

**Chemical Reaction, Heat and Mass Transfer in Nonlinear Hydromagnetic Flow over a Stretching Surface****T.Arun Kumar and L.Anand Babu**Department of Mathematics, University college of science, Osmania University, Hyderabad  
500 007Email: [arun.oumaths@gmail.com](mailto:arun.oumaths@gmail.com).**Abstract**

The boundary layer flow due to a surface stretching with a power law distribution in the presence of a transverse magnetic field is studied. A approximate Numerical solution for the flow problem has been obtained by solving the governing equations using Numerical Technique. A magnetic field is applied transversely to the direction of the flow. Adopting the similarity transformation, governing non linear partial differential equation of the problem are transformed to non linear ordinary differential equations. Then the numerical solution of the problem is derived using Quasilinearization method, for different values of the dimensionless parameter. The results obtained show that the flow field is influenced appreciably by the presence of chemical reaction and magnetic field.

**Keywords:** Chemical reaction, Magnetic field and Quasilinearization.**1. INTRODUCTION**

The problem concerned with nonlinear Hydromagnetic flow and heat and mass transfer over a stretching surface may find applications in polymer technology and metallurgy where Hydromagnetic techniques have been used. Numerous studies have been performed in recent years to investigate the effects of various physical phenomena on boundary layer heat and mass transfer in fluid flow over a stretching surface media, which is largely due to the extensive applications of this flow in metal casting technology, underground aquifer-energy storage, geothermal hydrology, ceramic engineering , soil mechanics and paper / textile technology. Recent years have also seen an increased interest in a natural convection heat and mass transfer from various geometries, particularly the vertical surface media. Free convection flow occurs frequently in nature. The temperature distribution and concentration varies from layer to layer and these types of flow have wide application in industry, agriculture and oceanography. Further they are especially used in dyeing- industries. The well-known Falknerskan transformation is used to reduce boundary-layer into ordinary differential equations for similar flows [1].various aspects of this problem have been studied by Sakiadis [2]. Sparrow and Cess [4] have studied the effect of temperature dependent heat sources or sinks in a stagnation point flow. Most studies have been concerned with constant surface temperature was analyzed by Crane [6] and Soundalgekar et al [7]. Gupta and Gupta

[8] and Soundalgekar and Ramamurthy [9] have investigated the problem with constant surface velocity and power law temperature variations.

The power-law temperature variations in the case of a stretching continuous surface was studied by Grubka and Bobba [10] and continuous flat plate was studied by Ericksm et al [11]. The effect of ;power-law surface temperature and power-law heat flux in the heat transfer characteristics of a continuous linear stretching surface was investigated by Chen and Char [12].processes involving the mass transfer effect have long been recognized as important principally in chemical processing equipment. Pera and Gebhart [13] and Gebhart [14] have analyzed the effect of different values of Prandtl number of the fluid along the surface. Recently, acharya et al [15] have studied heat and mass transfer over an accelerating surface with heat source in the presence of suction and blowing.

Here we analyse the nonlinear hydromagnetic flow and chemical reaction, heat and mass transfer over a surface stretching with power-law velocity. Nonlinear heat and mass transfer and the boundary layer flow due to the surface stretching with a power-law velocity distribution in the presence of a transverse magnetic field is studied. The similarity transformation has been utilized to convert the governing partial differential equation into ordinary differential equations and then numerical solution of the problem is derived using Quasilinearization method for different values of dimensionless parameters entering the problem under consideration have been obtained for the purpose of illustrating the results graphically. Examination of such flow models reveals the influence of magnetic field on velocity, temperature and concentration profiles. The analysis of the results obtained show that the flow field is influenced appreciably by the presence of applied magnetic field and chemical reaction.

## 2. MATHEMATICAL FORMULATION

Consider a steady viscous and electrically conducting Boussinesq fluid flowing over a stretching surface with a power-law velocity in the presence of applied magnetic field. The problem is considered to be of MHD laminar boundary layer type and two-dimensional. According to the coordinate system, the x-axis is chosen parallel to the vertical surface and the y-axis is taken normal to it. A transverse magnetic field of strength  $B_0$  is applied parallel to the y-axis. The fluid properties are assumed to be constant in a limited temperature range. The concentration of species far from the wall,  $C_\infty$  is infinitesimally small and hence the Soret and Dufour effects are negligible the value of  $C_\infty$  is set as zero in the problem. The chemical reactions are taking place in the flow and the physical properties  $\rho$ ,  $\mu$ ,  $D$  (chemical molecular Diffusivity) and the rate of chemical reaction,  $k_1$  are constant throughout the fluid. In writing the following equations, it is assumed that the induced magnetic field, the external electric field and the electric field due to polarization of charges are negligible. Under these conditions, the governing boundary layer equations of momentum, energy and diffusion for free convection flow with Joule's dissipation neglecting viscous and under Boussinesq's approximation are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta (T - T_\infty) - \left( \frac{\sigma B_0^2}{\rho} \right) u + g\beta^* (C - C_\infty) \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - k_1 C \quad (4)$$

The boundary conditions are

$$u=U(x) = ax^m, \quad v=0, \quad C=C_w(x), \quad T=T_w(x) \quad \text{at } y=0 \quad (5)$$

where a constant and m is pressure gradient parameter.

$$u=0, \quad C=C_\infty, \quad T = T_\infty(x) \quad \text{as } y \rightarrow \infty \quad (6)$$

As in [6] the following changes of variables are introduced:

$$\psi = \left( \frac{2vxU(x)}{(m+1)} \right)^{\frac{1}{2}} f(\eta) \quad (7)$$

$$\eta = \left( \frac{(m+1)U(x)}{2vx} \right)^{\frac{1}{2}} y$$

The velocity components are given by

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \quad (8)$$

It can be easily verified that the continuity Eq. (1) is identically satisfied. Similarity solution exist if we assume that  $U(x) = ax^m$ , the magnetic field B has the special form  $B(x) = B_0 x^{(m+1)/2}$  ( where  $B_0$  is Strength of applied magnetic field) and set  $C_\infty = 0$  and introduce the non-dimensional form of temperature and concentration as

$$\theta = \frac{T - T_\infty}{T_w - T_\infty},$$

$$\phi = \frac{C - C_\infty}{C_w - C_\infty},$$

$$Re_x = \frac{Ux}{\nu} \quad (\text{Reynolds number}),$$

$$Gr_x = \frac{g\beta\nu(C_w - T_\infty)}{U^3} \quad (\text{Grashof number}),$$

$$Gc_x = \frac{g\beta^* \nu(C_w - C_\infty)}{U^3} \quad (\text{Modified Grashof number}),$$

$$P_r = \frac{\mu c_p}{K} \quad (\text{Prandtl number}),$$

$$S_c = \frac{\nu}{D} \quad (\text{Schmidt number}),$$

$$M^2 = \frac{\sigma B_0^2}{\rho a} \quad (\text{Magnetic parameter}),$$

$$\gamma = \frac{\nu k_1}{U^2} \quad (\text{Chemical reaction parameter}) \quad (9)$$

A considerable amount of work has been done on the boundary layer flow over the surface. In many cases, similarity analysis is applicable, in which the dependence of the stream function  $\psi$  on  $x$  and  $y$  is converted into its dependence only on a single independent variable  $\eta$ , called the similarity variable. Therefore, the governing partial differential equations become ordinary differential equations. For a vertical surface, with the surface temperature variation given by the power-law temperature,  $T_w - T_\infty = Nx^n$  where  $N$  and  $n$  are constants. Also concentration variation is given by  $C_w - C_\infty = N_1 x^{n_1}$  where  $N_1$  and  $n_1$  are constants. The nonlinear equations and boundary conditions are obtained as

$$f''' + \left( \frac{2}{1+m} \right) \left( Gc_x \text{Re}_x \phi + Gr_x \text{Re}_x \theta - m(f')^2 - \left( \frac{M^2}{\text{Re}_x} \right) f' \right) + ff'' = 0 \quad (10)$$

$$\theta'' - \left( \frac{2n}{1+m} \right) \text{Pr} f' \theta + \text{pr} f \theta' = 0 \quad (11)$$

$$\phi'' - \left( \frac{2}{1+m} \right) \text{Sc} (\theta \gamma \text{Re}_x + f' \phi) + \text{Sc} f \phi' = 0 \quad (12)$$

The boundary conditions are given by

$$f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1, \quad \phi(0) = 1 \quad (13)$$

$$f'(\infty) = 0, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0$$

### 3. METHOD OF SOLUTION:

Eqs. (10) – (12) with boundary conditions (13) is solved using Quasilinearization of Newton's method. Flow field temperature and concentration are analyzed in detail for different reaction and magnetic parameter.

Quasi-linearization [16] of all non-linear terms in equation (10) gives

$$f''' + Ff'' - \left( \left( \frac{4m}{1+m} \right) F' + \left( \frac{2m^2}{1+m} \right) \left( \frac{1}{\text{Re}_x} \right) \right) f' + F''f = \left( \frac{2}{1+m} \right) \left( Gc_x \text{Re}_x \phi + Gr_x \text{Re}_x \theta \right) - \frac{2m}{1+m} F'^2 - FF'' \quad (14)$$

Where F is assumed to be a known function. In order to facilitate the application of finite difference scheme, we transform Equation (4) to a new system of co-ordinates where in the indefinite limit of integration in  $\eta$  is replaced by a finite limit. Employing the transformation

$$z = 1 - e^{-b\eta} \quad (15)$$

Where b is a 'constant' that can be used as a scaling factor to provide an optimum distribution at the nodal points across the boundary-layer. Eqn (2.1.14) reduces to:

$$Q^3 f''' + F_1 f'' + F_2 f' + F_3 f = F_4 \quad (16)$$

Where

$$Q = b(1 - z), \quad F_1 = -3bQ^2 + Q^2 F',$$

$$F_2 = -2Q^2 F' + b^2 Q - \left( \frac{2}{1+m} \right) (M^2 / \text{Re}_x) Q - QbF', \quad F_3 = Q^2 F'' - QbF',$$

$$F_4 = Q^2 F'' F - \left( \frac{2m}{1+m} \right) Q^2 F'^2 - QbF' F - \left( \frac{2}{1+m} \right) Gc_x \text{Re}_x \phi - \left( \frac{2}{1+m} \right) Gr_x \text{Re}_x \theta$$

Here prime denotes the differentiation with respect to z.

Using forward difference formulae in the equation (2.1.15), we get,

$$A_1(i)f(i+2) + A_2(i)f(i+1) + A_3(i)f(i) + A_4(i)f(i-1) = A_5(i) \quad (17).$$

$$\text{Where } A_1(i) = Q^3(i);$$

$$A_2(i) = h F_1(i) + h^2 F_2(i) - 3 Q^3(i);$$

$$A_3(i) = 3 A_1(i) - 2h F_1(i) - h^2 F_2(i) + h^3 F_3(i);$$

$$A_4(i) = h F_1(i) - A_1(i);$$

$$A_5(i) = h^3 F_4(i)$$

Here h represents the mesh size in z-direction. Equation (17) has been solved to obtain the numerical solutions. Here the numerical solutions of f are considered as the nth order iterative solutions and F are the (n - 1)<sup>th</sup> order solutions. After each cycle of iteration the convergence

check is performed, the tolerance set at  $10^{-6}$ , i.e.,  $|F - f| < 10^{-6}$  is satisfied at all points, then  $f$  is considered as convergent solution. Otherwise  $f$  becomes the new  $F$  and another cycle of iteration is carried out.

#### 4. RESULTS AND DISCUSSIONS:

In order to get a clear insight of the physical problem, numerical results are displayed with the help of graphical illustrations.

Figs. (1) to (3) depicts the effect of chemical reaction keeping all other parameters fixed. The effect of increasing chemical reaction parameter is to decelerate the velocity and concentration where as its effect is to enhance the temperature of the fluid.

The effect of magnetic field over the velocity, temperature and concentration of the fluid is displayed through figures (4) to (6). The effect of increasing magnetic field parameter is to decelerate the velocity while its effect is to enhance the temperature and concentration of the fluid.

Figs. (7) to (9) depicts the effect of Prandtl number parameter keeping all other parameters fixed. The effect of Prandtl number is to accelerate the temperature and decelerate the velocity and concentration of the fluid.

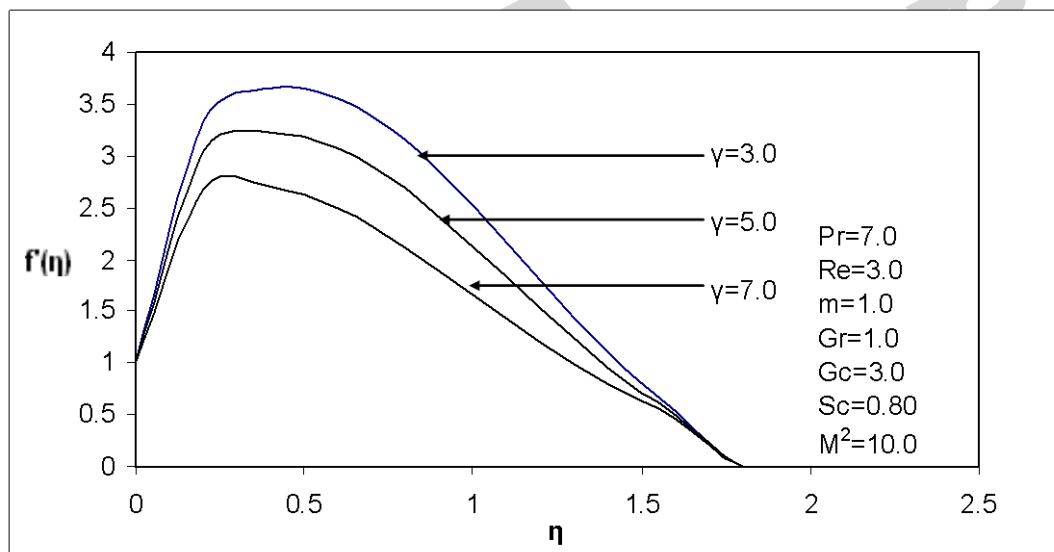


Figure. (1). Effects of chemical reaction over the velocity profiles.

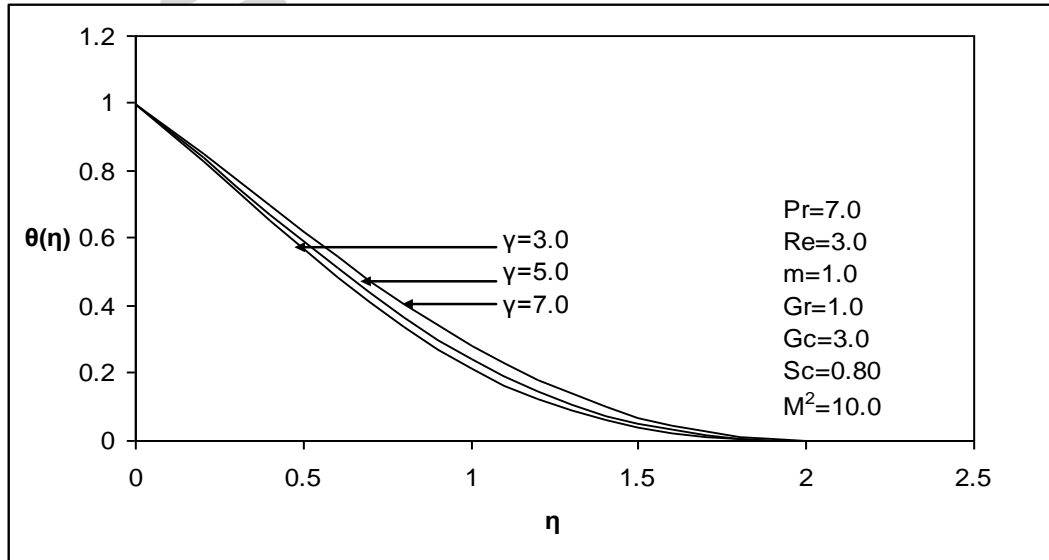


Figure. (2). Effects of chemical reaction over the temperature profiles.

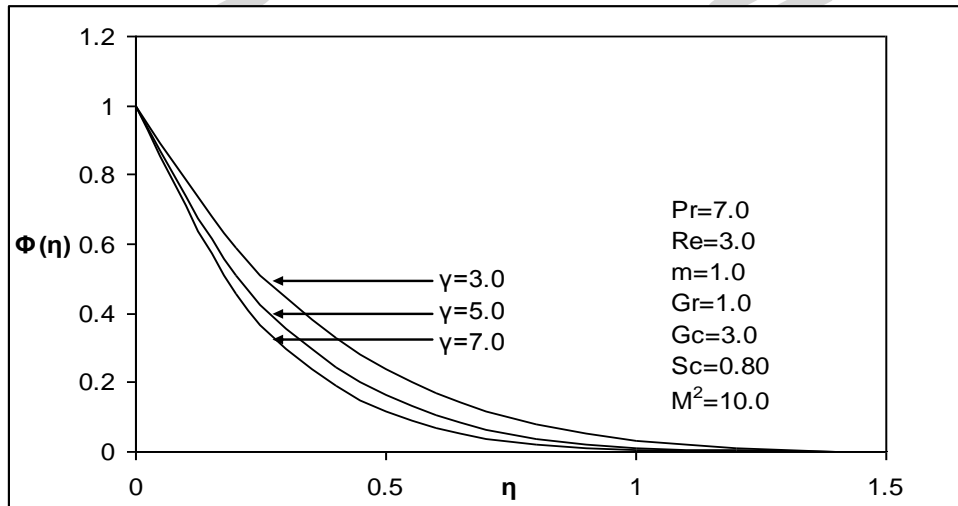


Figure. (3). Effects of chemical reaction over the concentration profiles.

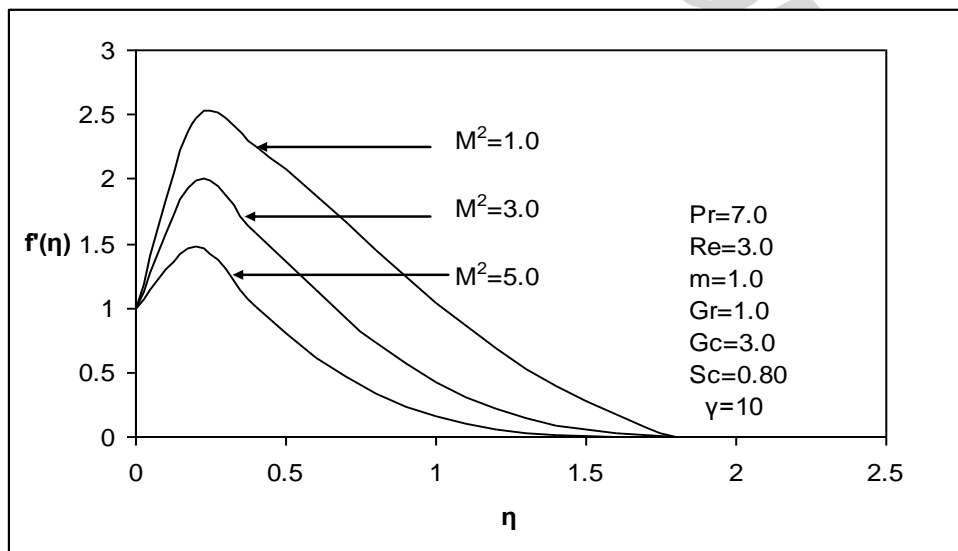


Figure (4). Influence of magnetic field over the velocity profiles.

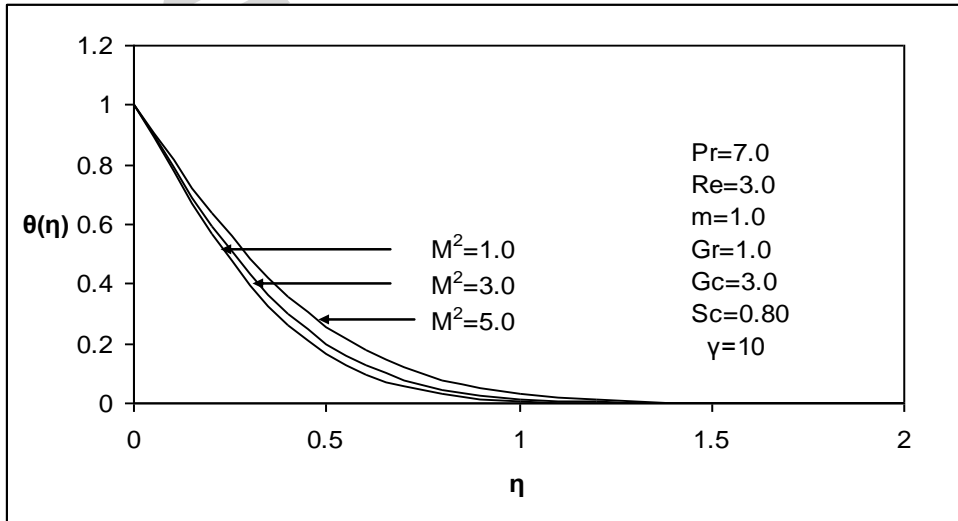


Figure (5). Influence of magnetic field over the temperature profiles.

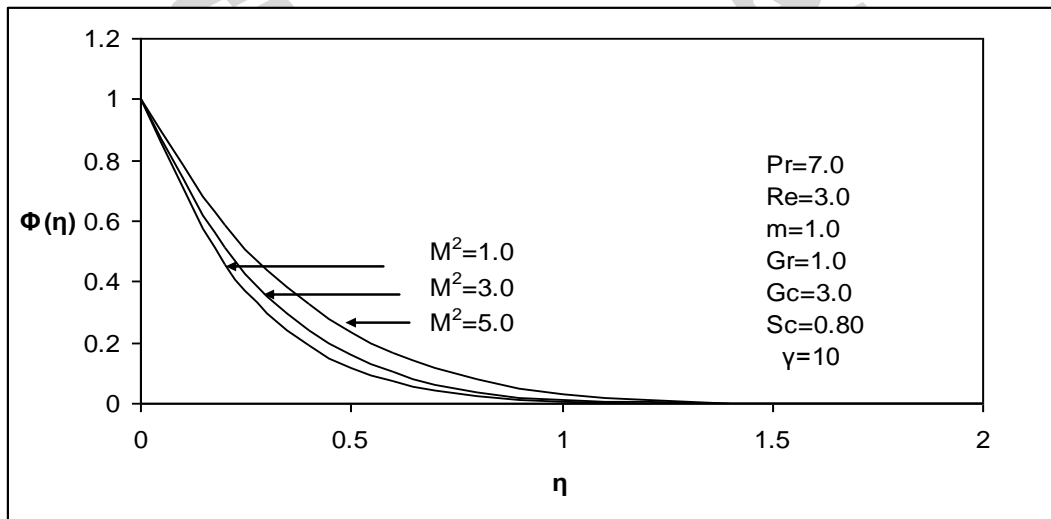


Figure (6). Influence of magnetic field over the concentration profiles.

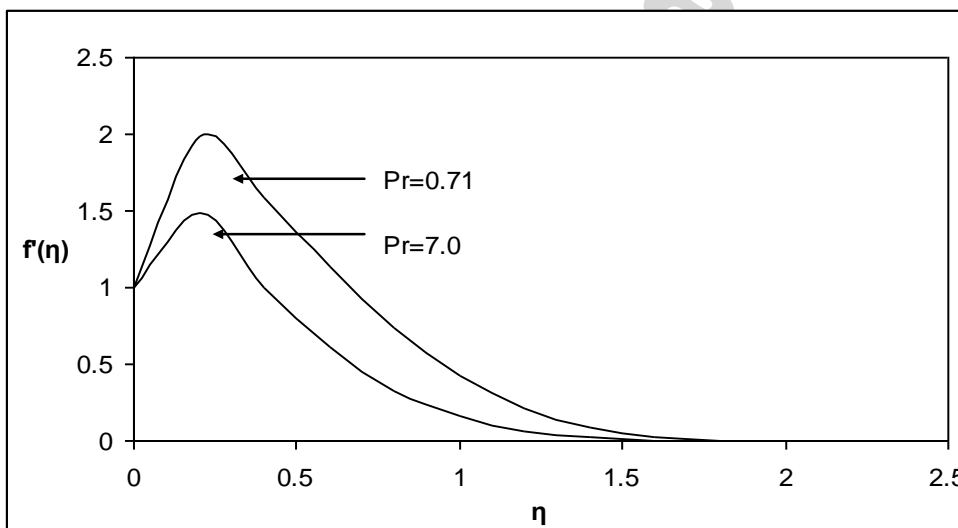


Figure (7). Effect of Prandtl number of velocity profile.



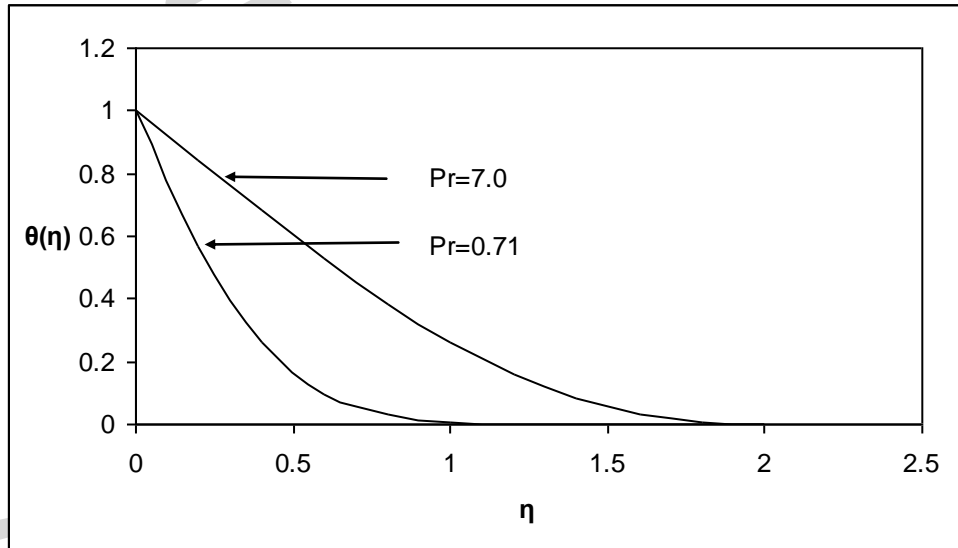


Figure (8). Effect of Prandtl number of temperature profile.

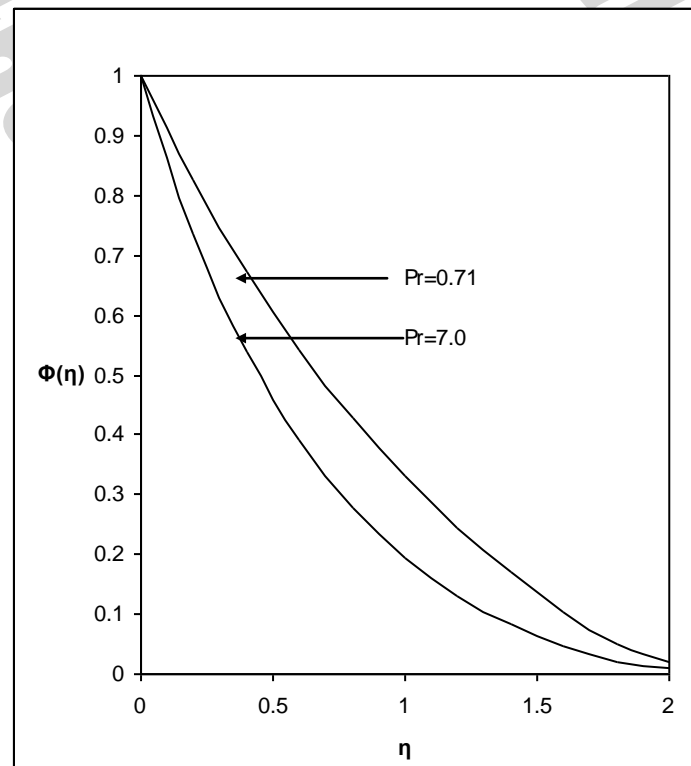


Figure (9). Effect of Prandtl number of concentration profile.

## 5. CONCLUSIONS

1. Due to the constant chemical reaction effects, an increase in the value of strength of applied magnetic field decreases the fluid velocity inside the boundary layer. So, the fluid particles are decelerated along the fluid motion. On the contrary, the temperature and concentration of the fluid increase as the strength of applied magnetic field increases. These trends are noted in both the cases of air and water.
2. Due to the uniform magnetic field, it is observed that the velocity and concentration of the fluid decreases and the temperature of the fluid increases with increase of chemical reaction in both the cases of air and water.

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