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Unsteady MHD Free Convective Flow Through Porous Media Past on Moving Vertical Plate with Variable Temperature and Viscous Dissipation

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ABSTRACT

The problem of an unsteady flow through porous media past on moving vertical plate with variable temperature in the presence of inclined magnetic field and viscous dissipation is studied. The governing equations are discritized by Galerkin finite-element method and are being solved by Crank-Nicolson method using C-Program. Numerical results for the velocity and temperature are presented graphically for various values of the parameter.

INTRODUCTION

The study of heat and mass transfer problems are important in many reactive chemicals and chemical formulations. The analysis of free convection flows is great concern due to its useful applications in many branches of engineering and sciences eg. Geophysics, agriculture and thermal insulation. Elbashbeshy [1] investigated heat and mass transfer along a vertical plate with variable surface temperature and concentration in the presence of magnetic field. Jai singh [2] studied viscous dissipation and chemical reaction effects on flow past a stretching porous surface in a porous medium. Helmy KA [3] studied the MHD unsteady free convection flow past a vertical porous plate. Chamkha AJ *et.al* [4] presented unsteady MHD free convection flow past an exponentially accelerated vertical plate with mass transfer, chemical reaction and thermal radiation. Alao FI *et.al* [5] analyzed the effects of thermal radiation, Soret and Dufour on an unsteady heat and mass transfer flow of a chemically reacting fluid past a semi – infinite vertical plate with viscous dissipation. Free convection heat and mass transfer MHD flow in a vertical channel in the presence of chemical reaction was presented by Barik RN [6]. Tanvir Ahmed and Md. Mahumad [7] investigated a finite difference solution of MHD mixed convection flow with heat generation and chemical reaction. Mazumdar MK and Deka RK [8] have analyzed MHD flow past an impulsive started infinite vertical plate in the presence of thermal radiation. Mohmed Abd Ei-Aziz [9] presented unsteady mixed convection heat transfer along a vertical stretching surface with variable viscosity and viscous dissipation. Raptis A *et.al* [10] examined the effect of thermal radiation on MHD flow.

The objective of this paper is to study an unsteady flow through porous media past on moving vertical plate with variable temperature in the presence of inclined magnetic field and viscous dissipation.

FORMULATION OF THE PROBLEM

An unsteady MHD flow of a viscous incompressible fluid by a vertical plate and the plate impulsively started moving with velocity u_0 taking into account in the presence of viscous dissipation. The x^* –axis is taken along the vertical plate in the upward direction and the flow is assumed to be in this direction and y^* - axis normal to the plate. A uniform magnetic field B_0 is assumed to be applied on the plate with angle α. Initially the plate and the fluid are at same temperature T_∞^*. At time $t^* > 0$, temperature of the plate is increased to T_w^*. The governing equations of flow field are as under follows:

Momentum equation:

$$\frac{\partial u^*}{\partial t^*} = \nu \frac{\partial^2 u^*}{\partial y^{*2}} + g\beta\left(T^* - T^*_\infty\right) + \frac{\sigma B_0^2 \sin^2 \alpha}{\rho} - \frac{\nu}{K} u$$

… 1

Energy equation:

$$\frac{\partial T^*}{\partial t^*} = \frac{k}{\rho C_p}\frac{\partial T^{*2}}{\partial y^{*2}} \frac{\mu}{\rho C_p}\left(\frac{\partial u^*}{\partial y^*}\right)^2$$

… 2

where $u^*, v^* -$ are the velocity components in $x^*, y^* -$directions, g is the gravitational acceleration, b is the thermal expansion coefficient respectively, r is the fluid density, $\mu -$

is the coefficient of viscosity, T^* is the thermal temperature inside the thermal boundary layer k is the thermal conductivity, K is the permeability of the medium C_p is the specific heat at constant pressure, σ is the electric conductivity, T is the temperature of the fluid far away from the plate, t^* - is time.

The boundary conditions are:

$$\left.\begin{array}{ll} t^* \le 0 : u^* = 0, T^* = T_\infty^* \quad \forall y & \\ t^* > 0 : u^* = u_0^*, T^* = T_\infty^* + \left(T_w^* - T_\infty^*\right)\dfrac{u_0^{*2} t^*}{v} & at \ y^* = 0 \\ u^* \to 0 \quad T^* \to T_\infty^* & as \ y^* \to \infty \end{array}\right\}$$

… 3

where u_p^* is the velocity of the fluid, T_w^* is the temperature of the wall respectively, t^* is the time. Introducing the following non- dimensional quantities,

$$\left.\begin{array}{l} u = \dfrac{u^*}{u_0}, y = \dfrac{u_0 y^*}{\nu}, \nu = \dfrac{\nu^*}{\nu_0}, \theta = \dfrac{T^* - T_\infty^*}{T_w^* - T_\infty^*}, \Pr = \dfrac{\nu \rho C_p}{k}, K = \dfrac{K u_0}{v^2}, \\ Gr = \dfrac{g \beta v\left(T_w^* - T_\infty^*\right)}{u_0^3}, t = \dfrac{t^* u_0^2}{v}, Ha = \dfrac{\sigma B_0^2 l^2}{\mu}, Ec = \dfrac{U_0^2}{c_p (T_w^* - T_\infty^*)} \end{array}\right\}$$

… 4

where Gr, Pr, Ha, Ec, and Pr are the thermal Grashof number, Prandtl number, Hartmann number, Eckert number, respectively.

With the help of non- dimensional quantities, equations (1) and (2) becomes

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gr\theta - \left(M + \frac{1}{K}\right)u$$

... 5 $\Pr \dfrac{\partial \theta}{\partial t} = \dfrac{\partial^2 \theta}{\partial y^2} + \Pr Ec \left(\dfrac{\partial u}{\partial y}\right)^2$

… 6

The corresponding dimensions less boundary condition are

$$\left.\begin{array}{l} t \le 0 : u = 0, \theta = 0 \quad \forall y \\ t > 0 : u = 1, \theta = t \ \ at \ \ y = 0 \\ u \to 0 \ \ \theta \to 0 \qquad as \ \ y^{*} \to \infty \end{array}\right\}$$

… 7

METHOD OF SOLUTION

By applying Galerkin finite element method for equation (5) over the element (e), $\left(y_j \le y \le y_k\right)$ is:

$$\int_{y_j}^{y_k} \left\{ N^{(e)T} \left[\frac{\partial^2 u^{(e)}}{\partial y^2} - \frac{\partial u^{(e)}}{\partial t} - Nu^{(e)} + P \right] \right\} dy = 0$$

... 8

Where $P = (Gr)\theta$, $N = M + \frac{1}{K}$

Integrating the first term in equation (8) by parts we obtain

$$N^{(e)T} \left\{ \frac{\partial u^{(e)^T}}{\partial y} \right\}_{y_j}^{y_k} - \int_{y_j}^{y_k} \left\{ \frac{\partial N^{(e)T}}{\partial y} \frac{\partial u^{(e)}}{\partial y} + N^{(e)T} \left(\frac{\partial u^{(e)}}{\partial t} + Nu^{(e)} - P \right) \right\} dy = 0$$

… 9

Neglecting the first term in equation (9), we get:

$$\int_{y_j}^{y_k} \left\{ \frac{\partial N^{(e)T}}{\partial y} \frac{\partial u^{(e)}}{\partial y} + N^{(e)T} \left(\frac{\partial u^{(e)}}{\partial t} + Nu^{(e)} - P \right) \right\} dy = 0$$

Let $u^{(e)} = N^{(e)} \phi^{(e)} =$ be the linear piecewise approximation solution over the element (e) ($y_j \le y \le y_k$) where $N^{(e)} = \left[N_j \quad N_k \right]$, $\phi^{(e)} = \left[u_j \quad u_k \right]^T$ and $N_j = \frac{y_k - y}{y_k - y_j}, N_k = \frac{y - y_j}{y_k - y_j}$ are the basis functions.

We obtain

$$\int_{y_j}^{y_k} \left\{ \begin{bmatrix} N_j' N_j' & N_j' N_k' \\ N_j' N_k' & N_k' N_k' \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} \right\} dy + \int_{y_j}^{y_k} \left\{ \begin{bmatrix} N_j N_j & N_j N_k \\ N_j N_k & N_k N_k \end{bmatrix} \begin{bmatrix} \dot{u}_j \\ \dot{u}_k \end{bmatrix} \right\} dy - B \int_{y_j}^{y_k} \left\{ \begin{bmatrix} N_j N_j' & N_j N_k' \\ N_j' N_k & N_k' N_k \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} \right\} dy = P \int_{y_j}^{y_k} \begin{bmatrix} N_j \\ N_k \end{bmatrix} dy$$

On simplifying we get

$$\frac{1}{l^{(e)}}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}u_j\\ u_k\end{bmatrix}+\frac{l^{(e)}}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\begin{bmatrix}\dot{u}_j\\ \dot{u}_k\end{bmatrix}+\frac{Nl^{(e)}}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\begin{bmatrix}u_j\\ u_k\end{bmatrix}=\frac{Pl^{(e)}}{2}\begin{bmatrix}1\\ 1\end{bmatrix}$$

where prime and dot denote differentiation with respect to y and time t respectively. Assembling the element equations for two consecutive elements $(y_{i-1} \le y \le y_i)$ and $(y_i \le y \le y_{i+1})$ following is obtained:

$$\frac{1}{l^{(e)^2}}\begin{bmatrix}1 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 1\end{bmatrix}\begin{bmatrix}u_{i-1}\\ u_i\\ u_{i+1}\end{bmatrix}+\frac{1}{6}\begin{bmatrix}2 & 1 & 0\\ 1 & 4 & 1\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}\dot{u}_{i-1}\\ \dot{u}_i\\ \dot{u}_{i+1}\end{bmatrix}+\frac{N}{6}\begin{bmatrix}2 & 1 & 0\\ 1 & 4 & 1\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}u_{i-1}\\ u_i\\ u_{i+1}\end{bmatrix}=\frac{P}{2}\begin{bmatrix}1\\ 2\\ 1\end{bmatrix}$$

… 10

Now put row corresponding to the node i to zero, from equation (10) the difference schemes with $l^{(e)} = h$ is:

$$\frac{1}{6}\left[\dot{u}_{i-1}+4\dot{u}_i+\dot{u}_{i+1}\right]+\frac{1}{h^2}\left[-u_{i-1}+2u_i-u_{i+1}\right]+\frac{N}{6}\left[u_{i-1}+4u_i+u_{i+1}\right]=P$$

Applying Crank – Nicholson method to the above equation, we get

$$A_1u_{i-1}^{j+1}+A_2u_i^{j+1}+A_3u_{i+1}^{j+1}=A_4u_{i-1}^{j}+A_5u_i^{j}+A_6u_{i+1}^{j}+P^*$$

… 11

where

$A_1 = 2-6r+Nk$ $\quad$ $A_4 = 2+6r-Nk$

$A_2 = 8+12r+4Nk$ $\quad$ $A_5 = 8-12r-4Nk$

$A_3 = 2-6r+Nk$ $\quad$ $A_6 = 2+6r+Nk$

$P^* = 12Pk = 12kGr\theta_i^j$

Now from equation (6) following equation is obtained:

$$B_1\theta_{i-1}^{j+1}+B_2\theta_i^{j+1}+B_3\theta_{i+1}^{j+1}=B_4\theta_{i-1}^{j}+B_5\theta_i^{j}+B_6\theta_{i+1}^{j}+P^{**}$$

… 12

Where

$B_1 = \Pr - 3r$ $\quad B_4 = \Pr + 3r$

$B_2 = 4\Pr + 6r$ $\quad B_5 = 4\Pr - 6r$

$B_3 = \Pr - 3r$ $\quad B_6 = \Pr + 3r$

$$P^{**} = 6P_1 k = 6k \Pr Ec \left(\frac{\partial u_i}{\partial y_i} \right)^2$$

Here, $r = \dfrac{k}{h^2}$ and h, k are mesh size along the y direction and the time direction respectively. Index i refers to the space, and j refers to the time. In the equations (11) and (12), taking $i = 1,...,n$ and using boundary conditions (7), the following system of equations are obtained:

$$A_i X_i = B_i, \qquad i = 1,...,n \qquad \dots 13$$

where A_i's are matrix of order n and X_i, B_i's column matrices having n components. The solutions of above systems of equations are obtained by using the Thomas algorithm for velocity, temperature and concentration. Also the numerical solutions are obtained by executing the C-program with the smaller values of h and k. No significant change was observed in u, and θ then the Galerkin finite element method is stable and convergent.

RESULTS AND DISCUSSIONS

The investigation of an unsteady flow through porous media past on moving vertical plate with variable temperature in the presence of inclined magnetic field and viscous dissipation has been carried out in the previous section. The numerical values were computed with respect to the physical parameters, like angle of inclination of magnetic field α, Hartmann number Ha, thermal Grashof number Gr, permeability K, Prandtl number $\Pr$, Eckert number Ec and time t is shown in figures.

Figure 1 represents the velocity profiles for different values of angle of inclination of magnetic field α. It is observed that an increase in α leads to a decrease in the value of velocity. The velocity profile decreases with an increase in Hartmann number as shown in figure 2.

Figure 3 and 4 illustrate the behavior of velocity profile for different values of Thermal Grashof number Gr and permeability parameter K. The numerical result shows that increasing in the values of Gr and K results velocity increases.

Figure 5 represent the velocity profile for different values of Eckert numbers. It observed that velocity increases with an increase in Ec values.

The effect of the Prandtl number (Pr) on the velocity and temperature are shown in figures 6 and 7, the velocity and temperature decreases as there is an increase in the Pr values.

Figures 8 and 9 illustrate the behavior of the velocity and temperature profiles for different values of *t*. As the time increases, the velocity and temperature increases.

CONCLUSIONS

In this paper, an unsteady MHD free convection flow of a viscous incompressible fluid by a moving vertical plate with variable temperature in the presence of inclined magnetic field and viscous dissipation has been studied numerically. From the present study we see that the momentum boundary layer decreases with an increase in angle of inclination of magnetic field, Hartmann number and Prandtl number, while there is an increases with an increase in thermal Grashof number, permeability, Eckert number and time. The thermal boundary layer thickness decreases with an increase in Prandtl number whereas it increases with an increase in time.

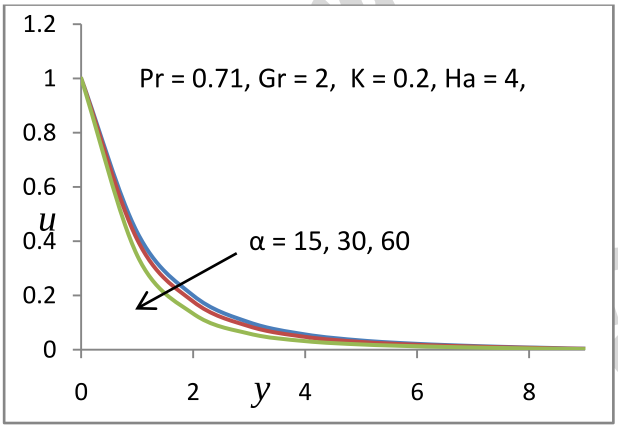

Figure 2: Effect of Ha on velocity profile

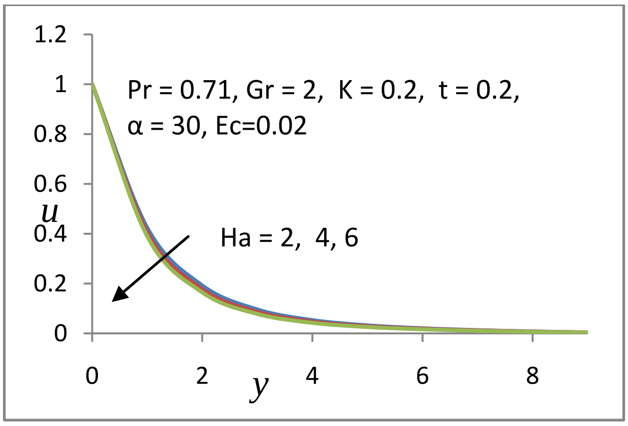

Figure 2.1: Effect of α on velocity profile

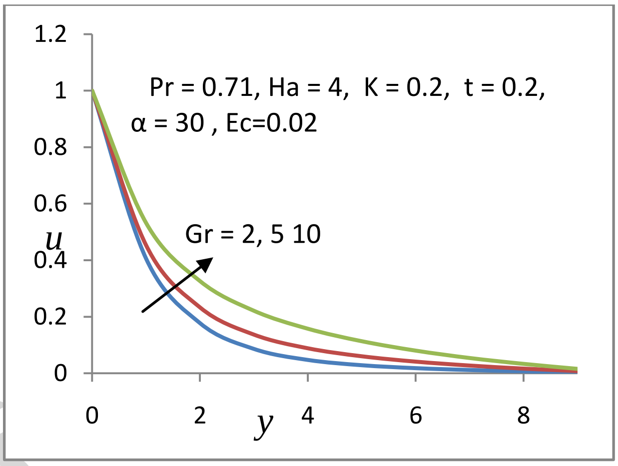

Figure 3: Effect of Gr on Velocity profile

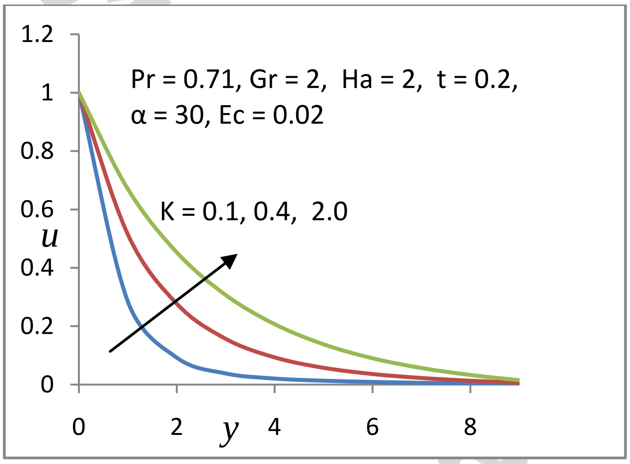

Figure 4: Effect of K on velocity profile

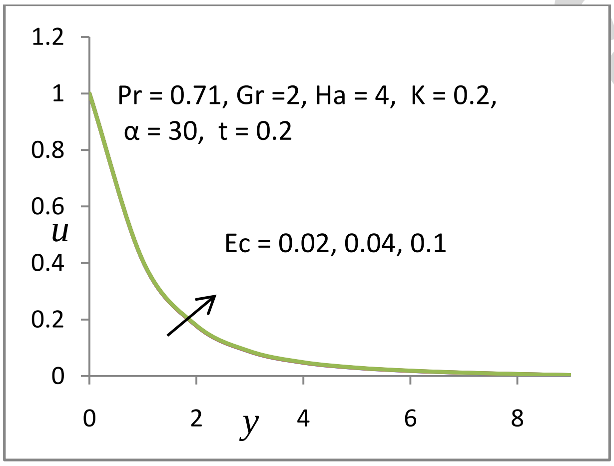

Figure 5: Effect of Ec on velocity profile

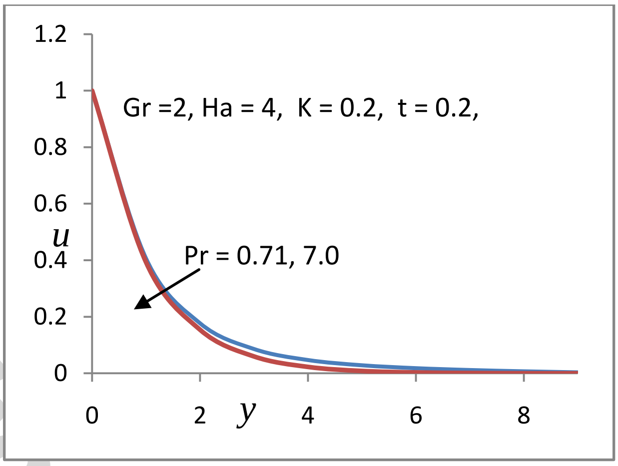

Figure 6: Effect of Pr on velocity profile

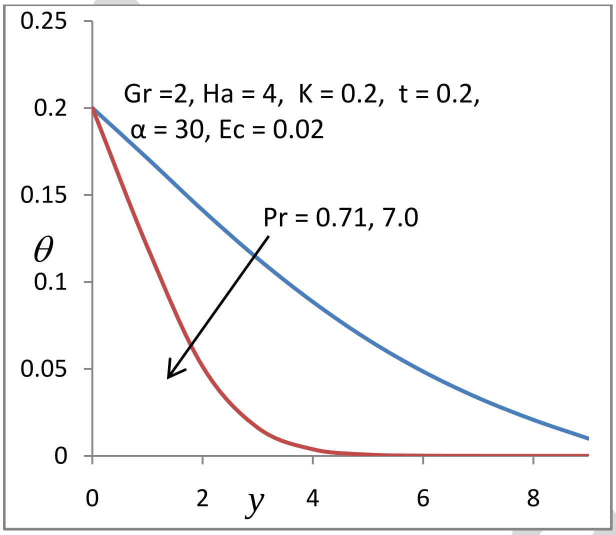

Figure 7: Effect of Pr on temperature profile

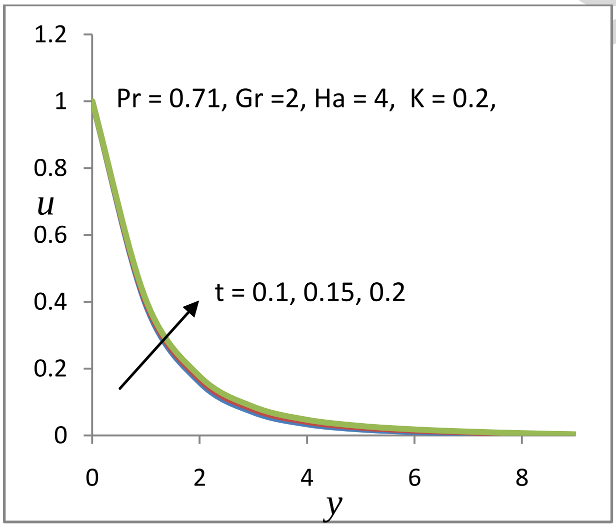

Figure 8: Effect of *t* on velocity profile

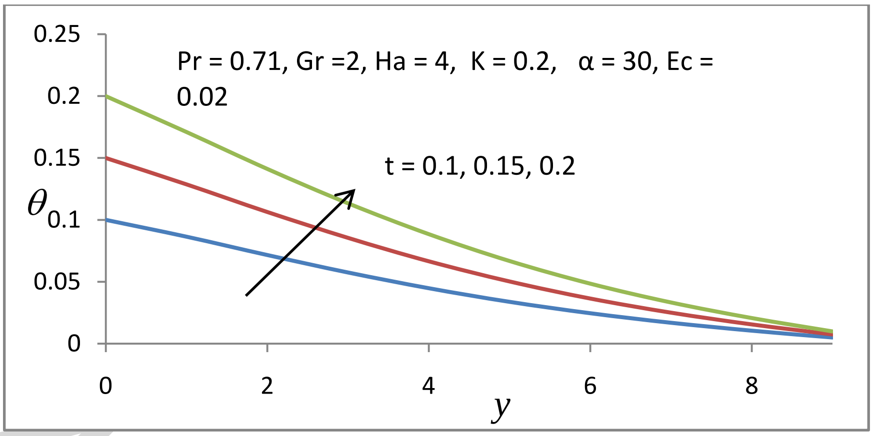

Figure 9: Effect of *t* on temperature profile

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