semigroups and primary and semiprimary ideals in semigroups. GIRI and WAZALWAR [6] intiated the study of prime radicals in semigroups. MADHUSUDHANA RAO, ANJANEYULU and GANGADHARA RAO [7], [8], [9], [10], [11] and [12] initiated the study of pseudo symmetric Γ-ideals, prime Γ-radicals and semipseudo symmetric Γ-ideals in Γ-semigroups and N(A)-semigroups pseudo Integral Γ-semigroups, Primary and semi primary Γ-ideals in Γ-semigroups. In this paper we introduce the notions of duo chained Γ-semigroup and characterize duo chained Γ-semigroup.

2. PRELIMINARIES:

DEFINITION 2.1 : Let S and Γ be any two non-empty sets. Then S is said to be a **Γ-*semigroup*** if there exist a mapping from $S \times \Gamma \times S$ to S which maps $(a, \gamma, b) \rightarrow a\,\gamma\,b$ satisfying the condition : $(a\alpha b)\beta c = a\alpha(b\beta c)$ for all $a, b, c \in S$ and $\alpha, \beta \in \Gamma$.

NOTE 2.2 : Let S be a Γ-semigroup. If A and B are two subsets of S, we shall denote the set $\{ a\gamma b : a \in A, b \in B \text{ and } \gamma \in \Gamma \}$ by AΓB.

DEFINITION 2.3: A nonempty subset A of a Γ-semigroup S is said to be a ***left* Γ-*ideal*** of S if $s \in S, a \in A, \alpha \in \Gamma$ implies $s\alpha a \in A$.

NOTE 2.4 : A nonempty subset A of a Γ-semigroup S is a left Γ- ideal of S iff SΓA⊆A.

DEFINITION 2.5: A nonempty subset A of a Γ-semigroup S is said to be a ***right* Γ-*ideal*** of S if $s \in S, a \in A, \alpha \in \Gamma$ implies $a\alpha s \in A$.

NOTE 2.6 : A nonempty subset A of a Γ-semigroup S is a right Γ- ideal of S iff AΓS⊆A.

DEFINITION 2.7 : A nonempty subset A of a Γ-semigroup S is said to be a ***two sided* Γ-*ideal*** or simply a **Γ- *ideal*** of S if $s \in S$, $a \in A$, $\alpha \in \Gamma$ imply $s\alpha a \in A$, $a\alpha s \in A$.

NOTE 2.8 : A nonempty subset A of a Γ-semigroup S is a two sided Γ-ideal iff it is both a left Γ-ideal and a right Γ- ideal of S.

THEOREM 2.9 : The nonempty intersection of any two (left or right) Γ-ideals of a Γ-semigroup S is a (left or right) Γ-ideal of S.

THEOREM 2.10 : The nonempty intersection of any family of (left or right) Γ-ideals of a Γ-semigroup S is a (left or right) Γ-ideal of S.

THEOREM 2.11 : The union of any two (left or right) Γ-ideals of a Γ-semigroup S is a (left or right) Γ-ideal of S.

THEOREM 2.12 : The union of any family of (left or right) Γ-ideals of a Γ-semigroup S is a(left or right) Γ-ideal of S.

DEFINITION 2.13 : A Γ- semigroup S is said to be a ***left duo Γ- semigroup*** provided every left Γ- ideal of S is a two sided Γ- ideal of S.

DEFINITION 2.14 : A Γ- semigroup S is said to be a ***right duo* Γ- *semigroup*** provided every right Γ- ideal of S is a two sided Γ- ideal of S.

DEFINITION 2.15 : A Γ- semigroup S is said to be a ***duo* Γ- *semigroup*** provided it is both a left duo Γ- Semigroup and a right duo Γ- semigroup.

THEOREM 2.16 : A Γ- semigroup S is a duo Γ- semigroup if and only if $x\Gamma S^1 = S^1\Gamma x$ for all $x \in S$.

THEOREM 2.17 : Let A be a Γ-ideal in a duo Γ-semigroup S and $a, b \in S$. Then $a\Gamma b \subseteq A$ if and only if $\langle a \rangle \Gamma \langle b \rangle \subseteq A$.

DEFINITION 2.18 : A Γ- ideal P of a Γ-semigroup S is said to be a ***completely prime* Γ- *ideal*** provided $x, y \in S$ and $x\Gamma y \subseteq P$ implies either $x \in P$ or $y \in P$.

DEFINITION 2.19 : A Γ- ideal P of a Γ-semigroup S is said to be a ***prime* Γ- *ideal*** provided A, B are two Γ-ideals of S and $A\Gamma B \subseteq P \Rightarrow$ either $A \subseteq P$ or $B \subseteq P$.

COROLLARY 2.20 : A Γ- ideal P of a Γ-semigroup S is a prime Γ- ideal iff $a, b \in S$ such that $a\Gamma S^1\Gamma b \subseteq P$, then either $a \in P$ or $b \in P$.

THEOREM 2.21 : Every completely prime Γ-ideal of a Γ-semigroup S is a prime Γ-ideal of S.

THEOREM 2.22 : Let S be a commutative Γ-semigroup. A Γ-ideal P of S is prime Γ-ideal if and only if P is a completely prime Γ-ideal.

DEFINITION 2.23: A Γ-ideal A of a Γ-semigroup S is said to be a ***completely semiprime* Γ-*ideal*** provided $x\Gamma x \subseteq A$; $x \in S$ implies $x \in A$.

THEOREM 2.24 : Every completely prime Γ-ideal of a Γ-semigroup S is a completely semiprime Γ-ideal of S.

THEOREM 2.25: The nonempty intersection of any family completely prime Γ-ideals of a Γ-semigroup S is a completely semiprime Γ-ideal of S.

DEFINITION 2.26: A Γ- ideal A of a Γ-semigroup S is said to be a ***semiprime* Γ- *ideal*** provided $x \in S$, $x\Gamma S^1\Gamma x \subseteq A$ implies $x \in A$.

THEOREM 2.27: Every completely semiprime Γ-ideal of a Γ-semigroup S is a semiprime Γ-ideal of S.

THEOREM 2.28 : Let S be a commutative Γ-semigroup. A Γ-ideal A of S is completely semiprime iff semiprime.

THEOREM 2.29 : Every prime Γ-ideal of a Γ-semigroup S is a semiprime Γ-ideal of S.

THEOREM 2.30: The nonempty intersection of any family of prime Γ-ideals of a Γ-semigroup S is a semiprime Γ-ideal of S.

NOTATION 2.31 : If A is a Γ-ideal of a Γ-semigroup S, then we associate the following four types of sets.

A_1 = The intersection of all completely prime Γ-ideals of S containing A.

$A_2 = \{x \in S : (x\Gamma)^{n-1}x \subseteq A$ for some natural number $n\}$

A_3 = The intersection of all prime ideals of S containing A.

$A_4 = \{x \in S : (\langle x \rangle\Gamma)^{n-1} \langle x \rangle \subseteq A$ for some natural number $n\}$

THEOREM 2.32 : If A is a Γ- ideal of a Γ-semigroup S, then $A \subseteq A_4 \subseteq A_3 \subseteq A_2 \subseteq A_1$.

THEOREM 2.33 : If A is a Γ-ideal in a duo Γ-semigroup S then $A_1 = A_2 = A_3 = A_4$.

DEFINITION 2.34 : If A is a Γ-ideal of a Γ-semigroup S, then the intersection of all prime Γ-ideals of S containing A is called ***prime Γ-radical*** or simply ***Γ-radical*** of A and it is denoted by **√A** or ***rad A***.

DEFINITION 2.35 : If A is a Γ-ideal of a Γ-semigroup S, then the intersection of all completely prime Γ-ideals of S containing A is called ***complete prime Γ-radical*** or simply ***complete Γ-radical*** of A and it is denoted by ***c. rad A***.

NOTE 2.36 : If A is a Γ-ideal of a Γ-semigroup S then ***rad* A = A_3** and ***c.rad* A = A_4.**

THEOREM 2.37 : If A is a Γ-ideal of a duo Γ-semigroup S, then *rad* A = *c.rad* A

DEFINITION 2.38 : A Γ-ideal A of a Γ-semigroup S is said to be a ***left primary Γ-ideal*** provided

i) If X, Y are two Γ-ideals of S such that $X\Gamma Y \subseteq A$ and $Y \nsubseteq A$ then $X \subseteq \sqrt{A}$.

ii) √A is a prime Γ-ideal of S.

DEFINITION 2.39 : A Γ-ideal A of a Γ-semigroup S is said to be a ***right primary Γ-ideal*** provided

i) If X, Y are two Γ-ideals of S such that $X\Gamma Y \subseteq A$ and $X \nsubseteq A$ then $Y \subseteq \sqrt{A}$.

ii) √A is a prime Γ-ideal of S.

DEFINITION 2.40 : A Γ-ideal A of a Γ-semigroup S is said to be a ***primary Γ-ideal*** provided A is both a left primary Γ-ideal and a right primary Γ-ideal.

THEOREM 2.41 : Let A be a Γ-ideal of a Γ-semigroup S. Then X, Y are two Γ-ideals of S such that $X\Gamma Y \subseteq A$ and $Y \nsubseteq A \Rightarrow X \subseteq \sqrt{A}$ if and only if $x, y \in S$, $\langle x \rangle \Gamma \langle y \rangle \subseteq A$ and $y \notin A \Rightarrow x \in \sqrt{A}$.

THEOREM 2.42 : Let A be a Γ-ideal of a Γ-semigroup S. Then X, Y are two Γ-ideals of S such that $X\Gamma Y \subseteq A$ and $X \nsubseteq A \Rightarrow Y \subseteq \sqrt{A}$ if and only if $x, y \in S$, $< x > \Gamma < y > \subseteq A$ and $x \notin A \Rightarrow y \in \sqrt{A}$.

DEFINITION 2.43 : A Γ-ideal A of a Γ-semigroup S is said to be ***semiprimary*** provided $\sqrt{A}$ is a prime Γ-ideal of S.

DEFINITION 2.44 : A Γ-semigroup S is said to be a ***semiprimary Γ-semigroup*** provided every Γ-ideal of S is a semiprimary Γ-ideal.

THEOREM 2.45 : Every left primary or right primary Γ-ideal of a Γ-semigroup is a semiprimary Γ-ideal.

DEFINITION 2.46 : An element a of Γ- semigroup S is said to be ***semisimple*** provided $a \in < a > \Gamma < a >$, that is, $< a > \Gamma < a > = < a >$.

DEFINITION 2.47 : A Γ- semigroup S is said to be ***semisimple Γ- semigroup*** provided every element is a semisimple.

DEFINITION 2.48 : An element a of Γ- semigroup S is said to be an ***α – idempotent*** if $a\alpha a = a$ for $\alpha \in \Gamma$.

DEFINITION 2.49 : An element a of Γ- semigroup S is said to be an ***idempotent*** or ***Γ-idempotent*** if $a\alpha a = a$ for all $\alpha \in \Gamma$.

NOTE 2.50 : a is an idempotent of S iff a is an α-idempotent for all $\alpha \in \Gamma$.

NOTE 2.51 : The set of all α-idempotent elements in a Γ- semigroup S is denoted by E_α.

NOTE 2.52 : If an element a of a Γ- semigroup S is *an* ***idempotent,*** then $a\Gamma a = a$.

DEFINITION 2.53 : A Γ-semigroup S is said to be an ***idempotent Γ-semigroup*** provided every element of S is an α–idempotent for some $\alpha \in \Gamma$.

DEFINITION 2.54 : A Γ- semigroup S is said to be a ***strongly idempotent Γ- semigroup*** provided every element in S is an idempotent.

DEFINITION 2.55 : An element a of a Γ-semigroup S is said to be ***regular*** provided $a = a\alpha x\beta a$, for some $x \in S$, $\alpha, \beta \in \Gamma$. i.e, $a \in a\Gamma S\Gamma a$.

DEFINITION 2.56 : A Γ- semigroup S is said to be *a* ***regular Γ- semigroup*** provided every element of S is regular.

DEFINITION 2.57 : A Γ-ideal A of a Γ-semigroup S is said to be a ***maximal Γ-ideal*** provided A is a proper Γ-ideal of S and A is not properly contained in any proper Γ-ideal of S.

THEOREM 2.58 : If S is a duo Γ-semigroup, then the following are equivalent for any element $a \in S$.

1) a is completely regular.

2) *a* is regular.
3) *a* is left regular.
4) *a* is right regular.
5) *a* is intra regular.
6) *a* is semisimple.

THEOREM 2.59 : If a Γ-semigroup S contains regular elements then S contains idempotents.

DEFINITION 2.60 : A Γ- semigroup S is said to be an ***archimedian* Γ- *semigroup*** provided for any $a,b \in$ S, there exists a natural number n such that $(a\Gamma)^{n-1}a \subseteq \langle b \rangle$.

DEFINITION 2.61 : A Γ-semigroup S is said to be a ***strongly archimedean* Γ-*semigroup*** provided for any a, $b \in$ S, there is a natural number n such that $(\langle a \rangle\Gamma)^{n-1} \langle a \rangle \subseteq \langle b \rangle$.

THEOREM 2.62 : If S is a duo Γ-semigroup, then the conditions (1) S is strongly Archimedean, (2) S is Archimedean and (3) S has no proper prime Γ-ideals are equivalent.

3. DUO CHAINED Γ-SEMIGROUP :

DEFINITION 3.1 : A Γ-semigroup S is said to be a ***chained* Γ*-semigroup*** if the Γ-ideals in S are linearly ordered by set inclusion.

THEOREM 3.2 : Let S be a duo chained Γ-semigroup and $x \in S$. If P is a prime Γ-ideal of S and $x \notin P$ then $P = \bigcap_{n=1}^{\infty}(x\Gamma)^n P$.

Proof **:** Since $x \notin P$ and P is prime, $(x\Gamma)^{n-1}x \not\subseteq P$ for all natural numbers n. Since $(x\Gamma)^{n-1}x \subseteq S$ and P is a Γ-ideal of S, it follows that $(x\Gamma)^{n-1}x\Gamma P \subseteq P$ for all natural numbers n and hence $(x\Gamma)^n P \subseteq P$ for all natural numbers n. Therefore $\bigcap_{n=1}^{\infty}(x\Gamma)^n P \subseteq P$. Since S is a duo chained Γ-semigroup, $(x\Gamma)^n S^1$ is a Γ-ideal of S. Since $(x\Gamma)^{n-1}x \not\subseteq P$, we get $(x\Gamma)^n S^1 \not\subseteq P$ and since S is a chained Γ-semigroup, $P \subseteq (x\Gamma)^n S^1$ for all natural numbers n. Let $y \in P$. Then $y \in (x\Gamma)^n S^1$. Therefore $y \in (x\Gamma)^n z$ for some $z \in S^1$. Therefore y = $x\alpha_1 x\alpha_2 .....x\alpha_n z$ for some $z \in S^1$ and $\alpha_1, \alpha_2, ..., \alpha_n \in \Gamma$. Since P is prime, y = $x\alpha_1 x\alpha_2 .....x\alpha_n z \in$ P, $x \notin$ P, we get $z \in$ P. Therefore $y \in (x\Gamma)^n P$ for all natural numbers n. Hence $P \subseteq \bigcap_{n=1}^{\infty}(x\Gamma)^n P$. Therefore $P = \bigcap_{n=1}^{\infty}(x\Gamma)^n P$.

THEOREM 3.3 : If S is a duo chained Γ-semigroup, then S is a semiprimary Γ-semigroup.

Proof **:** Let A be a Γ-ideal of S. We have $\sqrt{A} = \bigcap_{\alpha \in \Delta} P_\alpha$ = Intersection of all prime Γ-ideals of S containing A. Since S is a duo chained Γ-semigroup, we have $\{P_\alpha : \alpha \in \Delta\}$

forms a chain. By Zorn`s lemma $\{P_\alpha : \alpha \in \Delta\}$ has a minimal element say P_β. Therefore $\sqrt{A} = P_\beta$ and P_β is a prime Γ-ideal of S and hence $\sqrt{A}$ is prime. Therefore A is a semiprimary Γ-ideal of S and hence S is a semiprimary Γ-semigroup.

NOTE 3.4 : If S is a Γ-semigroup and $a \in$ S then we denote $< a >^w = \bigcap_{n=1}^{\infty} (< a > \Gamma)^{n-1} < a >$.

NOTE 3.5 : If S is a duo Γ-semigroup then $< a >^w = \bigcap_{n=1}^{\infty} < (a\Gamma)^{n-1} a > = \bigcap_{n=1}^{\infty} (a\Gamma)^{n-1} a \Gamma S^1$

THEOREM 3.6 : Let S be duo chained Γ-semigroup. If $a \in S$ is a semisimple element of S, then $< a >^w \neq \emptyset$.

***Proof* :** Suppose that *a* is a semisimple element of S. Therefore $a \in < a > \Gamma < a >$, implies that $< a > = < a > \Gamma < a >$. Therefore $a \in < a > = (< a > \Gamma)^{n-1} < a >$ for all natural numbers *n*. Hence $a \in \bigcap_{n=1}^{\infty} (< a > \Gamma)^{n-1} < a > = < a >^w$ and hence $< a >^w \neq \emptyset$.

THEOREM 3.7 : Let S be a duo chained Γ-semigroup. If $< a >^w = \emptyset$ for all $a \in$S, then S has no semisimple elements.

***Proof* :** Suppose that $< a >^w = \emptyset$ for all $a \in$ S. Suppose if possible S has a semisimple element *a*. By theorem 3.6, $< a >^w \neq \emptyset$. It is a contradiction. Therefore S has no semisimple elements.

THEOREM 3.8 : Let S be a duo chained Γ-semigroup. If S has no Γ-idempotents elements, then for any $a \in$ S, $< a >^w = \emptyset$ or $< a >^w$ is a prime Γ-ideal of S.

***Proof* :** Suppose that S has no Γ-idempotent elements and $a \in$ S. We have $< a >^w = \bigcap_{n=1}^{\infty} (< a > \Gamma)^{n-1} < a >$. Assume that $< a >^w \neq \emptyset$. If possible suppose that $<a>^w$ is not prime. Then there exists $x, y \in$ S such that $x\Gamma y \subseteq <a>^w$, $x \notin < a >^w$ and $y \notin < a >^w$. By theorem 2.17, $< x > \Gamma < y > = < x\Gamma y > \subseteq < a >^w$.
Now $x, y \notin < a >^w$, implies that there exists natural numbers *n*, *m* such that

$x \notin (<a>\Gamma)^{n-1} < a >$, $y \notin (<a>\Gamma)^{m-1} < a >$. Consider $k = \min\{n, m\}$.

Then $x, y \notin (<a>\Gamma)^{k-1} < a >$. Since S is a duo chained Γ-semigroup,

we have $(<a>\Gamma)^{k-1} < a > \subseteq < x >$ and $(<a>\Gamma)^{k-1} < a > \subseteq < y >$.

Therefore $(<a>\Gamma)^{2k-1} < a > = (<a>\Gamma)^{k-1} < a > \Gamma (<a>\Gamma)^{k-1} < a > \subseteq < x > \Gamma < y >$

$\subseteq < x\Gamma y > \subseteq < a >^w \subseteq (<a>\Gamma)^{4k-1} < a > \subseteq (<a>\Gamma)^{2k-1} < a > \Gamma (<a>\Gamma)^{2k-1} < a >$. Therefore

$(<a>\Gamma)^{2k-1} < a > \subseteq (<a>\Gamma)^{2k-1} < a > \Gamma (<a>\Gamma)^{2k-1} < a >$.

Therefore a^{2k} is a semisimple element of S. By theorem 2.58, a^{2k} is a regular element of S. Therefore $a^{2k} = a^{2k}\Gamma x\Gamma a^{2k}$ for some $x \in$ S, implies that $(a^{2k}\Gamma x)\Gamma(a^{2k}\Gamma x) = a^{2k}\Gamma x$ and hence $a^{2k}\Gamma x$ is a Γ-idempotent of S. So S has Γ-idempotent elements.

It is a contradiction. Hence $< a >^{w}$ is a prime Γ-ideal of S.

DEFINITION 3.9 : An element a of a Γ-semigroup S is said to be ***left α-cancellative*** *provided* for $\alpha \in \Gamma$, $a\alpha b = a\alpha c$ implies $b = c$.

DEFINITION 3.10 : An element a of a Γ-semigroup S is said to be ***right α-cancellative*** provided for $\alpha \in \Gamma$, $b\alpha a = c\alpha a$ implies $b = c$.

DEFINITION 3.11 : An element a of a Γ-semigroup S is said to be ***α-cancellative*** provided a is both a left α-cancellative element and a right α-cancellative element.

DEFINITION 3.12: An element a of a Γ-semigroup S is said to be ***left Γ-cancellative*** provided a is left α-cancellative for all $\alpha \in \Gamma$.

DEFINITION 3.13: An element a of a Γ-semigroup S is said to be ***right Γ-cancellative*** provided a is right α-cancellative for all $\alpha \in \Gamma$.

DEFINITION 3.14: An element a of a Γ-semigroup S is said to be ***Γ-cancellative*** provided a is a both left Γ-cancellative and Γ-cancellative.

DEFINITION 3.15: An element a of a Γ-semigroup S is said to be ***strongly left Γ-cancellative*** provided $a\Gamma b = a\Gamma c$ implies $b = c$.

NOTE 3.16: An element a of a Γ-semigroup S is said to be ***strongly left Γ-cancellative*** provided $a\alpha b = a\beta c$, $\alpha, \beta \in \Gamma \Rightarrow b = c$.

DEFINITION 3.17: An element a of a Γ-semigroup S is said to be ***strongly right Γ-cancellative*** provided $b\Gamma a = c\Gamma a$ implies $b = c$.

NOTE 3.18: An element a of a Γ-semigroup S is said to be ***strongly right Γ-cancellative*** provided $b\alpha a = c\beta a$, $\alpha, \beta \in \Gamma \Rightarrow b = c$.

DEFINITION 3.19: An element a of a Γ-semigroup S is said to be ***strongly Γ-cancellative*** provided a is a both strongly left **Γ**-*cancellative* and strongly right Γ-*cancellative*.

DEFINITION 3.20: An element a of a Γ-semigroup S is said to be a ***left identity*** of S provided $a\alpha s = s$ for all $s \in S$ and $\alpha \in \Gamma$.

DEFINITION 3.21: An element 'a' of a Γ-semigroup S is said to be a ***right identity*** of S provided $s\alpha a = s$ for all $s \in S$ and $\alpha \in \Gamma$.

DEFINITION 3.22: An element 'a' of a Γ-semigroup S is said to be a ***two sided identity*** or an ***identity*** provided it is both a left identity and a right identity of S.

THEOREM 3.23: **If a is a left identity and b is a right identity of a Γ-semigroup S, then $a = b$.**

Proof: Since a is a left identity of S, $a\alpha s = s$ for all $s \in$ S and $\alpha \in \Gamma$ and hence $a\alpha b = b$ for all $\alpha \in \Gamma$. Since b is a right identity of S, $s\alpha b = s$ for all $s \in$ S and $\alpha \in \Gamma$ and hence $a\alpha b = a$ for all $\alpha \in \Gamma$. Now $a = a\alpha b = b$.

THEOREM 3.24 : Any Γ-semigroup S has at most one identity.

***Proof* :** Let a, b be two identity elements of the Γ-semigroup S. Now a can be considered as a left identity and b can be considered as a right identity of S. By theorem 3.23, $a = b$. Then S has atmost one identity.

NOTE 3.25 : The identity (if exists) of a Γ-semigroup is usually denoted by e or 1.

DEFINITION 3.26 : An element b of a Γ-semigroup S is said to be a ***left α-inverse*** of a of a Γ-semigroup S provided for $\alpha \in \Gamma$, $b\alpha a = e$.

DEFINITION 3.27 : An element b of a Γ-semigroup S is said to be a ***right α-inverse*** of a of a Γ-semigroup S provided for $\alpha \in \Gamma$, $a\alpha b = e$.

THEOREM 3.28 : If *b* is a left α-inverse and *c* is a right α-inverse of an element *a* of a Γ-semigroup S, then *b* = *c*.

***Proof* :** Since b is a left α-inverse of an element a in S, $b\alpha a = e$
and c is a right α- inverse of an element a in S, $a\alpha c = e$ for $\alpha \in \Gamma$.
Now $b = b\alpha e = b\alpha(a\alpha c) = (b\alpha a)\alpha c = e\alpha c = c$.

DEFINITION 3.29 : An element b of a Γ-semigroup S is said to be a ***α-inverse*** of a of a Γ-semigroup S provided for $\alpha \in \Gamma$, $a\alpha b = b\alpha a = e$.

THEOREM 3.30 : The α-inverse of an element *a* in a Γ-semigroup S (if exists) is unique.

***Proof* :** Let b, c be two α-inverse elements of an element a in a Γ-semigroup S. If b is a α – inverse of a then $a\alpha b = b\alpha a = e$ and if c is a α-inverse of a then $a\alpha c = c\alpha a = e$.
Now $b = b\alpha e = b\alpha(a\alpha c) = (b\alpha a)\alpha c = e\alpha c = c$.

DEFINITION 3.31 : An element b of a Γ-semigroup S is said to be a ***left Γ-inverse*** of a of a Γ-semigroup S provided $b\alpha a = e$ for all $\alpha \in \Gamma$.

NOTE 3.32 : An element b of a Γ-semigroup S is said to be a ***left Γ-inverse*** of a of a Γ-semigroup S provided $b\Gamma a = e$.

DEFINITION 3.33 : An element b of a Γ-semigroup S is said to be a ***right Γ-inverse*** of a of a Γ-semigroup S provided $a\alpha b = e$ for all $\alpha \in \Gamma$.

NOTE 3.34 : An element b of a Γ-semigroup S is said to be a ***right Γ-inverse*** of a of a Γ-semigroup S provided $a\Gamma b = e$.

THEOREM 3.35 : If *b* is a left Γ-inverse and *c* is a right Γ-inverse of an element *a* of a Γ-semigroup S, then *b* = *c*.

Proof **:** Since b is a left Γ-inverse of an element a in S, $b\Gamma a = e$
and c is a right Γ- inverse of an element a in S, $a\Gamma c = e$.
Now $b = b\Gamma e = b\Gamma(a\Gamma c) = (b\Gamma a)\alpha c = e\Gamma c = c$.

DEFINITION 3.36 : An element b of a Γ-semigroup S is said to be a ***Γ-inverse*** of a of a Γ-semigroup S provided b is both a left Γ-inverse and a right Γ-inverse of a in S.

NOTE 3.37 : An element b of a Γ-semigroup S is said to be a ***Γ-inverse*** of a of a Γ-semigroup S provided $a\Gamma b = b\Gamma a = e$.

THEOREM 3.38 : The Γ-inverse of an element *a* in a Γ-semigroup S (if exists) is unique.

Proof **:** Let b, c be two Γ-inverse elements of an element a in a Γ-semigroup S. If b is a Γ–inverse of a then $a\Gamma b = b\Gamma a = e$ and if c is a Γ-inverse of a then $a\Gamma c = c\Gamma a = e$.
Now $b = b\Gamma e = b\Gamma(a\Gamma c) = (b\Gamma a)\Gamma c = e\Gamma c = c$.

DEFINITION 3.39 : An element a of a Γ-semigroup S is said to be a ***unit*** if it has Γ- inverse.

THEOREM 3.40 : If S is a duo chained strongly cancellative Γ-semigroup with an identity then for every nonunit *a*, $<a>^w$ is either empty or a prime Γ-ideal of S.

Proof **:** Suppose that a is a nonunit in S. If $<a>^w = \emptyset$ then the proof is trivial.
Let $<a>^w \neq \emptyset$. If possible suppose that $<a>^w$ is not a prime **Γ**-ideal of S.

Then there exists $x, y \in S$ such that $x\Gamma y \subseteq <a>^w$ and $x, y \notin <a>^w$.
By theorem 2.17, $<x>\Gamma<y> = <x\Gamma y> \subseteq <a>^w$. Now $x, y \notin <a>^w$, implies that there exists natural numbers n, m such that $x \notin (<a>\Gamma)^{n-1}<a>$ and $y \notin (<a>\Gamma)^{m-1}<a>$. Consider $k = \min\{n, m\}$. Then $x, y \notin (<a>\Gamma)^{k-1}<a>$.
Since S is duo chained **Γ**-semigroup, we have $(<a>\Gamma)^{k-1}<a> \subseteq <x>$ and $(<a>\Gamma)^{k-1}<a> \subseteq <y>$. Therefore $(<a>\Gamma)^{2k-1}<a> = (<a>\Gamma)^{k-1}<a>\Gamma$
$(<a>\Gamma)^{k-1}<a> \subseteq <x>\Gamma<y>$
$\subseteq <x\Gamma y> \subseteq <a>^w \subseteq (<a>\Gamma)^{4k-1}<a>$.
Then $(<a>\Gamma)^{2k-1}<a> \subseteq (<a>\Gamma)^{4k-1}<a> \subseteq (<a>\Gamma)^{2k-1}<a>\Gamma(<a>\Gamma)^{2k-1}<a>$
and hence $a^{2k} \in (<a>\Gamma)^{2k-1}<a>\Gamma(<a>\Gamma)^{2k-1}<a>$.
Therefore a^{2k} is a semisimple element of S.
By theorem 2.58, a^{2k} is a regular element of S.
Therefore $a^{2k} = a^{2k}\Gamma x\Gamma a^{2k}$ for some $x \in S$ implies that $(a^{2k}\Gamma x)\Gamma(a^{2k}\Gamma x) = a^{2k}\Gamma x$
and hence S has Γ-idempotent elements. Since S is strongly cancellative and
$a^{2k}\Gamma x\Gamma e = (a^{2k}\Gamma x)\Gamma(a^{2k}\Gamma x)$ implies that $a^{2k}\Gamma x = e$ and hence $a\Gamma(a^{2k-1}\Gamma x) = e$.
Hence a is a unit in S. It is a contradiction. Thus $<a>^w$ is a prime Γ-ideal of S. Hence $<a>^w = \emptyset$ or $<a>^w$ is a prime Γ-ideal of S.

THEOREM 3.41: Let S be a duo chained Γ-semigroup. If $S \neq S\Gamma S$ then $S \backslash S\Gamma S = \{x\}$ for some $x \in S$.

Proof **:** Suppose if possible x, y ∈ S\SΓS and $x \neq y$. Since S is a chained Γ-semigroup, $< x > \subseteq < y >$ or $< y > \subseteq < x >$. If $< x > \subseteq < y >$ then $x \in < y >$ and hence $x \in y\Gamma s$ for some $s \in S$. Therefore $x \in S\Gamma S$, which is not true. If $< y > \subseteq < x >$, then $y \in < x >$ and hence $y \in x\Gamma s$ for some $s \in S$. Therefore $y \in S\Gamma S$, which is not true. It is not a contradiction. Therefore x = y. So there exists unique x ∈ S such that $x \notin S\Gamma S$. Therefore S\ SΓS = { x } for some x ∈ S.

THEOREM 3.42 : Let S be a duo chained Γ-semigroup with S\ SΓS = { x } for some x ∈ S. Then S\{ x } is a Γ-ideal of S.

Proof **:** Let a ∈ S\{ x } and s ∈ S. Since $\{x\} \not\subseteq S\Gamma S$ we have $a\Gamma s \neq \{x\}$ and hence a Γs ⊆S\{ x }. Therefore S\{ x } is a right Γ-ideal of S. Since S is a duo Γ-semigroup, S\{ x } is a Γ-ideal of S.

THEOREM 3.43 : Let S be a duo chained Γ-semigroup. If S ≠ SΓS such that S\SΓS = { x } for some x ∈ S then S = x ΓS^1 = S^1 Γx and S ΓS = x ΓS = S Γx is the unique maximal Γ-ideal of S.

Proof **:** Since S\SΓS = { x }, S ΓS = S\{ x }. Now x ΓS^1 is a Γ-ideal of S and SΓS is a Γ-ideal of S. Since $\{x\} \not\subseteq S\Gamma S$ and since S is a chained Γ-semigroup, S ΓS ⊆ x ΓS^1. So x ΓS ⊆ S ΓS ⊆(x ΓS) ∪{ x } and $\{x\} \not\subseteq S\Gamma S$. Thus S ΓS = x ΓS. So S = x ΓS^1 = S^1 Γx and S ΓS = x ΓS = S Γx.
Since S ΓS is trivial, S ΓS = x ΓS = S Γx is a maximal Γ-ideal.

Since S is a chained Γ-semigroup, S ΓS = x ΓS = S Γx is the unique maximal Γ-ideal of S.

THEOREM 3.44 : Let S be a duo chained Γ-semigroup with S ≠ SΓS such that S\SΓS = { x } for some x ∈ S. If a∈S and a ∉ < x >^{w} then $a \in (x\Gamma)^{n-1}x$ for some natural number n > 1.

Proof **:** Since S is a duo chained Γ-semigroup with S ≠ SΓS such that S\SΓS = { x } for some x ∈ S, by theorem 3.28, S ΓS = x ΓS = S Γx = S\{ x }. Since $a \notin < x >^w = \bigcap_{n=1}^{\infty} < (x\Gamma)^{n-1}x >$, there exists a natural number k such that $a \notin < (x\Gamma)^{k-1}x >$. Let n be the least positive integer such that $a \notin < (x\Gamma)^{n-1}x >$ and $a \in < (x\Gamma)^{n-2}x >$. Now $a \in (x\Gamma)^{n-2}x\Gamma S^1$ and $a \notin (x\Gamma)^{n-1}x\Gamma S^1$.
Now $a \in (x\Gamma)^{n-2}x\Gamma S^1 \Rightarrow a \in (x\Gamma)^{n-2}x\Gamma S$ for some s ∈ S. s ∈ S, $s \neq x \Rightarrow s \in x$ ΓS

Therefore $a \in (x\Gamma)^{n-2}x\Gamma x\Gamma S = (x\Gamma)^{n-1}x\Gamma S$. It is a contradiction

and hence $s = x$. Therefore $a \in (x\Gamma)^{n-2}x\Gamma x = (x\Gamma)^{n-1}x$.

THEOREM 3.45 : Let S be a duo chained Γ-semigroup with S ≠ SΓS such that S\SΓS = { x } for some x ∈ S. If a∈S and a ∈ < x >^{w} then $a \in (x\Gamma)^{r-1}x$ for some natural number r or $a \in (x\Gamma)^{n-1}x\Gamma s_n$, $s_n \in < x >^w$ for all natural numbers n.

Proof **:** Since S is a duo chained Γ-semigroup with S $\neq$ SΓS such that $x \in$ S\S$\Gamma$S, by theorem 3.44, S ΓS = x ΓS = S Γx = S\{ x }. Let $a \in$S. Suppose that $a \in < x >^{w}$ Then $a \in$ $\bigcap_{n=1}^{\infty}(x\Gamma)^{n-1}x\Gamma S^{1}$. Therefore $a \in (x\Gamma)^{n-1}x\Gamma S^{1}$ for all natural numbers n. Hence $a \in (x\Gamma)^{n-1}x$ or $a \in (x\Gamma)^{n-1}x\Gamma s_n$ for some $s_n \in$ S. If $s_n \notin < x >^{w}$ then by theorem 3.44, $s_n \in (x\Gamma)^{r-1}x$ for some natural number r
and hence $a \in (x\Gamma)^{n-1}x\Gamma(x\Gamma)^{r-1}x = (x\Gamma)^{n+r-1}x$. If $s_n \in < x >^{w}$ then $a \in (x\Gamma)^{n-1}x\Gamma s_n$.

THEOREM 3.46 : Let S be a duo chained Γ-semigroup with S\S$\Gamma$S = { x } for some x $\in$ S. If S contains strongly cancelable elements then x is a strongly cancellable element and $< x >^{w}$ is either empty or a prime Γ-ideal of S.

Proof **:** Suppose if possible x is not strongly cancellable element in S. Let Z be the set of all non strongly cancellable elements of S. Clearly $x \in$ Z. So Z is nonempty subset of S. Let a $\in$ Z and $s \in$ S. Since $a \in$Z**,** a is not strongly cancellable in S. So there exists b, $c \in$ S such that $a\Gamma b = a\Gamma c$ and $b \neq c$. Now $a\Gamma b = a\Gamma c$ implies $s\Gamma(a\Gamma b) = s\Gamma(a\Gamma c)$ and hence $(s\Gamma a)\Gamma b = (s\Gamma a)\Gamma c$ and $b \neq c$.
Therefore $s\Gamma a$ is a set of nonstrongly cencellable elements of S.
Therefore $s\Gamma a \subseteq$ Z and hence Z is left Γ-ideal of S. Since S is a duo Γ-semigroup, Z is a Γ-ideal of S. Since S\S$\Gamma$S = { x }, by theorem 3.27, S = $x\Gamma S^{1}$. Since $x \in$Z and Z is a Γ-ideal of S, Z = S. It is a contradiction. Therefore x is a strongly cancellable element in S. Suppose that $< x >^{w} \neq \emptyset$. Let $a, b \in$ S and $a\Gamma b \subseteq < x >^{w}$.
Suppose if possible $a \notin < x >^{w}$ and $b \notin < x >^{w}$. Now a, $b \notin < x >^{w}$, by theorem 3.28, $a \in$ $(x\Gamma)^{n-1}x$, $b \in (x\Gamma)^{m-1}x$ for some natural numbers n, m.

Therefore $(x\Gamma)^{n+m-1}x = \left[(x\Gamma)^{n-1}x\right]\Gamma\left[(x\Gamma)^{m-1}x\right] = a\Gamma b \subseteq < x >^{w} \subseteq (x\Gamma)^{n+m}S$.

Therefore $x \in x\Gamma$S $\in$ S ΓS. It is a contradiction.

Therefore either $a \in < x >^{w}$ or $b \in < x >^{w}$ and hence $< x >^{w}$ is a prime Γ-ideal of S.

THEOREM 3.47 : Let S be a duo chained Γ-semigroup. Then S is an archemedian Γ-semigroup without Γ-idempotents if and only if $< a >^{w} = \emptyset$ for every $a \in S$.

Proof **:** Suppose that S is an archemedian Γ-semigroup without Γ-idempotents. If possible suppose that $< a >^{w} \neq \emptyset$ for some $a \in S$. By theorem 3.8, $< a >^{w}$ is a prime Γ-ideal of S. Since S is an archemedian duo Γ-semigroup, by theorem 2.57, S has no proper prime Γ-ideals. Therefore $< a >^{w}$ = S. Now $a \in < a >^{w} \subseteq < a > \Gamma < a >$. Thus a is semisimple element. By theorem 2.55, a is regular element. By theorem 2.56, S has Γ-idempotent elements. It is a contradiction. Hence $< a >^{w} = \emptyset$ for every $a \in S$. Conversely suppose that $< a >^{w} = \emptyset$ for every $a \in S$. Since $< a >^{w} = \emptyset$ for every $a \in S$, by corollary 3.6, S has no semi simple elements. By theorem 2.55, S has no regular elements. By theorem 2.59, S has no Γ-idempotent elements. If possible, suppose that P is proper prime Γ-ideal of S. Let $x \in S$ such that $x \notin P$. Since $x \notin P$ by theorem 3.2, $P = \bigcap_{n=1}^{\infty}(x\Gamma)^{n}P$**.** Therefore P $\subseteq < a >^{w} = \emptyset$. It is a contradiction. Hence S has no proper prime Γ-ideals. By theorem 2.62, S is an archemedian Γ-semigroup.

DEFINITION 3.48: A Γ-semigroup S is said to be a ***Γ-group*** if
(1) $\exists\ e \in S \ni a\Gamma e = e\Gamma a = a$ for all $a \in S$.
(2) every element $a \in S$ has a α-inverse in S for some $\alpha \in \Gamma$.

THEOREM 3.49 : Let S be a strongly cancellative archemedian duo chained Γ-semigroup with $< a >^{w} \neq \emptyset$ for some $a \in S$, then S is a Γ-group.

Proof **:** Suppose that S is a strongly cancellative archemedian duo chained Γ-semigroup with $< a >^{w} \neq \emptyset$ for some $a \in S$. Suppose if possible S has no Γ-idempotent elements. Since $< a >^{w} \neq \emptyset$, by theorem 3.7, $< a >^{w}$ is a prime Γ-ideal of S. Since S is an archemedian duo Γ-semigroup, by theorem 2.64, S has no proper prime Γ-ideals. It is a contradiction. Hence S has Γ-idempotent elements. Let e be a Γ-idempotent element in S. Then $x\alpha(e\beta e) = x\alpha e$ for every $x \in S$ and $\alpha, \beta \in \Gamma$. Since S is strongly cancellative, we have $x\alpha e = x$ for every $x \in S$, $\alpha \in \Gamma$. Similarly $e\alpha x = x$ for every $x \in S$, $\alpha \in \Gamma$. Therefore $e\Gamma x = x\Gamma e = x$. Hence e is the identity element in S. Let $a \in S$. Since $e, a \in S$ and S is archemedian Γ-semigroup, $(e\Gamma)^{n-1} e \subseteq S\Gamma a\Gamma S$. Therefore $e \in S\Gamma a\Gamma S$. Since S is duo Γ-semigroup $S\Gamma a\Gamma S = (S\Gamma S)\Gamma a = a\Gamma(S\Gamma S)$. Therefore $e \in (S\Gamma S)\Gamma a$ and hence $e = x\alpha a$ for some $x \in S.\Gamma S \subseteq S$ and $\alpha \in \Gamma$. Now $e = e\alpha e = (x\alpha a)\alpha(x\alpha a)$ implies that $x\alpha(a\alpha x)\alpha a = e = x\alpha a = x\alpha(e\alpha a)$. Since S is strongly cancelllative, we have $a\alpha x = e$. Similarily $x\alpha a = e$ Therefore $x\alpha a = a\alpha x = e$ and hence x is the α-inverse of a in S. Therefore S is a Γ-group.

THEOREM 3.50 : Let S be a duo chained Γ-semigroup containing strongly cancellative elements and $< a >^{w} = \emptyset$ for every $a \in S$, then S is a strongly cancellative Γ-semigroup.

Proof **:** Let S be a duo chained Γ-semigroup containing strongly cancellable elements. Suppose that $< a >^{w} = \emptyset$ for every $a \in S$. Let Z be the set of all non strongly Γ-cancellative elements in S. Suppose if possible Z is a nonempty subset of S. If $x \in Z$, then there exists $y, z \in S$, $\alpha, \beta \in \Gamma$ such that $x\alpha y = x\beta z$ and $y \neq z$. Therefore for any $s \in S$, $\gamma \in \Gamma$ $s\gamma(x\alpha y) = s\gamma(x\beta z) \Rightarrow (s\gamma x)\alpha y = (s\gamma x)\beta z$ and $y \neq z$. Hence $s\gamma x \in Z$. Therefore Z is a left Γ-ideal of S and hence Z is a Γ-ideal of S. If possible, suppose that Z is not prime. Then there exists $a, b \in S$ such that $a\gamma b \in Z$ and $a, b \notin Z$. Since $a \notin Z$, $b\alpha c = b\beta d$ for some $c, d \in S$, $\alpha, \beta \in \Gamma$. Since $b \notin Z$, $c = d$. It is a contradiction. Therefore Z is a prime Γ-ideal of S. Since $< a >^{w} = \emptyset$ for every $a \in S$, by theorem 3.47, we have S is an archimedian Γ-semigroup without Γ-idempotents. Therefore by theorem 2.59, S has no prime Γ-ideals and hence Z = S. It is a contradiction to S contains strongly cancellable elements. Hence $Z = \emptyset$. Thus S is a strongly cancellative Γ-semigorup.

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