A Study on Discrete Model of Three Species Syn-Eco-System with Unlimited Resources for the First Species

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ABSTRACT: In this paper, the three species syn eco-system comprises of a commensal (S₁), two hosts S₂ and S₃ i.e., S₂ and S₃ both benefit S₁, without getting themselves effected either positively or adversely. Further S₂ is a commensal of S₃, S₃ is a host of both S₁, S₂ and the first species has unlimited resources. The basic equations for this model constitute as three first order non-linear coupled ordinary difference equations. All possible equilibrium points are identified based on the model equations at two stages and criteria for their stability are discussed. Further the numerical solutions are computed for specific values of the various parameters and the initial conditions.

Keywords: Commensal, equilibrium state, host, stable, oscillatory.

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1. INTRODUCTION

Ecology, a branch evolutionary biology, deals with living species that coexist in a physical environment sustain themselves on common resources. It is a common observations that the species of same nature can not flourish is isolation without any interaction with species of different kinds. Syn-ecology is an ecosystem comprising of two or more distinct species. Species interact with each other in one way or other. The Ecological interactions can be broadly classified as Ammensalism, Competition, Commensalism, Neutralism, Mutualism, Predation, Parasitism and so on. Lotka[7], Svirezhev et al [22] and Volterra [25] pioneered theoretical ecology significantly and opened new eras in the field of life and biological sciences. The authors Rogers et al [18], Varma [23] and Veilleux [24] discussed prey-predator, competing ecological models. Colinvaux [3], Smith [20] and Wangersky [26] studied basic concepts of population models in ecology. Mathematical Modeling is a vital
role in providing insight into the mutual relationships (positive, negative) between the interacting species. The general concepts of modeling have been discussed by several authors Kapur [4], Kushing [6], Meyer [8] and Pieiou [15]. Srinivas [21] studied competitive ecosystem of two species and three species with limited and unlimited resources. Later, Laxminarayan et al [9] studied prey-predator ecological models with partial cover for the prey and alternate food for the predator. Stability analysis of competitive species was carried out by ([17], [19]), while Ravindra Reddy [16] investigated mutualism between two species. Acharyulu et al ([11], [12]) derived some productive results on various mathematical models of ecological Ammensalism with multifarious resources in the manifold directions. Further, Kumar [5] studied some mathematical models of ecological commensalism. The present investigation is a discrete model of three species (S1, S2, S3) syn-ecosystem with unlimited resources for the first species. The system comprises of a commensal (S1), two hosts S2 and S3. Further S2 is a commensal of S3, S3 is a host of both S1 and S2.

Commensalism is a symbiotic interaction between two populations where one population (S1) gets benefit from (S2) while the other (S2) is neither harmed nor benefited due to the interaction with (S1). The benefited species (S1) is called the commensal and the other, the helping one (S2) is called the host species. A common example is an animal using a tree for shelter-tree (host) does not get any benefit from the animal (commensal).

2. BASIC EQUATIONS OF THE MODEL

2.1 Notation Adopted

\[ N_i(t) \] : The population strength of \( S_i \) at time \( t \), \( i = 1, 2, 3 \)

\( t \) : Time instant

\( a_i \) : Natural growth rate of \( S_i \), \( i = 1, 2, 3 \)

\( a_{ii} \) : Self inhibition coefficients of \( S_i \), \( i = 2, 3 \)

\( a_{12}, a_{13} \) : Interaction coefficients of \( S_1 \) due to \( S_2 \) and \( S_1 \) due to \( S_3 \)

\( a_{23} \) : Interaction coefficient of \( S_2 \) due to \( S_3 \)

Further the variables \( N_1, N_2, N_3 \) are non-negative and the model parameters \( a_1, a_2, a_3, a_{12}, a_{22}, a_{33}, a_{13}, a_{23} \) are assumed to be non-negative constants.

2.2 Basic equations

Consider the growth of the species during the time interval \((t, t + 1)\).

(i) Equation for the first species (\( N_1 \)):

\[ N_1(t+1) = N_1(t) + a_1 N_1(t) + a_{12} N_1(t) N_2(t) + a_{13} N_1(t) N_3(t) \]  \hspace{1cm} (2.1)

(ii) Equation for the second species (\( N_2 \)):

\[ N_2(t+1) = N_2(t) + a_2 N_2(t) - a_{22} N_2^2(t) + a_{23} N_2(t) N_3(t) \]  \hspace{1cm} (2.2)

(iii) Equation for the third species (\( N_3 \)):
\[ N_3(t+1) = N_3(t) + a_3N_3^2(t) \]  

(2.3)

2.3 Species-growth equations in the discrete form

Consider the nonlinear autonomous system of discrete equations

\[ N_i(t+1) = \alpha_i N_i(t) + a_{i2} N_i(t) N_j(t) + a_{i3} N_k(t) N_j(t) \]  

(2.4)

\[ N_2(t+1) = \alpha_2 N_2(t) - a_{22} N_2^2(t) + a_{23} N_3(t) N_j(t) \]  

(2.5)

\[ N_3(t+1) = \alpha_3 N_3(t) - a_{33} N_3^2(t) \]  

(2.6)

where \( \alpha_i = a_i + 1, i = 1, 2, 3 \)  

(2.7)

3. EQUILIBRIUM STATES

For a continuous model the equilibrium states are defined by \( \frac{dN_i}{dt} = 0, i = 1, 2, 3 \), the equilibrium states for a discrete model are defined in terms of the period of no growth.

i.e, \( N_i(t+r) = N_i(t), r = 1, 2, 3, \ldots \), where \( r \) is the period of the equilibrium state.


3.1 One period equilibrium states (Stage-I)

\[ N_i(t+1) = N_i(t), i = 1, 2, 3 \]  

(3.1)

The system under investigation has five equilibrium states given by

(i) Fully washed out state

\( E_1: \bar{N}_1 = 0, \bar{N}_2 = 0, \bar{N}_3 = 0 \)

(ii) The state in which only the first species survives

\[ E_2: \bar{N}_1 = 0, \bar{N}_2 = 0, \bar{N}_3 = \frac{\alpha_3-1}{a_{33}}, \text{ when } \alpha_3 > 1 \]

\[ E_3: \bar{N}_1 = 0, \bar{N}_2 = \frac{\alpha_2-1}{a_{22}}, \bar{N}_3 = 0, \text{ when } \alpha_2 > 1 \]

\[ E_4: \bar{N}_1 = 0, \bar{N}_2 = \frac{1}{a_{22}} \left[ (\alpha_2-1) + a_{21} \left( \frac{\alpha_3-1}{a_{33}} \right) \right], \bar{N}_3 = \frac{\alpha_3-1}{a_{33}}, \text{ when } \alpha_2 > 1 \text{ and } \alpha_3 > 1 \]

\[ E_5: \bar{N}_1 = 0, \bar{N}_2 = a_{23} \left( \frac{\alpha_3-1}{a_{22}a_{33}} \right), \bar{N}_3 = \frac{\alpha_3-1}{a_{33}}, \text{ when } \alpha_2 = 1 \text{ and } \alpha_3 > 1 \]
3.1.1 Stability of equilibrium states

**Stability of** \( E_1(0,0,0) \):

\[
N_1(t) = N_1(t+1) = N_1(t+2) = \ldots = 0; \quad N_2(t) = N_2(t+1) = N_2(t+2) = \ldots = 0
\]

\[
N_3(t) = N_3(t+1) = N_3(t+2) = \ldots = 0
\]

i.e, \( N_i(t+r) = 0 \), where \( r \) is an integer and \( i = 1, 2, 3 \)

Hence, \( E_1(0,0,0) \) is stable.

**Stability of** \( E_2 \):

\[
N_1(t) = N_1(t+1) = N_1(t+2) = \ldots = 0; \quad N_2(t) = N_2(t+1) = N_2(t+2) = \ldots = 0
\]

\[
N_3(t) = N_3(t+1) = N_3(t+2) = \ldots = \frac{\alpha_i - 1}{a_{33}}
\]

i.e, \( N_i(t+r) = 0, \quad N_3(t+r) = \frac{\alpha_i - 1}{a_{33}} \), where \( r \) is an integer and \( i = 1, 2 \)

Hence, \( E_2 \) is stable.

**Stability of** \( E_3 \):

\[
N_1(t) = N_1(t+1) = N_1(t+2) = \ldots = 0; \quad N_3(t) = N_3(t+1) = N_3(t+2) = \ldots = 0
\]

\[
N_2(t) = N_2(t+1) = N_2(t+2) = \ldots = \frac{\alpha_2 - 1}{a_{22}}
\]

i.e, \( N_i(t+r) = 0, \quad N_2(t+r) = \frac{\alpha_2 - 1}{a_{22}} \), where \( r \) is an integer and \( i = 1, 3 \)

Hence, \( E_3 \) is stable.

**Stability of** \( E_4 \):

\[
N_1(t) = N_1(t+1) = N_1(t+2) = \ldots = 0; \quad N_3(t) = N_3(t+1) = N_3(t+2) = \ldots = \frac{\alpha_i - 1}{a_{33}}
\]

\[
N_2(t) = N_2(t+1) = N_2(t+2) = \ldots = \frac{1}{a_{22}} \left[ \left( \alpha_2 - 1 \right) + a_{23} \left( \frac{\alpha_3 - 1}{a_{33}} \right) \right]
\]

i.e, \( N_1(t+r) = 0, \quad N_2(t+r) = \frac{1}{a_{22}} \left[ \left( \alpha_2 - 1 \right) + a_{23} \left( \frac{\alpha_3 - 1}{a_{33}} \right) \right] \), \( N_3(t+r) = \frac{\alpha_3 - 1}{a_{33}} \)

where \( r \) is an integer

Hence, \( E_4 \) is stable.

**Stability of** \( E_5 \):
\[ N_i(t) = N_i(t+1) = N_i(t+2) = \ldots = 0 \; ; \; N_2(t) = N_2(t+1) = N_2(t+2) = \ldots = a_{23} \left( \frac{\alpha_3 - 1}{a_{22}a_{33}} \right) \]

\[ N_3(t) = N_3(t+1) = N_3(t+2) = \ldots = \frac{\alpha_3 - 1}{a_{33}} \]

i.e, \[ N_1(t+r) = 0, \; N_2(t+r) = a_{23} \left( \frac{\alpha_3 - 1}{a_{22}a_{33}} \right), \; N_3(t+r) = \frac{\alpha_3 - 1}{a_{33}}, \] where \( r \) is an integer

Hence, \( E_5 \) is stable.

At this stage all the five equilibrium states are stable.

### 3.2 Two period equilibrium states (Stage-II)

\[ N_i(t+2) = N_i(t), \; i = 1, 2, 3 \]  

(3.2)

The system under investigation has twenty five equilibrium states given by

(i) **Fully washed out state**

\[ E_1 : \bar{N}_1 = 0, \; \bar{N}_2 = 0, \; \bar{N}_3 = 0. \]

(ii) **States in which only the third species survives**

\[ E_2 : \bar{N}_1 = 0, \; \bar{N}_2 = 0, \; \bar{N}_3 = \frac{\alpha_3 - 1}{a_{33}}, \] when \( \alpha_3 > 1 \)

\[ E_3 : \bar{N}_1 = 0, \; \bar{N}_2 = 0, \; \bar{N}_3 = \frac{(\alpha_3 + 1) + \sqrt{(\alpha_3 + 1)(\alpha_3 - 3)}}{2a_{33}}, \] when \( \alpha_3 > 3 \)

\[ E_4 : \bar{N}_1 = 0, \; \bar{N}_2 = 0, \; \bar{N}_3 = \frac{(\alpha_3 + 1) - \sqrt{(\alpha_3 + 1)(\alpha_3 - 3)}}{2a_{33}}, \] when \( \alpha_3 > 3 \)

\[ E_5 : \bar{N}_1 = 0, \; \bar{N}_2 = 0, \; \bar{N}_3 = \frac{2}{a_{33}}, \] when \( \alpha_3 = 3 \)

The states \( E_3 \) and \( E_4 \) coincide when \( \alpha_3 = 3 \) and do not exist when \( \alpha_3 < 3 \).

(iii) **States in which only the second species survives**

\[ E_6 : \bar{N}_1 = 0, \; \bar{N}_2 = \frac{\alpha_2 - 1}{a_{22}}, \; \bar{N}_3 = 0, \] when \( \alpha_2 > 1 \)

\[ E_7 : \bar{N}_1 = 0, \; \bar{N}_2 = \frac{(\alpha_2 + 1) + \sqrt{(\alpha_2 + 1)(\alpha_2 - 3)}}{2a_{22}}, \; \bar{N}_3 = 0, \] when \( \alpha_2 > 3 \)

\[ E_8 : \bar{N}_1 = 0, \; \bar{N}_2 = \frac{(\alpha_2 + 1) - \sqrt{(\alpha_2 + 1)(\alpha_2 - 3)}}{2a_{22}}, \; \bar{N}_3 = 0, \] when \( \alpha_2 > 3 \)
\[ E_6 : \bar{N}_1 = 0, \bar{N}_2 = \frac{2}{a_{22}}, \bar{N}_3 = 0, \text{ when } \alpha_2 = 3 \]

The states \( E_7 \) and \( E_8 \) coincide when \( \alpha_2 = 3 \) and do not exist when \( \alpha_2 < 3 \).

(iv) States in which only the second and third species survives

\[ E_{10} : \bar{N}_1 = 0, \bar{N}_2 = \frac{\beta_2 - 1}{a_{22}}, \bar{N}_3 = \frac{\alpha_3 - 1}{a_{33}}, \text{ when } \beta_2, \alpha_3 > 1 \]

where \( \beta_2 = \alpha_2 + a_{23} \left( \frac{\alpha_3 - 1}{a_{33}} \right) \)

\[ E_{11} : \bar{N}_1 = 0, \bar{N}_2 = \frac{(\beta_2 + 1) + \sqrt{(\beta_2 + 1)(\beta_2 - 3)}}{2a_{22}}, \bar{N}_3 = \frac{\alpha_3 - 1}{a_{33}}, \text{ when } \beta_2 > 3, \alpha_3 > 1 \]

\[ E_{12} : \bar{N}_1 = 0, \bar{N}_2 = \frac{(\beta_2 + 1) - \sqrt{(\beta_2 + 1)(\beta_2 - 3)}}{2a_{22}}, \bar{N}_3 = \frac{\alpha_3 - 1}{a_{33}}, \text{ when } \beta_2 > 3, \alpha_3 > 1 \]

\[ E_{13} : \bar{N}_1 = 0, \bar{N}_2 = \frac{2}{a_{22}}, \bar{N}_3 = \frac{\alpha_3 - 1}{a_{33}}, \text{ when } \beta_2 = 3, \alpha_3 > 1 \]

The states \( E_{11} \) and \( E_{12} \) coincide when \( \beta_2 = 3 \) and do not exist when \( \beta_2 < 3 \).

\[ E_{14} : \bar{N}_1 = 0, \bar{N}_2 = \frac{\gamma_2 - 1}{a_{22}}, \bar{N}_3 = \frac{(\alpha_3 + 1) + \sqrt{(\alpha_3 + 1)(\alpha_3 - 3)}}{2a_{33}}, \text{ when } \gamma_2 > 1, \alpha_3 > 3 \]

where \( \gamma_2 = \alpha_2 + a_{23} \left( \frac{(\alpha_3 + 1) + \sqrt{(\alpha_3 + 1)(\alpha_3 - 3)}}{2a_{33}} \right) \)

\[ E_{15} : \bar{N}_1 = 0, \bar{N}_2 = \frac{(\gamma_2 + 1) + \sqrt{(\gamma_2 + 1)(\gamma_2 - 3)}}{2a_{22}}, \bar{N}_3 = \frac{(\alpha_3 + 1) + \sqrt{(\alpha_3 + 1)(\alpha_3 - 3)}}{2a_{33}}, \text{ when } \gamma_2, \alpha_3 > 3 \]

\[ E_{16} : \bar{N}_1 = 0, \bar{N}_2 = \frac{(\gamma_2 + 1) - \sqrt{(\gamma_2 + 1)(\gamma_2 - 3)}}{2a_{22}}, \bar{N}_3 = \frac{(\alpha_3 + 1) + \sqrt{(\alpha_3 + 1)(\alpha_3 - 3)}}{2a_{33}}, \text{ when } \gamma_2, \alpha_3 > 3 \]

\[ E_{17} : \bar{N}_1 = 0, \bar{N}_2 = \frac{2}{a_{22}}, \bar{N}_3 = \frac{(\alpha_3 + 1) + \sqrt{(\alpha_3 + 1)(\alpha_3 - 3)}}{2a_{33}}, \text{ when } \gamma_2 = 3, \alpha_3 > 3 \]

The states \( E_{15} \) and \( E_{16} \) coincide when \( \gamma_2 = 3 \) and do not exist when \( \gamma_2 < 3 \).
$E_{18}: \bar{N}_1 = 0, \bar{N}_2 = \frac{\mu_2 - 1}{a_{22}}, \bar{N}_3 = \frac{\alpha_3 + 1 - \sqrt{\alpha_3 + 1)(\alpha_3 - 3)}}{2a_{33}}$, when $\mu_2 > 1, \alpha_3 > 3$

where $\gamma_2 = \alpha_3 + a_{23} \left[ \frac{\alpha_3 + 1 - \sqrt{\alpha_3 + 1)(\alpha_3 - 3)}}{2a_{33}} \right]$

$E_{19}: \bar{N}_1 = 0, \bar{N}_2 = \frac{(\mu_2 + 1) + \sqrt{(\mu_2 + 1)(\mu_2 - 3)}}{2a_{22}}, \bar{N}_3 = \frac{\alpha_3 + 1 - \sqrt{\alpha_3 + 1)(\alpha_3 - 3)}}{2a_{33}}$, when $\mu_2, \alpha_3 > 3$

$E_{20}: \bar{N}_1 = 0, \bar{N}_2 = \frac{(\mu_2 + 1) - \sqrt{(\mu_2 + 1)(\mu_2 - 3)}}{2a_{22}}, \bar{N}_3 = \frac{\alpha_3 + 1 - \sqrt{\alpha_3 + 1)(\alpha_3 - 3)}}{2a_{33}}$, when $\mu_2, \alpha_3 > 3$

$E_{21}: \bar{N}_1 = 0, \bar{N}_2 = \frac{2}{a_{22}}, \bar{N}_3 = \frac{\alpha_3 + 1 - \sqrt{\alpha_3 + 1)(\alpha_3 - 3)}}{2a_{33}}$, when $\mu_2 = 3, \alpha_3 > 3$

The states $E_{19}$ and $E_{20}$ coincide when $\mu_2 = 3$ and do not exist when $\mu_2 < 3$.

$E_{22}: \bar{N}_1 = 0, \bar{N}_2 = \frac{\delta_2 - 1}{a_{22}}, \bar{N}_3 = \frac{2}{a_{33}}$, when $\delta_2 > 1, \alpha_3 = 3$

where $\delta_2 = \alpha_3 + \frac{2a_{23}}{a_{33}}$

$E_{23}: \bar{N}_1 = 0, \bar{N}_2 = \frac{(\delta_2 + 1) + \sqrt{(\delta_2 + 1)(\delta_2 - 3)}}{2a_{22}}, \bar{N}_3 = \frac{2}{a_{33}}$, when $\delta_2 > 3, \alpha_3 = 3$

$E_{24}: \bar{N}_1 = 0, \bar{N}_2 = \frac{(\delta_2 + 1) - \sqrt{(\delta_2 + 1)(\delta_2 - 3)}}{2a_{22}}, \bar{N}_3 = \frac{2}{a_{33}}$, when $\delta_2 > 3, \alpha_3 = 3$

$E_{25}: \bar{N}_1 = 0, \bar{N}_2 = \frac{2}{a_{22}}, \bar{N}_3 = \frac{2}{a_{33}}$, when $\delta_2 = 3, \alpha_3 = 3$

The states $E_{23}$ and $E_{24}$ coincide when $\delta_2 = 3$ and do not exist when $\delta_2 < 3$.

3.2.1 Stability of Equilibrium States

The equilibrium states $E_1, E_2$ and $E_6$ are stable as established in 3.1.1. Now we will discuss the stability of other equilibrium states except these three states.

**Stability of $E_3$:**

$N_1(t) = N_1(t + 1) = N_1(t + 2) = \ldots = 0; \ N_2(t) = N_2(t + 1) = N_2(t + 2) = \ldots = 0$
i.e, \( N_i(t+r) = 0 \), where \( r \) is an integer and \( i = 1, 2 \)

\[
N_3(t) = N_3(t+2) = N_3(t+4) = \ldots = \frac{(\alpha_3 + 1) + \sqrt{(\alpha_3 + 1)(\alpha_3 - 3)}}{2a_{33}}
\]

\[
N_3(t+1) = N_3(t+3) = N_3(t+5) = \ldots = \frac{(\alpha_3 + 1) - \sqrt{(\alpha_3 + 1)(\alpha_3 - 3)}}{2a_{33}}
\]

\[
N_3(t+2r) = \frac{(\alpha_3 + 1) + \sqrt{(\alpha_3 + 1)(\alpha_3 - 3)}}{2a_{33}} ;
\]

\[
N_3(t+2r+1) = \frac{(\alpha_3 + 1) - \sqrt{(\alpha_3 + 1)(\alpha_3 - 3)}}{2a_{33}}
\]

where \( r \) is an integer.

Hence, \( E_3 \) oscillates between \( \frac{(\alpha_3 + 1) + \sqrt{(\alpha_3 + 1)(\alpha_3 - 3)}}{2a_{33}} \) and \( \frac{(\alpha_3 + 1) - \sqrt{(\alpha_3 + 1)(\alpha_3 - 3)}}{2a_{33}} \), when \( \alpha_3 > 3 \) and is stable when \( \alpha_3 = 3 \).

**Stability of \( E_4 \):**

\[
N_4(t) = N_4(t+1) = N_4(t+2) = \ldots = 0 ;
N_4(t) = N_4(t+1) = N_4(t+2) = \ldots = 0
\]

i.e, \( N_4(t+r) = 0 \), where \( r \) is an integer and \( i = 1, 2 \)

\[
N_4(t) = N_4(t+2) = N_4(t+4) = \ldots = \frac{(\alpha_3 + 1) - \sqrt{(\alpha_3 + 1)(\alpha_3 - 3)}}{2a_{33}}
\]

\[
N_4(t+1) = N_4(t+3) = N_4(t+5) = \ldots = \frac{(\alpha_3 + 1) + \sqrt{(\alpha_3 + 1)(\alpha_3 - 3)}}{2a_{33}}
\]

\[
N_4(t+2r) = \frac{(\alpha_3 + 1) - \sqrt{(\alpha_3 + 1)(\alpha_3 - 3)}}{2a_{33}} ;
\]

\[
N_4(t+2r+1) = \frac{(\alpha_3 + 1) + \sqrt{(\alpha_3 + 1)(\alpha_3 - 3)}}{2a_{33}}
\]

where \( r \) is an integer.

Hence, \( E_4 \) oscillates between \( \frac{(\alpha_3 + 1) - \sqrt{(\alpha_3 + 1)(\alpha_3 - 3)}}{2a_{33}} \) and \( \frac{(\alpha_3 + 1) + \sqrt{(\alpha_3 + 1)(\alpha_3 - 3)}}{2a_{33}} \), when \( \alpha_3 > 3 \) and is stable when \( \alpha_3 = 3 \).

**Stability of \( E_5 \):**

\[
N_5(t) = N_5(t+1) = N_5(t+2) = \ldots = 0 ;
N_5(t) = N_5(t+1) = N_5(t+2) = \ldots = 0
\]

\[
N_5(t) = N_5(t+1) = N_5(t+2) = \ldots = \frac{2}{a_{33}}
\]
i.e, $N_i(t+r) = 0$, $N_3(t+r) = \frac{2}{a_{33}}$, where $r$ is an integer and $i = 1, 2$

Hence, $E_3$ is stable.

**Stability of $E_7$**:

$N_i(t) = N_i(t+1) = N_i(t+2) = \ldots = 0$; $N_3(t) = N_3(t+1) = N_3(t+2) = \ldots = 0$

i.e, $N_i(t+r) = 0$, where $r$ is an integer and $i = 1, 3$

$N_2(t) = N_2(t+2) = N_2(t+4) = \ldots = \frac{(\alpha_2 + 1) + \sqrt{(\alpha_2 + 1)(\alpha_2 - 3)}}{2a_{22}}$

$N_2(t+1) = N_2(t+3) = N_2(t+5) = \ldots = \frac{(\alpha_2 + 1) - \sqrt{(\alpha_2 + 1)(\alpha_2 - 3)}}{2a_{22}}$

i.e, $N_2(t+2r) = \frac{(\alpha_2 + 1) + \sqrt{(\alpha_2 + 1)(\alpha_2 - 3)}}{2a_{22}}$; $N_2(t+2r+1) = \frac{(\alpha_2 + 1) - \sqrt{(\alpha_2 + 1)(\alpha_2 - 3)}}{2a_{22}}$

where $r$ is an integer.

Hence, $E_7$ oscillates between $\frac{(\alpha_2 + 1) + \sqrt{(\alpha_2 + 1)(\alpha_2 - 3)}}{2a_{22}}$ and $\frac{(\alpha_2 + 1) - \sqrt{(\alpha_2 + 1)(\alpha_2 - 3)}}{2a_{22}}$, when \(\alpha_2 > 3\) and is stable when \(\alpha_2 = 3\).

**Stability of $E_8$**:

$N_i(t) = N_i(t+1) = N_i(t+2) = \ldots = 0$; $N_3(t) = N_3(t+1) = N_3(t+2) = \ldots = 0$

i.e, $N_i(t+r) = 0$, where $r$ is an integer and $i = 1, 3$

$N_2(t) = N_2(t+2) = N_2(t+4) = \ldots = \frac{(\alpha_2 + 1) - \sqrt{(\alpha_2 + 1)(\alpha_2 - 3)}}{2a_{22}}$

$N_2(t+1) = N_2(t+3) = N_2(t+5) = \ldots = \frac{(\alpha_2 + 1) + \sqrt{(\alpha_2 + 1)(\alpha_2 - 3)}}{2a_{22}}$

i.e, $N_2(t+2r) = \frac{(\alpha_2 + 1) - \sqrt{(\alpha_2 + 1)(\alpha_2 - 3)}}{2a_{22}}$; $N_2(t+2r+1) = \frac{(\alpha_2 + 1) + \sqrt{(\alpha_2 + 1)(\alpha_2 - 3)}}{2a_{22}}$

where $r$ is an integer.

Hence, $E_8$ oscillates between $\frac{(\alpha_2 + 1) - \sqrt{(\alpha_2 + 1)(\alpha_2 - 3)}}{2a_{22}}$ and $\frac{(\alpha_2 + 1) + \sqrt{(\alpha_2 + 1)(\alpha_2 - 3)}}{2a_{22}}$, when \(\alpha_2 > 3\) and is stable when \(\alpha_2 = 3\).
Stability of $E_9$:
\[
N_1(t) = N_1(t+1) = N_1(t+2) = \ldots = 0; \quad N_3(t) = N_3(t+1) = N_3(t+2) = \ldots = 0
\]
\[
N_2(t) = N_2(t+1) = N_2(t+2) = \ldots = \frac{2}{a_{22}}
\]

i.e, $N_i(t+r) = 0$, $N_2(t+r) = \frac{2}{a_{22}}$, where $r$ is an integer and $i = 1,3$

Hence, $E_9$ is stable.

Stability of $E_{10}$:
\[
N_1(t) = N_1(t+1) = N_1(t+2) = \ldots = 0; \quad N_3(t) = N_3(t+1) = N_3(t+2) = \ldots = \frac{\beta_2 - 1}{a_{22}}
\]
\[
N_4(t) = N_4(t+1) = N_4(t+2) = \ldots = \frac{\alpha_3 - 1}{a_{33}}
\]

i.e, $N_1(t+r) = 0$, $N_2(t+r) = \frac{\beta_2 - 1}{a_{22}}$, $N_3(t+r) = \frac{\alpha_3 - 1}{a_{33}}$, where $r$ is an integer

Hence, $E_{10}$ is stable.

Stability of $E_{11}$:
\[
N_1(t) = N_1(t+1) = N_1(t+2) = \ldots = 0; \quad N_3(t) = N_3(t+1) = N_3(t+2) = \ldots = \frac{\alpha_3 - 1}{a_{33}}
\]

i.e, $N_1(t+r) = 0$, $N_3(t+r) = \frac{\alpha_3 - 1}{a_{33}}$, where $r$ is an integer

\[
N_2(t) = N_2(t+2) = N_2(t+4) = \ldots = \frac{(\beta_2 + 1) + \sqrt{(\beta_2 + 1)(\beta_2 - 3)}}{2a_{22}}
\]
\[
N_2(t+1) = N_2(t+3) = N_2(t+5) = \ldots = \frac{(\beta_2 + 1) - \sqrt{(\beta_2 + 1)(\beta_2 - 3)}}{2a_{22}}
\]

i.e, $N_2(t+2r) = \frac{(\beta_2 + 1) + \sqrt{(\beta_2 + 1)(\beta_2 - 3)}}{2a_{22}}$; $N_2(t+2r+1) = \frac{(\beta_2 + 1) - \sqrt{(\beta_2 + 1)(\beta_2 - 3)}}{2a_{22}}$

where $r$ is an integer.

Hence, $E_{11}$ oscillates between $\frac{(\beta_2 + 1) + \sqrt{(\beta_2 + 1)(\beta_2 - 3)}}{2a_{22}}$ and $\frac{(\beta_2 + 1) - \sqrt{(\beta_2 + 1)(\beta_2 - 3)}}{2a_{22}}$, when $\beta_2 > 3$ and is stable when $\beta_2 = 3$. 

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Stability of $E_{12}$:

$N_1(t) = N_1(t+1) = N_1(t+2) = ... = 0$; $N_3(t) = N_3(t+1) = N_3(t+2) = ... = \frac{\alpha_1 - 1}{a_{33}}$

i.e, $N_1(t + r) = 0$, $N_3(t + r) = \frac{\alpha_1 - 1}{a_{33}}$, where $r$ is an integer

$N_2(t) = N_2(t+2) = N_2(t+4) = ... = \frac{(\beta_2 + 1) - \sqrt{(\beta_2 + 1)(\beta_2 - 3)}}{2a_{22}}$

$N_2(t+1) = N_2(t+3) = N_2(t+5) = ... = \frac{(\beta_2 + 1) + \sqrt{(\beta_2 + 1)(\beta_2 - 3)}}{2a_{22}}$

i.e, $N_2(t + 2r) = \frac{(\beta_2 + 1) - \sqrt{(\beta_2 + 1)(\beta_2 - 3)}}{2a_{22}}$; $N_2(t + 2r + 1) = \frac{(\beta_2 + 1) + \sqrt{(\beta_2 + 1)(\beta_2 - 3)}}{2a_{22}}$

where $r$ is an integer.

Hence, $E_{12}$ oscillates between $\frac{(\beta_2 + 1) - \sqrt{(\beta_2 + 1)(\beta_2 - 3)}}{2a_{22}}$ and $\frac{(\beta_2 + 1) + \sqrt{(\beta_2 + 1)(\beta_2 - 3)}}{2a_{22}}$, when $\beta_2 > 3$ and is stable when $\beta_2 = 3$.

Stability of $E_{13}$:

$N_1(t) = N_1(t+1) = N_1(t+2) = ... = 0$; $N_3(t) = N_3(t+1) = N_3(t+2) = ... = \frac{2}{a_{22}}$

$N_3(t) = N_3(t+1) = N_3(t+2) = ... = \frac{\alpha_1 - 1}{a_{33}}$

i.e, $N_1(t + r) = 0$, $N_3(t + r) = \frac{\alpha_1 - 1}{a_{33}}$, where $r$ is an integer

Hence, $E_{13}$ is stable.

Stability of $E_{14}$:

$N_1(t) = N_1(t+1) = N_1(t+2) = ... = 0$, i.e, $N_1(t + r) = 0$, where $r$ is an integer

$N_2(t) = N_2(t+1) = \frac{\gamma_2 - 1}{a_{22}}$, but $N_2(t+2) \neq N_2(t+3) \neq N_2(t+4) \neq ... \neq \frac{\gamma_2 - 1}{a_{22}}$

i.e, $N_2(t + r) \neq \frac{\gamma_2 - 1}{a_{22}}$, where $r$ is an integer except 0 and 1

$N_3(t) = N_3(t+2) = N_3(t+4) = ... = \frac{(\alpha_3 + 1) + \sqrt{(\alpha_3 + 1)(\alpha_3 - 3)}}{2a_{33}}$
\[ N_3(t+1) = N_3(t+3) = N_3(t+5) = \ldots = \frac{(\alpha_3 + 1) - \sqrt{(\alpha_3 + 1)(\alpha_3 - 3)}}{2a_{33}} \]

i.e, \[ N_3(t+2r) = \frac{(\alpha_3 + 1) + \sqrt{(\alpha_3 + 1)(\alpha_3 - 3)}}{2a_{33}}; \quad N_3(t + 2r + 1) = \frac{(\alpha_3 + 1) - \sqrt{(\alpha_3 + 1)(\alpha_3 - 3)}}{2a_{33}} \]

where \( r \) is an integer.

Hence, \( E_{14} \) is unstable, when \( \alpha_3 > 3 \) and is stable when \( \alpha_3 = 3 \).

**Stability of \( E_{15} \):**

\[ N_3(t) = N_3(t+1) = N_3(t+2) = \ldots = 0, \text{ i.e, } N_3(t+r) = 0, \text{ where } r \text{ is an integer} \]

\[ N_2(t+1) \neq N_2(t+2) \neq N_2(t+3) \neq \ldots \neq N_2(t) \]

i.e, \[ N_3(t+r) = \frac{(\gamma_2 + 1) - \sqrt{(\gamma_2 + 1)(\gamma_2 - 3)}}{2a_{22}}, \text{ where } r \text{ is an integer except 0} \]

\[ N_3(t) = N_3(t+2) = N_3(t+4) = \ldots = \frac{(\alpha_3 + 1) + \sqrt{(\alpha_3 + 1)(\alpha_3 - 3)}}{2a_{33}} \]

\[ N_3(t+1) = N_3(t+3) = N_3(t+5) = \ldots = \frac{(\alpha_3 + 1) - \sqrt{(\alpha_3 + 1)(\alpha_3 - 3)}}{2a_{33}} \]

i.e, \[ N_3(t+2r) = \frac{(\alpha_3 + 1) + \sqrt{(\alpha_3 + 1)(\alpha_3 - 3)}}{2a_{33}}; \quad N_3(t + 2r + 1) = \frac{(\alpha_3 + 1) - \sqrt{(\alpha_3 + 1)(\alpha_3 - 3)}}{2a_{33}} \]

where \( r \) is an integer.

Hence, \( E_{15} \) is unstable, when \( \alpha_3 > 3 \) and is oscillatory when \( \alpha_3 = 3 \).

**Stability of \( E_{16} \):**

\[ N_3(t) = N_3(t+1) = N_3(t+2) = \ldots = 0, \text{ i.e, } N_3(t+r) = 0, \text{ where } r \text{ is an integer} \]

\[ N_2(t+1) \neq N_2(t+2) \neq N_2(t+3) \neq \ldots \neq N_2(t) \]

i.e, \[ N_3(t+r) = \frac{(\gamma_2 + 1) + \sqrt{(\gamma_2 + 1)(\gamma_2 - 3)}}{2a_{22}}, \text{ where } r \text{ is an integer except 0} \]

\[ N_3(t) = N_3(t+2) = N_3(t+4) = \ldots = \frac{(\alpha_3 + 1) + \sqrt{(\alpha_3 + 1)(\alpha_3 - 3)}}{2a_{33}} \]

\[ N_3(t+1) = N_3(t+3) = N_3(t+5) = \ldots = \frac{(\alpha_3 + 1) - \sqrt{(\alpha_3 + 1)(\alpha_3 - 3)}}{2a_{33}} \]

i.e, \[ N_3(t+2r) = \frac{(\alpha_3 + 1) + \sqrt{(\alpha_3 + 1)(\alpha_3 - 3)}}{2a_{33}}; \quad N_3(t + 2r + 1) = \frac{(\alpha_3 + 1) - \sqrt{(\alpha_3 + 1)(\alpha_3 - 3)}}{2a_{33}} \]

where \( r \) is an integer.
Hence, $E_{16}$ is unstable, when $\alpha_\gamma > 3$ and is oscillatory when $\alpha_\gamma = 3$.

**Stability of $E_{17}$:**

$N_1(t) = N_1(t+1) = N_1(t+2) = ... = 0$, i.e., $N_1(t+r) = 0$, where $r$ is an integer

$N_2(t) = \frac{2}{a_{22}}$, but $N_2(t+2) \neq N_2(t+3) \neq N_2(t+4) \neq ... \neq \frac{2}{a_{22}}$

i.e., $N_2(t+r) \neq \frac{2}{a_{22}}$, where $r$ is an integer except 0 and 1

$N_3(t) = N_3(t+2) = N_3(t+4) = ... = \frac{(\alpha_\gamma + 1) + \sqrt{(\alpha_\gamma + 1)(\alpha_\gamma - 3)}}{2a_{33}}$

$N_3(t+1) = N_3(t+3) = N_3(t+5) = ... = \frac{(\alpha_\gamma + 1) - \sqrt{(\alpha_\gamma + 1)(\alpha_\gamma - 3)}}{2a_{33}}$

i.e., $N_3(t + 2r) = \frac{(\alpha_\gamma + 1) + \sqrt{(\alpha_\gamma + 1)(\alpha_\gamma - 3)}}{2a_{33}}$; $N_3(t + 2r + 1) = \frac{(\alpha_\gamma + 1) - \sqrt{(\alpha_\gamma + 1)(\alpha_\gamma - 3)}}{2a_{33}}$

where $r$ is an integer.

Hence, $E_{17}$ is unstable, when $\alpha_\gamma > 3$ and is stable when $\alpha_\gamma = 3$.

**Stability of $E_{18}$:**

$N_1(t) = N_1(t+1) = N_1(t+2) = ... = 0$, i.e., $N_1(t+r) = 0$, where $r$ is an integer

$N_2(t) = N_2(t+1) = \frac{\mu_2 - 1}{a_{22}}$, but $N_2(t+2) \neq N_2(t+3) \neq N_2(t+4) \neq ... \neq \frac{\mu_2 - 1}{a_{22}}$

i.e., $N_2(t+r) \neq \frac{\mu_2 - 1}{a_{22}}$, where $r$ is an integer except 0 and 1

$N_3(t) = N_3(t+2) = N_3(t+4) = ... = \frac{(\alpha_\gamma + 1) - \sqrt{(\alpha_\gamma + 1)(\alpha_\gamma - 3)}}{2a_{33}}$

$N_3(t+1) = N_3(t+3) = N_3(t+5) = ... = \frac{(\alpha_\gamma + 1) + \sqrt{(\alpha_\gamma + 1)(\alpha_\gamma - 3)}}{2a_{33}}$

i.e., $N_3(t + 2r) = \frac{(\alpha_\gamma + 1) - \sqrt{(\alpha_\gamma + 1)(\alpha_\gamma - 3)}}{2a_{33}}$; $N_3(t + 2r + 1) = \frac{(\alpha_\gamma + 1) + \sqrt{(\alpha_\gamma + 1)(\alpha_\gamma - 3)}}{2a_{33}}$

where $r$ is an integer.

Hence, $E_{18}$ is unstable, when $\alpha_\gamma > 3$ and is stable when $\alpha_\gamma = 3$.

**Stability of $E_{19}$:**

$N_1(t) = N_1(t+1) = N_1(t+2) = ... = 0$, i.e., $N_1(t+r) = 0$, where $r$ is an integer
\( N_2(t+1) \neq N_2(t+2) \neq N_2(t+3) \neq ... \neq N_2(t) \)

i.e, \( N_2(t+r) \neq \frac{(\mu_2+1)+\sqrt{(\mu_2+1)(\mu_2-3)}}{2a_{22}} \), where \( r \) is an integer except 0

\( N_3(t) = N_3(t+2) = N_3(t+4) = ... = \frac{(\alpha_3 + 1) - \sqrt{((\alpha_3+1)(\alpha_3-3)}}{2a_{33}} \)

\( N_3(t+1) = N_3(t+3) = N_3(t+5) = ... = \frac{(\alpha_3 + 1) + \sqrt{((\alpha_3+1)(\alpha_3-3)}}{2a_{33}} \)

i.e, \( N_3(t+2r) = \frac{(\alpha_3 + 1) - \sqrt{((\alpha_3+1)(\alpha_3-3)}}{2a_{33}} ; N_3(t + 2r+1) = \frac{(\alpha_3 + 1) + \sqrt{((\alpha_3+1)(\alpha_3-3)}}{2a_{33}} \)

where \( r \) is an integer.

Hence, \( E_{19} \) is unstable, when \( \alpha_3 > 3 \) and is oscillatory when \( \alpha_3 = 3 \).

**Stability of \( E_{20} \):**

\( N_i(t) = N_i(t+1) = N_i(t+2) = ... = 0 \), i.e, \( N_i(t+r) = 0 \), where \( r \) is an integer

\( N_2(t+1) \neq N_2(t+2) \neq N_2(t+3) \neq ... \neq N_2(t) \)

i.e, \( N_2(t+r) \neq \frac{(\mu_2+1)+\sqrt{(\mu_2+1)(\mu_2-3)}}{2a_{22}} \), where \( r \) is an integer except 0

\( N_3(t) = N_3(t+2) = N_3(t+4) = ... = \frac{(\alpha_3 + 1) - \sqrt{((\alpha_3+1)(\alpha_3-3)}}{2a_{33}} \)

\( N_3(t+1) = N_3(t+3) = N_3(t+5) = ... = \frac{(\alpha_3 + 1) + \sqrt{((\alpha_3+1)(\alpha_3-3)}}{2a_{33}} \)

i.e, \( N_3(t+2r) = \frac{(\alpha_3 + 1) - \sqrt{((\alpha_3+1)(\alpha_3-3)}}{2a_{33}} ; N_3(t + 2r+1) = \frac{(\alpha_3 + 1) + \sqrt{((\alpha_3+1)(\alpha_3-3)}}{2a_{33}} \)

where \( r \) is an integer.

Hence, \( E_{20} \) is unstable, when \( \alpha_3 > 3 \) and is oscillatory when \( \alpha_3 = 3 \).

**Stability of \( E_{21} \):**

\( N_i(t) = N_i(t+1) = N_i(t+2) = ... = 0 \), i.e, \( N_i(t+r) = 0 \), where \( r \) is an integer

\( N_2(t) = N_2(t+1) = \frac{2}{a_{22}} \), but \( N_2(t+2) \neq N_2(t+3) \neq N_2(t+4) \neq ... \neq \frac{2}{a_{22}} \)

i.e, \( N_2(t+r) \neq \frac{2}{a_{22}} \), where \( r \) is an integer except 0 and 1
\[ N_3(t) = N_3(t+2) = N_3(t+4) = \ldots = \frac{(\alpha_3 + 1) - \sqrt{((\alpha_3+1)(\alpha_3 - 3)}}{2a_{33}} \]

\[ N_3(t+1) = N_3(t+3) = N_3(t+5) = \ldots = \frac{(\alpha_3 + 1) + \sqrt{((\alpha_3+1)(\alpha_3 - 3)}}{2a_{33}} \]

i.e, \[ N_3(t + 2r) = \frac{(\alpha_3 + 1) - \sqrt{((\alpha_3+1)(\alpha_3 - 3)}}{2a_{33}}; \quad N_3(t + 2r+1) = \frac{(\alpha_3 + 1) + \sqrt{((\alpha_3+1)(\alpha_3 - 3)}}{2a_{33}} \]

where \( r \) is an integer.

Hence, \( E_{21} \) is unstable, when \( \alpha_3 > 3 \) and is stable when \( \alpha_3 = 3 \).

**Stability of \( E_{22} \):**

\[ N_4(t) = N_4(t+1) = N_4(t+2) = \ldots = 0; \quad N_2(t) = N_2(t+1) = N_2(t+2) = \ldots = \frac{\delta_2 - 1}{a_{22}} \]

\[ N_3(t) = N_3(t+1) = N_3(t+2) = \ldots = \frac{2}{a_{33}} \]

i.e, \[ N_1(t + r) = 0, \quad N_2(t + r) = \frac{\delta_2 - 1}{a_{22}}, \quad N_3(t + r) = \frac{2}{a_{33}} \], where \( r \) is an integer

Hence, \( E_{22} \) is stable.

**Stability of \( E_{23} \):**

\[ N_4(t) = N_4(t+1) = N_4(t+2) = \ldots = 0; \quad N_3(t) = N_3(t+1) = N_3(t+2) = \ldots = \frac{2}{a_{33}} \]

i.e, \[ N_1(t + r) = 0, \quad N_3(t + r) = \frac{2}{a_{33}} \], where \( r \) is an integer

\[ N_2(t) = N_2(t+2) = N_2(t+4) = \ldots = \frac{(\delta_2 + 1) + \sqrt{(\delta_2+1)(\delta_2 - 3)}}{2a_{22}} \]

\[ N_2(t+1) = N_2(t+3) = N_2(t+5) = \ldots = \frac{(\delta_2 + 1) - \sqrt{(\delta_2+1)(\delta_2 - 3)}}{2a_{22}} \]

i.e, \[ N_2(t + 2r) = \frac{(\delta_2 + 1) + \sqrt{(\delta_2+1)(\delta_2 - 3)}}{2a_{22}}; \quad N_2(t + 2r+1) = \frac{(\delta_2 + 1) - \sqrt{(\delta_2+1)(\delta_2 - 3)}}{2a_{22}} \]

where \( r \) is an integer.
Hence, $E_{23}$ oscillates between $\frac{(\delta_2 + 1) - \sqrt{(\delta_2 + 1)(\delta_2 - 3)}}{2a_{22}}$ and $\frac{(\delta_2 + 1) + \sqrt{(\delta_2 + 1)(\delta_2 - 3)}}{2a_{22}}$, when 

$\delta_2 > 3$ and is stable when $\delta_2 = 3$.

**Stability of $E_{24}$:**

$N_1(t) = N_1(t+1) = N_1(t+2) = \ldots = 0$; $N_3(t) = N_3(t+1) = N_3(t+2) = \ldots = \frac{2}{a_{33}}$,

i.e, $N_1(t+r) = 0, N_3(t+r) = \frac{2}{a_{33}}$, where $r$ is an integer

$N_2(t) = N_2(t+2) = N_2(t+4) = \ldots = \frac{(\delta_2 + 1) - \sqrt{(\delta_2 + 1)(\delta_2 - 3)}}{2a_{22}}$.

$N_2(t+1) = N_2(t+3) = N_2(t+5) = \ldots = \frac{(\delta_2 + 1) + \sqrt{(\delta_2 + 1)(\delta_2 - 3)}}{2a_{22}}$.

i.e, $N_2(t+2r) = \frac{(\delta_2 + 1) - \sqrt{(\delta_2 + 1)(\delta_2 - 3)}}{2a_{22}}$; $N_2(t+2r+1) = \frac{(\delta_2 + 1) + \sqrt{(\delta_2 + 1)(\delta_2 - 3)}}{2a_{22}}$

where $r$ is an integer.

Hence, $E_{24}$ oscillates between $\frac{(\delta_2 + 1) - \sqrt{(\delta_2 + 1)(\delta_2 - 3)}}{2a_{22}}$ and $\frac{(\delta_2 + 1) + \sqrt{(\delta_2 + 1)(\delta_2 - 3)}}{2a_{22}}$, when 

$\delta_2 > 3$ and is stable when $\delta_2 = 3$.

**Stability of $E_{25}$:**

$N_1(t) = N_1(t+1) = N_1(t+2) = \ldots = 0$; $N_2(t) = N_2(t+1) = N_2(t+2) = \ldots = \frac{2}{a_{22}}$

$N_3(t) = N_3(t+1) = N_3(t+2) = \ldots = \frac{2}{a_{33}}$

i.e, $N_1(t+r) = 0, N_2(t+r) = \frac{2}{a_{22}}, N_3(t+r) = \frac{2}{a_{33}}$, where $r$ is an integer

Hence, $E_{25}$ is stable, when $\delta_2 = 3, \alpha_3 = 3$.

At this stage, in all twenty five equilibrium states, only the nine states $E_1, E_2, E_5, E_6, E_9, E_{10}, E_{13}, E_{22}, E_{25}$ are stable and $E_3, E_4, E_7, E_8, E_{11}, E_{12}, E_{23}, E_{24}$ are oscillatory and remaining all eight are unstable.

4. **NUMERICAL EXAMPLES**

The numerical solutions of the discrete model equations computed for specific values of the various parameters and the initial conditions. The results are illustrated in Figures 4.1 to 4.4.
Figure 4.1: Variation of $N_1, N_2$ and $N_3$ against time($t$) for $\alpha_1 = 1.9, \alpha_2 = 3.6, \alpha_3 = 2.8, a_{12} = 0.3, a_{13} = 4.7, a_{22} = 0.8, a_{33} = 0.5, a_{23} = 1.2,$ $N_1(0) = 0, N_2(0) = 9.71, N_3(0) = 3.6$

Figure 4.2: Variation of $N_1, N_2$ and $N_3$ against time($t$) for $\alpha_1 = 2.5, \alpha_2 = 1.6, \alpha_3 = 3, a_{12} = 0.53, a_{13} = 3.7, a_{22} = 0.76, a_{33} = 0.4, a_{23} = 1.2,$ $N_1(0) = 0, N_2(0) = 10, N_3(0) = 5$
Figure 4.3: Variation of $N_1, N_2$ and $N_3$ against time($t$) for
\[\alpha_1 = 0.8, \alpha_2 = 1.3, \alpha_3 = 4.5, a_{12} = 2.8, a_{13} = 3.3, a_{22} = 1.4, a_{33} = 0.6, a_{23} = 0.2,\]
\[N_1(0) = 0, N_2(0) = 1.21, N_3(0) = 6.98\]

Figure 4.4: Variation of $N_1, N_2$ and $N_3$ against time($t$) for
\[\alpha_1 = 3.6, \alpha_2 = 4.8, \alpha_3 = 1.1, a_{12} = 0.3, a_{13} = 2, a_{22} = 0.35, a_{33} = 6.3, a_{23} = 3.2,\]
\[N_1(0) = 0, N_2(0) = 14.47, N_3(0) = 0\]
5. CONCLUSION

The present paper deals with an investigation on a discrete model of three species syn ecosystem with unlimited resources for the first species. The system comprises of a commensal ($S_1$) common to two hosts $S_2$ and $S_3$ ie., $S_2$ and $S_3$ both benefit $S_1$, without getting themselves effected either positively or adversely. Further $S_2$ is a commensal of $S_3$, $S_3$ is a host of both $S_1$, $S_2$. All possible equilibrium points of the model are identified based on the model equations at two stages.

Stage-I : $N_i(t+1) = N_i(t); i=1,2,3$
Stage-II : $N_i(t+2) = N_i(t); i=1,2,3$

In Stage-I there are only five equilibrium points, while the Stage-II there would be twenty five equilibrium points. All the five equilibrium points in Stage-I are found to be stable while in stage-II only nine are stable. Further the numerical solutions for the discrete model equations are computed.

REFERENCES


