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Optimization of Transportation Logistics: A Model for the Delivery Routes Problems

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ABSTRACT

Logistics is a key element of economic development. It is a set of activities which consists of managing physical, financial and information flows of an organization. Its objective is to optimize procurement, production, management of stocks and transportation, in order to avoid many losses. In our study, we focus on transportation management by treating the problem of optimizing delivery routes. It is about optimizing all the components in order to deliver to customers the products on requested quantity in the right time and requested location at a minimum cost. The different obtained results are; firstly, the development of a model that combines the different constraints of time, distance, capacity and compatibility. Secondly, the realization of an example for solving the shortest path problem, in the form of a comparison between two vehicle routing problem (VRP) methods [3], which are the Branch and Bound method and the Clarke and Wright algorithm [1] with the deviation method, applied to real data extracted from a geographic information system (GIS).

Key words and phrases: Logistics, management of stocks and transportation, optimization of delivery routes, shortest path problem, VRP methods, branch and bound method, Clarke and Wright algorithm, deviation method.

MSC (2010): 90B06, 90B20, 90C27, 90C30, 90C26.

1. INTRODUCTION

The problem can be defined as a problem that involves many clients to be served from a single depot with known demands of distinct products and heterogeneous vehicles. Mathematically, it can be defined on a path $T = (VL, A)$ where $VL = \{vl_0, \dots, vl_N\}$ represents all the cities to be visited and $A = \{(vli, vlj) : vli, vlj \in VL, i < j\}$ represents the set of possible arcs. The point vl_0 represents the supplier depot which is the point of departure and arrival of all deliveries. A d_{ij} distance is associated to each arc $(i, j) \in A$, with $d_{ij} = d_{ji}$ ∀ i, j ∈ A. The fleet is composed of K vehicles where the recharging capacity is Q_k for product delivery. The maximum duration of a path is t_{max}. The customer demand for the product i is q_i. Another constraint that also adds is the compatibility between products and vehicles.

2. MAIN RESULTS

2.1. Mathematical Model

2.1.1. Compatibility constraints

In a delivery planning, we must be taking into consideration that vehicles and products may have constraints of incompatibility or compatibility and conditioning to be respected.

Depending on the nature of product, there are conditions that it requires to be transported; it must be transported on specific vehicles. For example:

- A product may require temperature conditions;
- A product must be introduced in a covered vehicle;
- A product may require specific equipments... etc.

Mathematically, we can define this constraint as follows:

Let $P = \{P_1, P_2, \dots, P_n\}$ be a set of products, V= $\{V_1, V_2, \dots, V_m\}$ a set of vehicles and let $C = \{C_1, C_2, \dots, C_q\}$ be a set of conditions between P and V. $\forall P_i \in P$ and $\forall V_j \in V$, if $\alpha = \{C_1, C_2, \dots, C_r\} \subset C$ are the conditions that the product P_i requires to be transported and $\beta = \{C_1, C_2, \dots, C_k\} \subset C$ are the characteristics of the vehicle V_j, in order to check whether the product P_i can be transported on the vehicle V_j, we pose:

$$\varphi_{ij} = \varphi(P_i, V_j) = \begin{cases} 1, & if \quad \alpha \subset \beta \ and \ \forall\, C_k \in \alpha,\ \alpha_{C_k} \equiv \beta_{C_k} \\ 0, & else \end{cases}$$

Therefore, φ is equal to 1 if the product i is delivered by the vehicle j (i.e. φ is equal to 1 if the conditions that require the product i to be transported are equivalent to the characteristics of the vehicle j which will transport it), and 0 otherwise.

2.1.2. Parameters

We consider the following parameters:

m: Number of available vehicles planned for delivery.

N: Number of cities to be visited (The cities are numbered from 1 to N (customer depot) and the departure city is number 0 (supplier depot)).

Q_k: Capacity of vehicle k.

q_i : Quantity of product i.

d_{ij}: Distance between city i and j.

t_{ij} : Duration between city i and city j.

t_{max} : Maximum duration of delivery.

2.1.3. Decision variables

φ_{ik}: Binary decision variable;

x_{ijk}: Binary decision variable that is equal to 1 if the vehicle k travels from the city i to city j and 0 otherwise.

2.1.4. Formulation of the Problem:

We consider the following minimization problem:

$$\text{Minimize } \left(\sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{k=1}^{m} d_{ij}\,x_{ijk}\right) \tag{2.1}$$

Subject to constraints:

$$\sum_{i=1}^{n} q_i\varphi_{ik} \le Q_k \qquad 1 \le k \le m \tag{2.2}$$

$$\sum_{j=1}^{N} x_{0jk} = 1 \qquad \forall\, 1 \le k \le m \tag{2.3}$$

$$\sum_{i=1}^{N} x_{i0k} = 1 \qquad \forall\, 1 \le k \le m \tag{2.4}$$

$$\sum_{i=0}^{N}\sum_{j=0}^{N} t_{ij}\,x_{ijk} \le t_{max} \quad 1 \le k \le m \tag{2.5}$$

$$\sum_{k=1}^{m} \sum_{j=1}^{N} x_{0jk} \leq m \qquad (2.6)$$

$$\sum_{i \in S} \sum_{j \in S} x_{ijk} \leq |S| - 1 \qquad (2.7)$$

$\forall\, S \subset VL \; and \; 2 \leq |S| \leq N-2,$

$k = 1, \dots, m$

$$x_{ijk} = 0 \; or \; 1 \qquad (2.8)$$

$i = 0, \dots, N; \; j = 0, \dots, N; \; k = 1, \dots, m.$

$$\varphi_{ik} = 0 \; or \; 1 \qquad (2.9)$$

$i = 1, \dots, n; \; k = 1, \dots, m$.

This formulation helps to minimize the travelled distance by all vehicles.

- The constraint (2.2) ensures that the loading of the vehicles respects their capacity.
- The constraints (2.3) and (2.4) ensure that every tour begins and ends at the supplier depot.
- The constraint (2.5) ensures that the tour time of delivery does not exceed the maximum duration planned for delivery.
- The constraint (2.6) ensures that the number of vehicles does not exceed the number of planned vehicles for delivery.
- The constraint (2.7) helps to avoid sub-towers. It is a used constraint for TSP [2].
- The constraints (2.8) and (2.9) are binary constraints.

2.1.5. Example for solving the Shortest Path Problem

In a delivery planning, after selecting the suitable vehicles to carry out the products with the requested quantities, it remains to define the optimal path of the ordered delivery.

2.1.5.1 Comparison between the Deviation Method and the Exact Method; Branch and Bound

Let D_0 be a supplier depot and D_1, D_2, D_3 and D_4 customer depots:

	D_0	D_1	D_2	D_3	D_4
D_0		11	9	10	8
D_1	11		11	6	7
D_2	9	11		9	10
D_3	10	6	9		7
D_4	8	7	10	7	

Tab. 1: Distance between depots

a. Resolution with the Deviation Method

The Deviation Method is a logistical tool used for managing delivery routes and organizing the distribution in an optimal way to minimize the cost of delivery. This method proceeds in several steps:

Step 1: Collect information

The table above represents the distance between the various depots.

Step 2: Calculate the deviations

- Number of deviations= $\frac{4*3}{2}$ = 6 deviations
- Deviations:

ΔD_1D_2	9
ΔD_1D_3	15
ΔD_1D_4	12
ΔD_2D_3	10
ΔD_2D_4	7
ΔD_3D_4	11

Step 3: Classify the deviations in descending order.

ΔD_1D_3	15
ΔD_1D_4	12
ΔD_3D_4	11
ΔD_2D_3	10
ΔD_1D_2	9
ΔD_2D_4	7

Step 4: Select the couples of points by avoiding those forming a fork or a loop with the already selected couples.

ΔD_1D_3	15
ΔD_1D_4	12
ΔD_3D_4	11
ΔD_2D_3	10
ΔD_1D_2	9
ΔD_2D_4	7

Step 5: Results

After selecting all the connections, each extremity must be added to the depot (D_0).

- Retained tour $D_0D_2D_3D_1D_4D_0$
- Final distance: 39 Km.

b. Resolution with Branch and Bound method:

The separation and evaluation algorithm, well known by Branch and Bound (B&B) [4], is based on the separation (branch) of the set of solutions in smaller subsets. The exploration of these solutions uses an optimistic evaluation to increase (bound) the subsets, which makes it possible to consider only those that could contain a potentially solution better than the current solution.

By applying this method on the data of the above table (tab.1), we obtain:

-	11	9	10	8
11	-	11	6	7
9	11	-	9	10
10	6	9	-	7
8	7	10	7	-

-	3	1	2	0
5	-	5	0	1
0	2	-	0	1
4	0	3	-	1
1	0	3	0	-

-	3	0	2	0
5	-	4	0	1
0	2	-	0	1
4	0	2	-	1
1	0	2	0	-

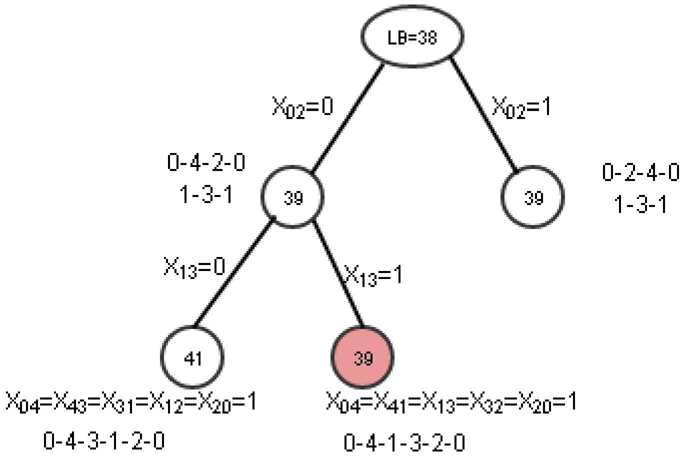
-	3	0	2	0
5	-	4	0	1
0	2	-	0	1
4	0	2	-	1
1	0	2	0	-

$\theta = 1$

-	4	0	3	0
4	-	3	0	0
0	2	-	1	1
3	0	1	-	0
0	0	1	0	-

We have $X_{02}=X_{20}= X_{13}=X_{34}=X_{41}=1$ with Z=9+9+6+7+7=38. We notice that there are two sub-towers in this solution: $X_{02}=X_{20}=1$ and $X_{13}=X_{34}=X_{41}=1$

So, Z is an unfeasible solution of the problem. In this case, we take Z as a lower bound, and we start the calculations:



- Total minimal distance: 39 km

  Retained tour: 041320

2.1.5.2. Comparison between the Deviation Method and the Heuristic Clark & Wright algorithm

We take D_0 Kénitra (city in Morocco) a supplier depot and D_1, D_2, ..., D_{10} customer depots:

	D_0	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9	D_{10}
D_0		39	134	358	468	606	131	552	192	231	499
D_1			95	320	478	567	149	513	154	254	460
D_2				233	527	480	253	409	106	255	390
D_3					502	247	459	256	237	327	204
D_4						674	335	753	417	290	299
D_5							706	173	485	575	383
D_6								665	233	154	562
D_7									494	584	456
D_8										147	340
D_9											437
D_{10}											

D_0	Kenitra
D_1	Rabat
D_2	Casablanca
D_3	Marrakech
D_4	Errachidia
D_5	Agadir
D_6	Meknes
D_7	Essaouira
D_8	Khouribga
D_9	Khenifra
D_{10}	Ouarzazate

Tab. 2: Distance between depots

a. Resolution with the Deviation Method

Step 1: Collect information

Fig. 1: Geographical location of depots

Step 2: Calculate the deviations

- Number of deviations = $\frac{10\times9}{2} = 45$ deviations
- Deviations:

ΔD_1D_2	77.9	ΔD_2D_3	259	ΔD_3D_4	324	ΔD_4D_5	400	ΔD_5D_6	31	ΔD_6D_7	18	ΔD_7D_8	250	ΔD_8D_9	276	ΔD_9D_{10}	293
ΔD_1D_3	77.3	ΔD_2D_4	75	ΔD_3D_5	717	ΔD_4D_6	264	ΔD_5D_7	985	ΔD_6D_8	90	ΔD_7D_9	199	ΔD_8D_{10}	351		
ΔD_1D_4	29.3	ΔD_2D_5	260	ΔD_3D_6	30	ΔD_4D_7	267	ΔD_5D_8	313	ΔD_6D_9	208	ΔD_7D_{10}	595				
ΔD_1D_5	78.3	ΔD_2D_6	12	ΔD_3D_7	654	ΔD_4D_8	243	ΔD_5D_9	262	ΔD_6D_{10}	68						
ΔD_1D_6	21.3	ΔD_2D_7	277	ΔD_3D_8	313	ΔD_4D_9	409	ΔD_5D_{10}	722								
ΔD_1D_7	78.3	ΔD_2D_8	220	ΔD_3D_9	262	ΔD_4D_{10}	668										
ΔD_1D_8	77.3	ΔD_2D_9	110	ΔD_3D_{10}	653												
ΔD_1D_9	16.3	ΔD_2D_{10}	243														
ΔD_1D_{10}	78.3																

Step 3: Classify the differences in descending order.

ΔD_5D_7	985	ΔD_9D_{10}	293	ΔD_6D_9	208	ΔD_3D_6	30
ΔD_5D_{10}	722	ΔD_2D_7	277	ΔD_7D_9	199	ΔD_1D_4	29.3
ΔD_3D_5	717	ΔD_8D_9	276	ΔD_2D_9	110	ΔD_1D_6	21.3
ΔD_4D_{10}	668	ΔD_4D_7	267	ΔD_6D_8	90	ΔD_6D_7	18
ΔD_3D_7	654	ΔD_4D_6	264	ΔD_1D_5	78.3	ΔD_1D_9	16.3
ΔD_3D_{10}	653	ΔD_3D_9	262	ΔD_1D_7	78.3	ΔD_2D_6	12
ΔD_7D_{10}	595	ΔD_5D_9	262	ΔD_1D_{10}	78.3		
ΔD_4D_9	409	ΔD_2D_5	260	ΔD_1D_2	77.9		
ΔD_4D_5	400	ΔD_2D_3	259	ΔD_1D_3	77.33		
ΔD_8D_{10}	351	ΔD_7D_8	250	ΔD_1D_8	77.3		
ΔD_3D_4	324	ΔD_2D_{10}	243	ΔD_2D_4	75		
ΔD_3D_8	313	ΔD_4D_8	243	ΔD_6D_{10}	68		
ΔD_5D_8	313	ΔD_2D_8	220	ΔD_5D_6	31		

Step 4: Select the couples of points by avoiding those forming a fork or a loop with the already selected couples.

ΔD_5D_7	985	ΔD_9D_{10}	293	ΔD_6D_9	208	ΔD_3D_6	30
ΔD_5D_{10}	722	ΔD_2D_7	277	ΔD_7D_9	199	ΔD_1D_4	29.3
ΔD_3D_5	717	ΔD_8D_9	276	ΔD_2D_9	110	ΔD_1D_6	21.3
ΔD_4D_{10}	668	ΔD_4D_7	267	ΔD_6D_8	90	ΔD_6D_7	18
ΔD_3D_7	654	ΔD_4D_6	264	ΔD_1D_5	78.3	ΔD_1D_9	16.3
ΔD_3D_{10}	653	ΔD_3D_9	262	ΔD_1D_7	78.3	ΔD_2D_6	12
ΔD_7D_{10}	595	ΔD_5D_9	262	ΔD_1D_{10}	78.3		
ΔD_4D_9	409	ΔD_2D_5	260	ΔD_1D_2	77.9		
ΔD_4D_5	400	ΔD_2D_3	259	ΔD_1D_3	77.33		
ΔD_8D_{10}	351	ΔD_7D_8	250	ΔD_1D_8	77.3		
ΔD_3D_4	324	ΔD_2D_{10}	243	ΔD_2D_4	75		
ΔD_3D_8	313	ΔD_4D_8	243	ΔD_6D_{10}	68		
ΔD_5D_8	313	ΔD_2D_8	220	ΔD_5D_6	31		

After the elimination of couples that form forks or loops, we have the following couples:

$$(D_5, D_7), (D_5, D_{10}), (D_4, D_{10}), (D_3, D_7), (D_4, D_9), (D_3, D_8), (D_6, D_9), (D_1, D_2), (D_2, D_8)$$

Step 5: Results

After selecting all the connections, each extremity must be added to the depot (D_0).

Fig. 2: Final track of delivery

- Retained tour: $D_0 D_1 D_2 D_8 D_3 D_7 D_5 D_{10} D_4 D_9 D_6 D_0$
- Total minimal distance: 2163 Km

b. Resolution with Clarke & Wright algorithm

The classical algorithm of Clarke and Wright [1] is one of the best known heuristic for VRP. It begins with a tour dedicated to each client to connect it to the single depot. The result is an initial solution. Then the goal is to merge the tours obtained to reduce the cost of the current solution.

By integrating the data from the table above (tab.2), into an application that implements this algorithm, we obtain:

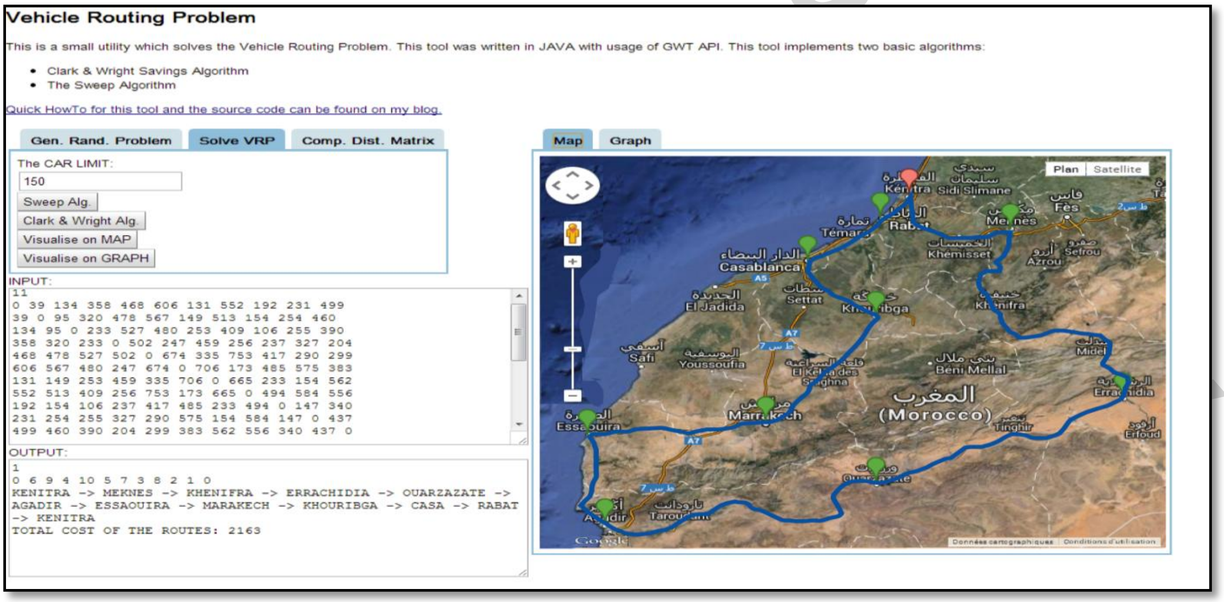


Fig. 3: Clarke & Wright Algorithm.

- Retained tour: Kénitra→ Méknes →Khénifra→Er-Rachidia→Ouarzazate→ Agadir →Essaouira→ Marrakech →Khouribga→ Casablanca → Rabat →Kenitra.
- Total minimal distance: 2163 Km.

3. DISCUSSION AND CONCLUSION

In this paper, we have been interested on transportation management by treating the problem of optimizing delivery routes.

Our first result is the elaboration of a mathematical model which combines a set of constraints: time, distance, capacity, and prohibition of sub-tours...with the addition of a new compatibility constraint between vehicles and products ($\varphi\left(P_i, V_j\right)$).

After planning the various constraints mentioned above, the choice of the method of optimization of the shortest path has been the object of two comparisons between three methods: the deviation method with the exact method (B&B) and the heuristic algorithm (Clarke and Wright [1]). The calculation was carried out over real city distances, extracted from a geographic information system (GIS). This comparison allowed us to have the same optimal results for similar study cases.

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