

## **Unsteady Heat and Mass transfer flow through a porous medium in rotating channel.**

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### **Abstract**

An attempt has been made to study the unsteady oscillatory convective heat and mass transfer of a viscous fluid through a porous medium in a rotating channel. The velocity, temperature and concentration has been analysed for various governing parameters.

**Keywords:** convective heat and mass transfer, porous medium, rotating channel.

### **1. Introduction:**

The combined rotational and buoyancy effects are very common in nature. Most flows has regions of rotations as well as stratification. Buoyancy and rotational effects are often comparable in geophysical process. Convective transport in a rotating atmosphere over a heated surface gives rise to typhoons and other rising atmosphere circulations. The unsteady flow of a rotating viscous fluid has been studied by several authors to analyse the growth and development of boundary layer associated with geothermal flows for applications in geophysical fluid dynamics. [1-7] Rao have made an initial value investigation of the combined free and forced convection effects in an unsteady hydro magnetic viscous incompressible rotating fluid between two discs under a uniform transfers magnetic field. Nagaraja [5] has investigated combined effects of heat and mass transfer flow of a viscous incompressible fluid through a porous medium in a rotating horizontal channel bounded by the flat walls. Prasad [7] has studied the mixed convective heat and mass transfer flow of a viscous fluid through a porous medium in a rotating parallel channel in the presence of a constant heat source.

In this paper we deal with the oscillatory flow of a combined effect of heat and mass transfer flow of a viscous incompressible fluid through a porous medium in a rotating horizontal channel bounded by flat walls. The perturbation in the flow is created by the non tensional oscillations of the lower plate. The solutions of velocity field, temperature and

concentration distributions are obtained. The shear stress, the rate of heat and mass transfer has been evaluated for different variations of the governing parameters.

## 2. Formulation of the problem:

We consider the unsteady flow of an incompressible viscous fluid through a porous medium bounded by two parallel plates. In the undisturbed states both the plates and the fluid rotate with the same angular velocity ( $\Omega$ ) and are maintained at constant temperature and concentration. The lower plate performs non tensional oscillations in its own plane.

The plates are cooled or heated by constant temperature gradient in some direction parallel to the plane of the plates. We choose a Cartesian coordinate system  $O(x,y,z)$  such that the plates are at  $z=0$  and  $z=1$  and the  $z$  axis coinciding with the axis of rotation of the plates. Neglect the soret and doffer effect, the unsteady hydrodynamic boundary layer equations of motions with respect to a rotating frame moving with angular velocity  $\Omega$  are the momentum equations

$$\frac{\partial u}{\partial t} - 2\Omega v = -\frac{1}{\rho} \left( \frac{\partial p}{\partial x} \right) + \nu \left( \frac{\partial^2 u}{\partial z^2} \right) - \left( \frac{v}{k} \right) u \quad (1)$$

$$\frac{\partial v}{\partial t} + 2\Omega u = -\frac{1}{\rho} \left( \frac{\partial p}{\partial y} \right) + \nu \left( \frac{\partial^2 v}{\partial z^2} \right) - \left( \frac{v}{k} \right) v \quad (2)$$

$$0 = -\frac{1}{\rho} \left( \frac{\partial p}{\partial z} \right) - g(1 - \beta(T - T_0) - \beta^*(C - C_0)) \quad (3)$$

The energy equation

$$\left( \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \right) (T - T_0) = \lambda \frac{\partial^2}{\partial z^2} (T - T_0) + \nu \left( \left( \frac{\partial u}{\partial z} \right)^2 + \left( \frac{\partial v}{\partial z} \right)^2 + \left( \frac{v}{k_1} \right) (u^2 + v^2) \right) \quad (4)$$

The diffusion equation

$$\left( \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \right) (C - C_0) = D_1 \frac{\partial^2}{\partial z^2} (C - C_0) \quad (5)$$

Where  $u, v$  are velocity components along  $x$  and  $y$  directions respectively,  $p$  is the pressure including the centrifugal force,  $\rho$  is the density,  $k$  is the permeability constant,  $\mu$  is the coefficient of viscosity,  $\lambda$  is the thermal diffusivity,  $D_1$  is the chemical molecular diffusivity,  $\beta$  is the coefficient of thermal expansion and  $\beta^*$  is the volumetric coefficient of expansion with mass fraction. Combining the equation ( 2.1) and( 2.2) we obtain

$$\frac{\partial q}{\partial t} - 2i\Omega q = -\frac{1}{\rho} \left( \frac{\partial p}{\partial x} + i \frac{\partial p}{\partial y} \right) + \nu \frac{\partial^2 q}{\partial z^2} - \left( \frac{v}{k} \right) q \quad (6)$$

Where  $q=u+iv$

Integrating equation ( 2.3) we obtain

$$\frac{p}{\rho} = -gz + \beta g \int (T - T_0) dz + \beta^* g \int (C - C_0) dz + \Phi(\xi, \xi) \quad (7)$$

Where

$$\xi = x - iy, \bar{\xi} = x + iy$$

Using (7), equation (6) can be written as

$$\frac{\partial q}{\partial t} - 2i\Omega q - v \frac{\partial q^2}{\partial z^2} + \left(\frac{v}{k}\right) q = -2\beta g \frac{\partial}{\partial \xi} (T - T_0) - 2\beta g \frac{\partial}{\partial \bar{\xi}} (C - C_0) \quad (8)$$

Since  $q = q(z, t)$ , equation (2.8) is valid if the temperature and concentration distributions are of the form

$$T - T_0 = \alpha_1 x + \beta_1 y + \theta_1(z, t)$$

$$C - C_0 = \alpha_2 x + \beta_2 y + \theta_2(z, t)$$

Where  $\alpha_1, \beta_1, \alpha_2, \beta_2$  are the gradients of the temperature and concentration along  $O(x, y)$  directions respectively,  $\theta_1(z, t), \theta_2(z, t)$  are the arbitrary functions of  $z$  and  $t$ . we take

$$T_0 + \alpha_1 x + \beta_1 y + \theta_1 w_1 \quad \text{and} \quad T_0 + \alpha_1 x + \beta_1 y + \theta_1 w_2, C_0 + \alpha_2 x + \beta_2 y + C_1 w_1$$

$$\text{And } C_0 + \alpha_2 x + \beta_2 y + C_1 w_2$$

To be temperature and concentration of lower and upper plates respectively, for  $t > 0$ .

Substituting (2.7) and (2.6) using (2.8) we get

$$\frac{\partial q}{\partial t} = 2i\Omega q + \frac{v}{k} q - \mu \frac{\partial q^2}{\partial z^2} + \beta g \bar{A} z + \beta g \bar{B} z = D_2 \quad (9)$$

$$\text{Where } D_2 = [\phi(\tau_Y, \bar{\tau}_Y)] \tau_Y$$

$$A = \alpha_1 + i\beta_1 \quad \text{and} \quad B = \alpha_2 + i\beta_2$$

Introducing non dimensional variables ( $z, t, q, \theta, c$ )

$$\hat{z} = \frac{z}{L}, \quad \hat{t} = \frac{tv}{L^2}, \quad \hat{q} = \frac{tv}{L^2}, \quad \hat{w} = \frac{wL^2}{v^2}$$

$$\hat{\theta} = \frac{\beta g L^3 (\theta_1 - \theta_{1w1})}{v^2},$$

$$\hat{c} = \frac{\beta g L^3 (C_1 - C_{1w1})}{v^2}$$

The governing equations in the non dimensional form are

$$q_{zz} - (D^{-1} - 2iE^{-1})q - q_1 = G(1 + N)z - R \quad (10)$$

$$P(\theta_1 + G_1 u + G_2 v) = \theta_{zz} + E_c D^{-1} q \cdot \bar{q} \quad (11)$$

$$S_c(C_1 + G_{1c}u + G_{2c}\mu) = C_{zz} \quad (12)$$

Where

$$E = \frac{\nu}{L^2\Omega} \text{ [Ekman number]}$$

$$D^{-1} = \frac{L^2}{k} \text{ [Darcy parameter]}$$

$$(G_1, G_2) = \frac{\beta GL^4}{\nu^2} (\alpha_1, \beta_1) \text{ [Grashof number]}$$

$$(G_{1c}, G_{2c}) = \frac{\beta GL^4}{\nu^2} (\alpha_2, \beta_2) \text{ [modified Grashof number]}$$

$$R = \frac{L^3 D}{\nu^2} \text{ [Reynolds number]}$$

$$P = \frac{\nu}{\lambda} \text{ [Prandtl number]}$$

$$D^{-1} = \frac{L^2}{k_1} \text{ [Darcy number]}$$

$$E_c = \beta g L P \text{ [Eckert number]}$$

$$S_c = \frac{\nu}{D} \text{ [Schmidt parameter]}$$

$$G = G_1 + iG_2, \quad G_c = G_{1c} + iG_{2c}$$

The boundary conditions in the non-dimensional form are

$$q(z, t) = ae^{i\omega t} + be^{-i\omega t} \text{ on } z = 0$$

$$q(z, t) = 0 \text{ on } z = 1$$

$$\theta(z, t) = 0, C(z, t) = 0 \text{ on } z = 0$$

$$\theta(z, t) = \frac{\beta g L^3 (\theta_{1w2} - \theta_{1w1})}{\nu^2} = \theta_0, C(z, t) = \frac{\beta g L^3 (C_{1w2} - C_{1w1}) = C_0}{\nu^2} \text{ on } z = 1 \quad (13)$$

In view of the boundary conditions (13) we have assumed the velocity, temperature and concentration distributions as follows (7 & 12)

$$q(z, t) = f(z) + f_1(z)e^{i\omega t} + f_2(z)e^{-i\omega t} + f_3(z)e^{2i\omega t} + f_4(z)e^{-2i\omega t} \quad (14)$$

$$\theta(z, t) = g(z) + g_1(z)e^{i\omega t} + g_2(z)e^{-i\omega t} + g_3(z)e^{2i\omega t} + g_4(z)e^{-2i\omega t}$$

$$C(z, t) = H(z) + H_1(z)e^{i\omega t} + H_2(z)e^{-i\omega t} + H_3(z)e^{2i\omega t} + H_4(z)e^{-2i\omega t}$$

Substituting (14) in (10)-(12) and comparing the corresponding terms the equation reduces to

$$f_{zz} - h^2 f = G(1 + N)z - R \quad (15)$$

$$g_{zz} = -P_1 |\hat{f}|^2 + 2\text{Re}(f^1_1 \bar{f}_2) - P_2 (|f|^2 + 2\text{Re} f_1 \bar{f}_2) + PG_1 \text{Re}(f) + PG_2 I_M(f) \quad (16)$$

$$H_{zz} = S_c (G_{1c} \text{Re}(f) + G_{1c} I_m(f)) \quad (17)$$

The corresponding boundary conditions are

$$f = 0; g = 0; H = 0 \text{ on } z = 0 \quad (18)$$

$$f = 0; g = 1; H = 1 \text{ on } z = 1$$

And

$$f_{1,zz} - (h^2 + iw)f_1 = 0 \quad (19)$$

$$g_{1,zz} - (iPw)g_1 = -2P_1 \text{Re}(\hat{f} \bar{f}_1) - P_2 2\text{Re}(f \bar{f}_1) + PG_1 \text{Re}(f_1) + PG_2 I_m(f_1) \quad (20)$$

$$H_{1,zz} - (iScw)H_1 = S_c G_{1c} \text{Re}(f_1) + S_c G_{2c} I_m(f_1) \quad (21)$$

The corresponding boundary conditions are

$$f_1 = a; g_1 = 0; H_1 = 0 \text{ on } z = 0$$

$$f_1 = 0; g_1 = 0; H_1 = 0 \text{ on } z = 1 \quad (22)$$

And

$$f_{2,zz} - (h^2 - iw)f_2 = 0 \quad (23)$$

$$g_{2,zz} + (iPw)g_2 = -2P_1 \text{Re}(\hat{f} \bar{f}_1) - P_2 2\text{Re}(f \bar{f}_1) + PG_1 \text{Re}(f_1) + PG_2 I_m(f_1) \quad (24)$$

$$H_{2,zz} + (iPw)H_2 = S_c (G_{1c} \text{Re}(f_2) + G_{2c} I_m(f_2)) \quad (25)$$

The corresponding boundary conditions are

$$f_2 = b; g_2 = 0; H_2 = 0 \text{ on } z = 0 \quad (26)$$

$$f_2 = 0; g_2 = 0; H_2 = 0 \text{ on } z = 1$$

$$f_{3,zz} - (h^2 + 2iw)f_3 = 0 \quad (27)$$

$$g_{3,zz} - (2iPw)g_3 = -E_c ((\hat{f}_1 \bar{f}_1) - D^{-1}(f_1 \bar{f}_1)) \quad (28)$$

$$H_{3,zz} - (2iPw)H_3 = S_c (G_{1c1} \text{Re}(f_3) + G_{2c2} I_m(f_3)) \quad (29)$$

The corresponding boundary conditions are

$$f_3 = 0; g_3 = 0; H_3 = 0 \text{ on } z = 0$$

$$f_3 = 0; g_3 = 0; H_3 = 0 \text{ on } z = 1 \quad (30)$$

And

$$f_{4,zz} - (h^2 - 2iw)f_4 = 0 \quad (31)$$

$$g_{4,zz} + (2iPw)g_4 = -E_c \left( (\dot{f}_2 \bar{f}_2) - D^{-1}(f_2 \bar{f}_2) \right) \quad (32)$$

$$H_{4,zz} + (2iS_c w)H_4 = S_c G_{1c1} \text{Re}(f_4) + G_{2c2} \text{Im}(f_4) \quad (33)$$

The corresponding boundary conditions are

$$f_4 = 0; g_4 = 0; H_4 = 0 \text{ on } z = 0$$

$$f_4 = 0; g_4 = 0; H_4 = 0 \text{ on } z = 1 \quad (34)$$

Solving the equations (15)-(17), (19)-(21), (23)-(25), and (26)-(28) with respect to the boundary conditions (18),(22),(26) and (29), the solutions are

$$f = (A_1 + iA_2)(z^2 - z)$$

$$f_1 = a(1 - z) + B_1(z^2 - z) + iB_2(z^2 - z)$$

$$f_2 = b(1 - z) + D_1(z^2 - z) + D_2i(z^2 - z)$$

$$f_3 = 0$$

$$f_4 = 0$$

$$g = c_5(z^2 - z) + z$$

$$g_1 = (E_1 + iE_2)(z^2 - z)$$

$$g_2 = c_7(z^2 - z)$$

$$g_3 = c_8(z^2 - z)$$

$$g_4 = c_9(z^2 - z)$$

$$H = c_{10}(z^2 - z) + z$$

$$H_1 = (L_1 + iL_2)(z^2 - z)$$

$$H_2 = (M_1 + iM_2)(z^2 - z)$$

$$H_3 = 0$$

$$H_4 = 0$$

### 3. Shear Stress, Nusselt Number and Sherwood Number

The non-dimensional shear stress  $\tau_x$  and  $\tau_y$  at the lower and upper plates are given by

$$(\tau_x + \tau_y)_{z=0} = \left(\frac{\partial q}{\partial z}\right)_{z=0}$$

$$(\tau_x + \tau_y)_{z=1} = \left(\frac{\partial q}{\partial z}\right)_{z=1}$$

The rate of heat transfer coefficient (nusselt number ) on the plates is given by

$$(\text{Nu})_{z=0} = \left(\frac{\partial \theta}{\partial z}\right)_{z=0} = \left(\frac{\partial g}{\partial z}\right)_{z=0} + \left(\frac{\partial g_1}{\partial z}\right)_{z=0} e^{i\omega t} + \left(\frac{\partial g_2}{\partial z}\right)_{z=0} e^{-i\omega t} + \left(\frac{\partial g_3}{\partial z}\right)_{z=0} e^{2i\omega t} + \left(\frac{\partial g_4}{\partial z}\right)_{z=0} e^{-i\omega t}$$

$$(\text{Nu})_{z=1} = \left(\frac{\partial \theta}{\partial z}\right)_{z=1} = \left(\frac{\partial g}{\partial z}\right)_{z=1} + \left(\frac{\partial g_1}{\partial z}\right)_{z=1} e^{i\omega t} + \left(\frac{\partial g_2}{\partial z}\right)_{z=1} e^{-i\omega t} + \left(\frac{\partial g_3}{\partial z}\right)_{z=1} e^{2i\omega t} + \left(\frac{\partial g_4}{\partial z}\right)_{z=1} e^{-i\omega t}$$

The rate of mass transfer (sherwood number) on the plates are given by

$$(\text{sh})_{z=0} = \left(\frac{\partial C}{\partial z}\right)_{z=0} = \left(\frac{\partial H}{\partial z}\right)_{z=0} + \left(\frac{\partial H_1}{\partial z}\right)_{z=0} e^{i\omega t} + \left(\frac{\partial H_2}{\partial z}\right)_{z=0} e^{-i\omega t} + \left(\frac{\partial H_3}{\partial z}\right)_{z=0} e^{2i\omega t} + \left(\frac{\partial H_4}{\partial z}\right)_{z=0} e^{-i\omega t}$$

$$(\text{Sh})_{z=1} = \left(\frac{\partial C}{\partial z}\right)_{z=1} = \left(\frac{\partial H}{\partial z}\right)_{z=1} + \left(\frac{\partial H_1}{\partial z}\right)_{z=1} e^{i\omega t} + \left(\frac{\partial H_2}{\partial z}\right)_{z=1} e^{-i\omega t} + \left(\frac{\partial H_3}{\partial z}\right)_{z=1} e^{2i\omega t} + \left(\frac{\partial H_4}{\partial z}\right)_{z=1} e^{-i\omega t}$$

### 4. Discussion on Numerical Results

The oscillatory solution for the velocity, temperature and concentration have been computed numerically for the governing parameters  $D^{-1}, N, w$  and their profiles are drawn in figs 1-13. for computational purpose we have assumed  $G$  to be real so that the applied pressure gradient in the  $y$ -direction is zero. Also the Prandtl number  $P$  is taken to be 0.71. Since thermal buoyancy balances the vertical pressure gradient in the absences of any other applied forces in the direction of rotation, the flow takes place in planes parallel to the boundary plates. The flow is three dimensional and all the perturbed variables have been obtained using boundary layer equations which would reduce to three coupled partial differential equations for a complex velocity, temperature and concentration.

Figs 1-7 corresponds to profiles of the axial velocity when one of the plates (lower) oscillates with given amplitude and the other at rest. The imposed pressure gradient along  $x$ -direction is chosen to be negative. The actual flow along  $x$ -axis remains negative for values of  $G$ . we find that in all cases,  $U$  rises from its prescribed value on the lower plate to the maximum attained in the lower half and gradually reduces to rest on the upper plate. The magnitude of  $U$  experiences a depreciation in  $N$  and  $w$  (figs.1-2). when the concentration buoyancy dominates over the thermal force  $U$  experiences an enhancement or a reduction according as the two forces either act in the same or in opposite directions (fig-2). In contrast to  $U$ , the transverse velocity  $V$  is positive for different  $N$  &  $w$ . we find that  $V$  decreases with  $w$ . The resultant velocity profiles like its components are bell shaped curves with their maximum attained at

$y=-150$  in the lower region. The magnitude of the resultant velocity decreases with  $w$  (fig 6). Also we observe that the resultant velocity increases with  $N(>0)$ .

The temperature profiles are plotted in figs.(7-9). We find that the temperature gradually decreases from its value on the lower plate to attain its minimum and then again increases to attain its prescribed value 1 at the upper plate. An increase in the permeability of porous medium the temperature in the lower half increases and decreases in the vicinity of the upper plate (fig.7)  $\Theta$  enhances with increase in the frequency  $w$  (fig .9). when the concentration buoyancy force dominates over the thermal force the temperature enhances with  $N$  irrespective of the directions of the buoyancy forces (fig.8).

The concentration distribution ( $C$ ) for different variations is exhibited in figs.(10-12)

The configuration chosen is such that the molecular diffusibility does not directly affect the flow field and hence the role of schmidt number  $Sc$  appears only in the variation of the concentration. We notice that the concentration increases with increase in  $D^{-1}$  or  $w$  (fig.10) when the concentration buoyancy force dominates over the thermal buoyancy force the concentration enhances or reduces according as the two buoyancy forces are in the same or opposite directions (fig 10). An

Increase in the molecular diffusivity increases the concentration for  $Sc \sim 0.6$  while for higher  $Sc \sim 1.3$   $C$  reduces in the fluid region (fig 12).

The shear stress ( $\tau$ ), the Nusselt number ( $Nu$ ) and the sherwood number on the plates are evaluated for different variations in the governing parameters are presented in tables (1-8). It is observed from tables 1 that the stress component  $\tau_x$  increases with a decrease in  $N$  and decreases with a decrease in  $w$  at  $z=0$ . At the upper plate the shear stress decreases with a decrease in  $N$  and  $w$ . As the permeability of the porous medium decreases the stress ( $\tau_x$ ) increases at the lower plate and decreases at the upper plate. (when the concentration buoyancy force dominates over thermal buoyancy the shear stress at both the plates enhances when the two forces are in the same direction and it decreases when they are in the opposite direction. shear stress  $\tau_x$  at both the plates decrease with  $N$  is decreasing and increase when  $w$  decreases.) from table 2 we find that the shear stress decreases when  $N$  and  $w$  both are decreasing. from table 3 we find that the shear stress  $\tau_y$  increases with decrease in  $N$  and decreases with decrease in  $w$ . An increase in permeability of the porous medium  $|\tau_y|$  increases at both the plates when the concentration buoyancy force dominates over the thermal buoyancy  $|\tau_y|$  increases when they are in the same direction and it decreases in opposite directions. At the upper plate  $|\tau_y|$  increases with  $N$  irrespective of the directions of the buoyancy forces.

The rate of heat transfer (Nusselt number) at both plates are presented in tables 5 & 6. The rate of heat transfer ( $Nu$ ) at both the plates increases with increase in thermal buoyancy ( $G$ ). A decrease in the permeability of the medium enhances  $Nu$  at the lower plate and  $|Nu|$  at the upper plate. when the concentration buoyancy dominates over the thermal buoyancy force the rate of heat transfer at the lower plate experiences an enhancement when the forces are in the



same direction and reduces when they act in opposite directions. We find a reversed effect at the upper plate. At the lower plate  $Nu$  decreases with  $N$ .

The rate of mass flux (Sherwood number) at the plates are exhibited in tables 7 & 8 for different variations in the governing parameters. We find that the rate of mass flux at lower plate decrease with  $N$  and  $w$  and increase at the upper plate. When the concentration buoyancy force dominates over thermal force the Sherwood number enhances or reduces at both the plates according as the two forces are either in the same or opposite directions. At the lower plate  $(sh)$  decreases with a decrease in  $N$  and increases with a decrease in  $w$  and  $(sh)$  increases with a decrease in  $N$  and  $w$ .

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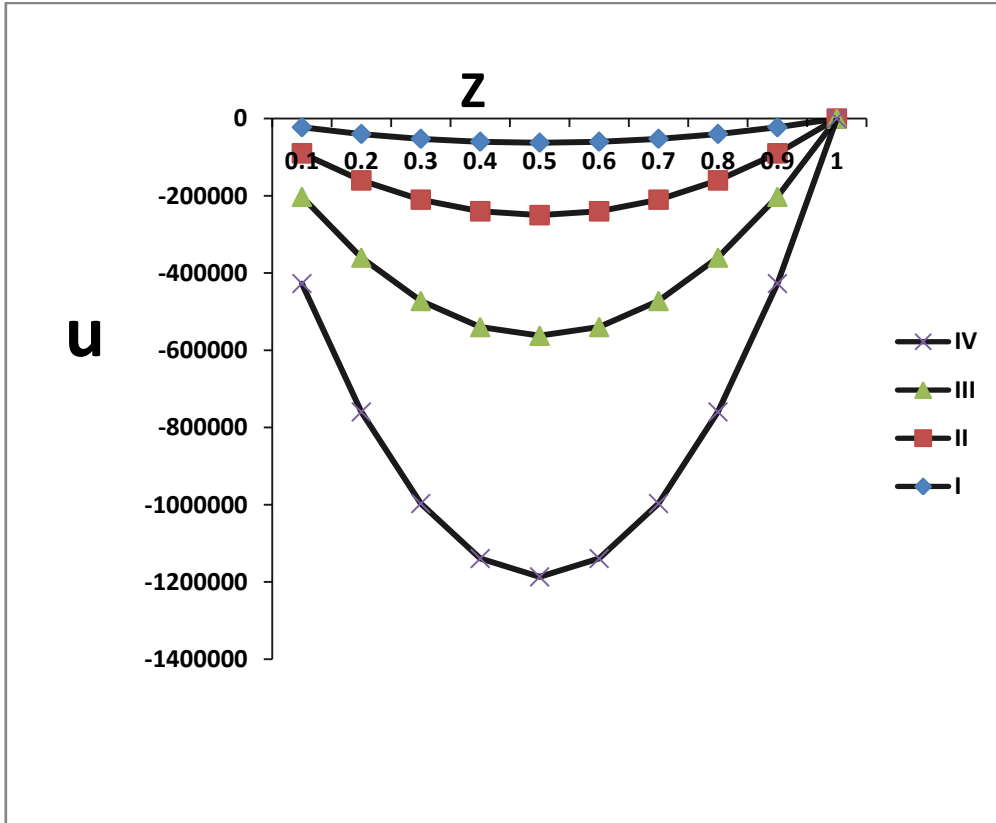
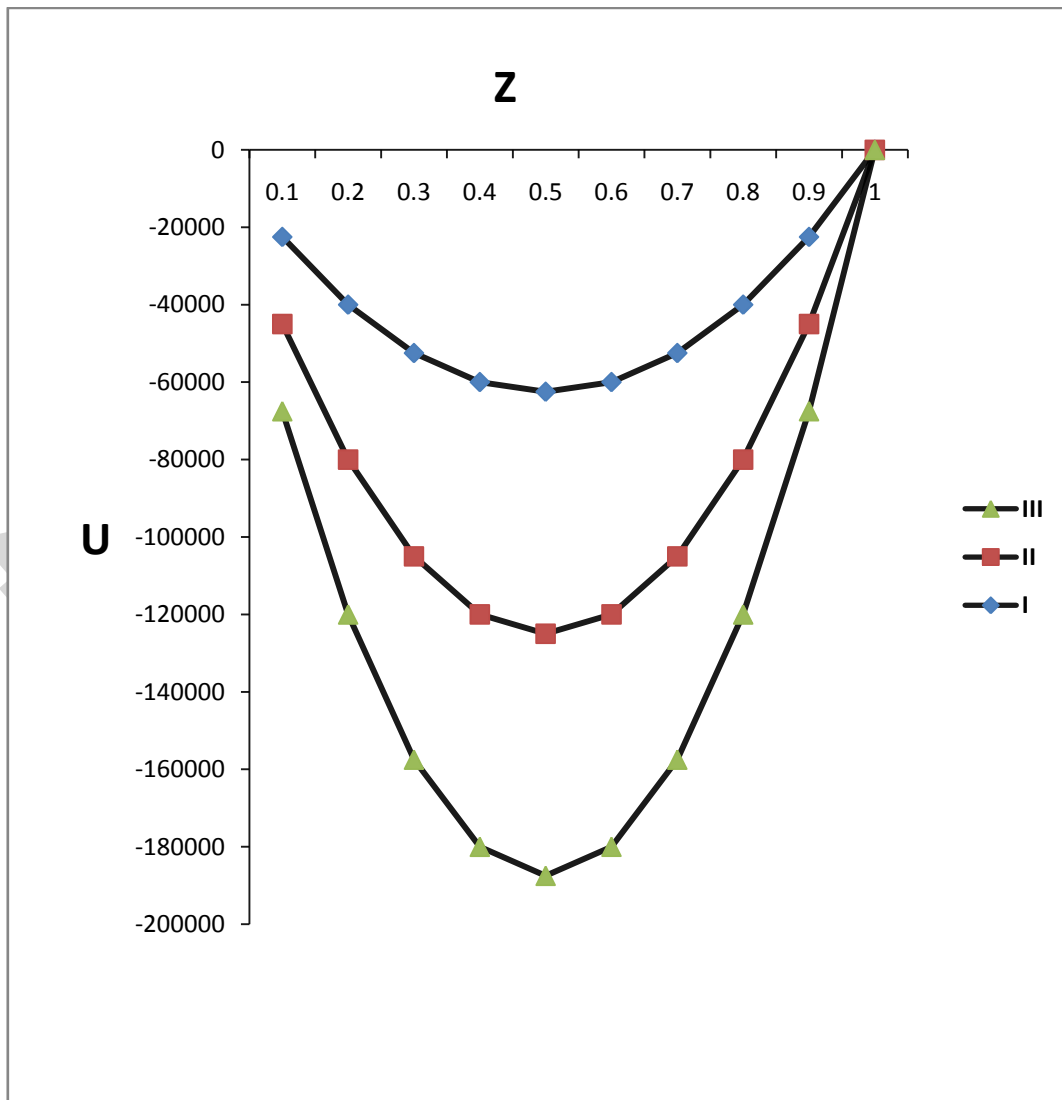


Fig.1. U with N

$G = 10^3, Sc = 1.3, w = 2$

N	I	II	III	IV
	$10^3$	$3 \times 10^3$	$5 \times 10^3$	$10^4$



**Fig.2. U with w**

**$G = 10^3$ ,  $Sc = 1.3$ ,  $N=1$**

W	I	II	III
	2	5	10

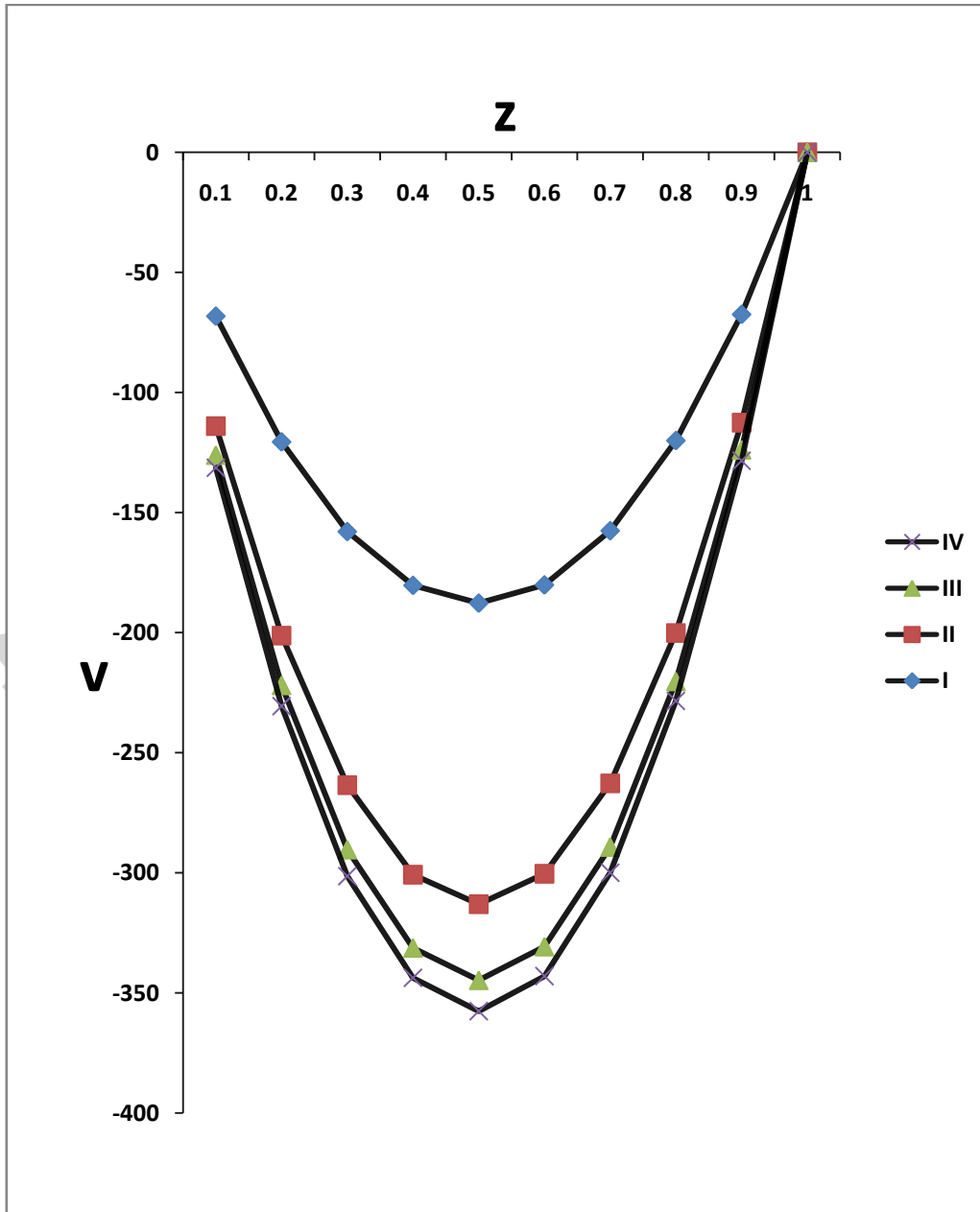
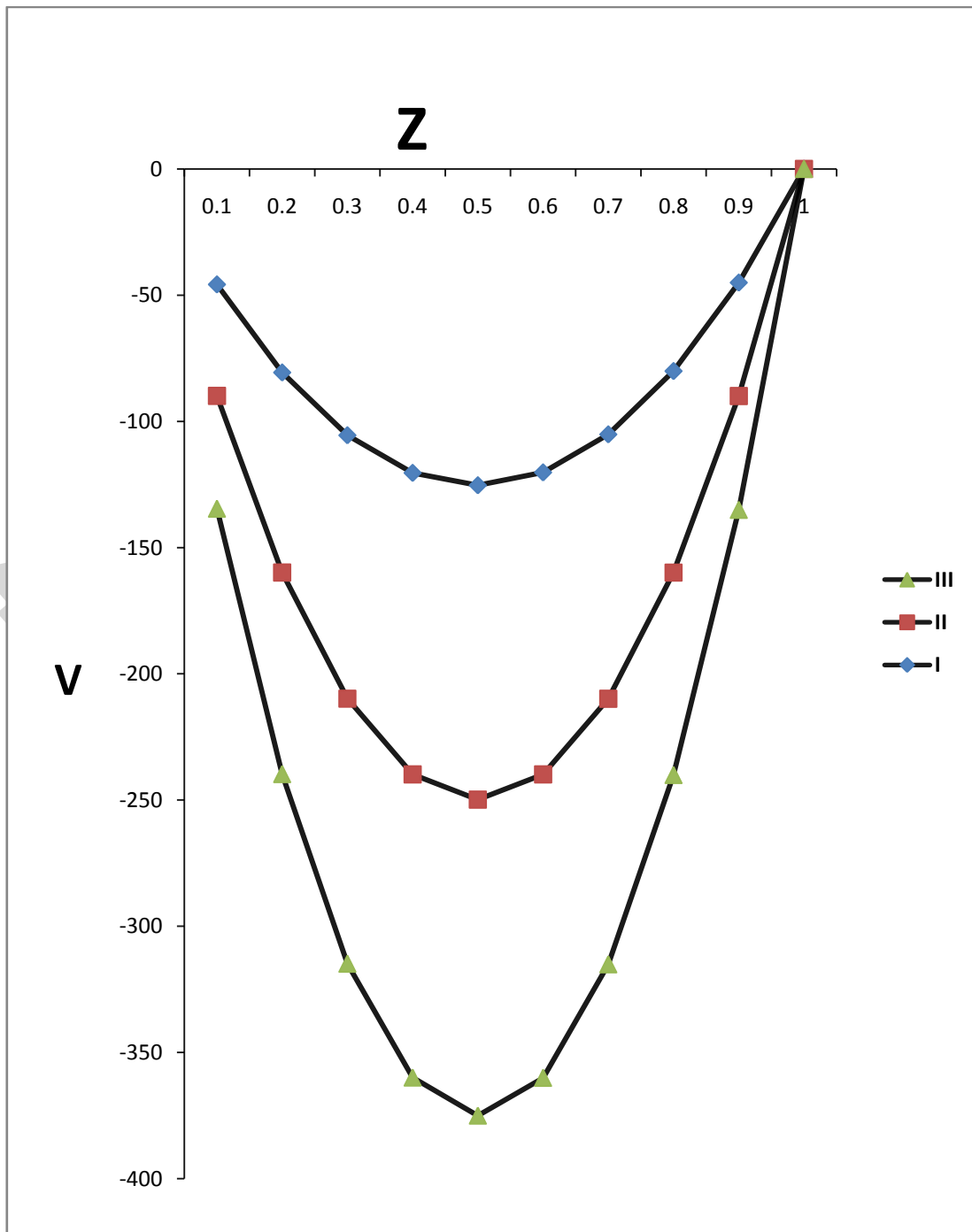


Fig.3. V with N

$G = 10^3, Sc = 1.3, w = 2$

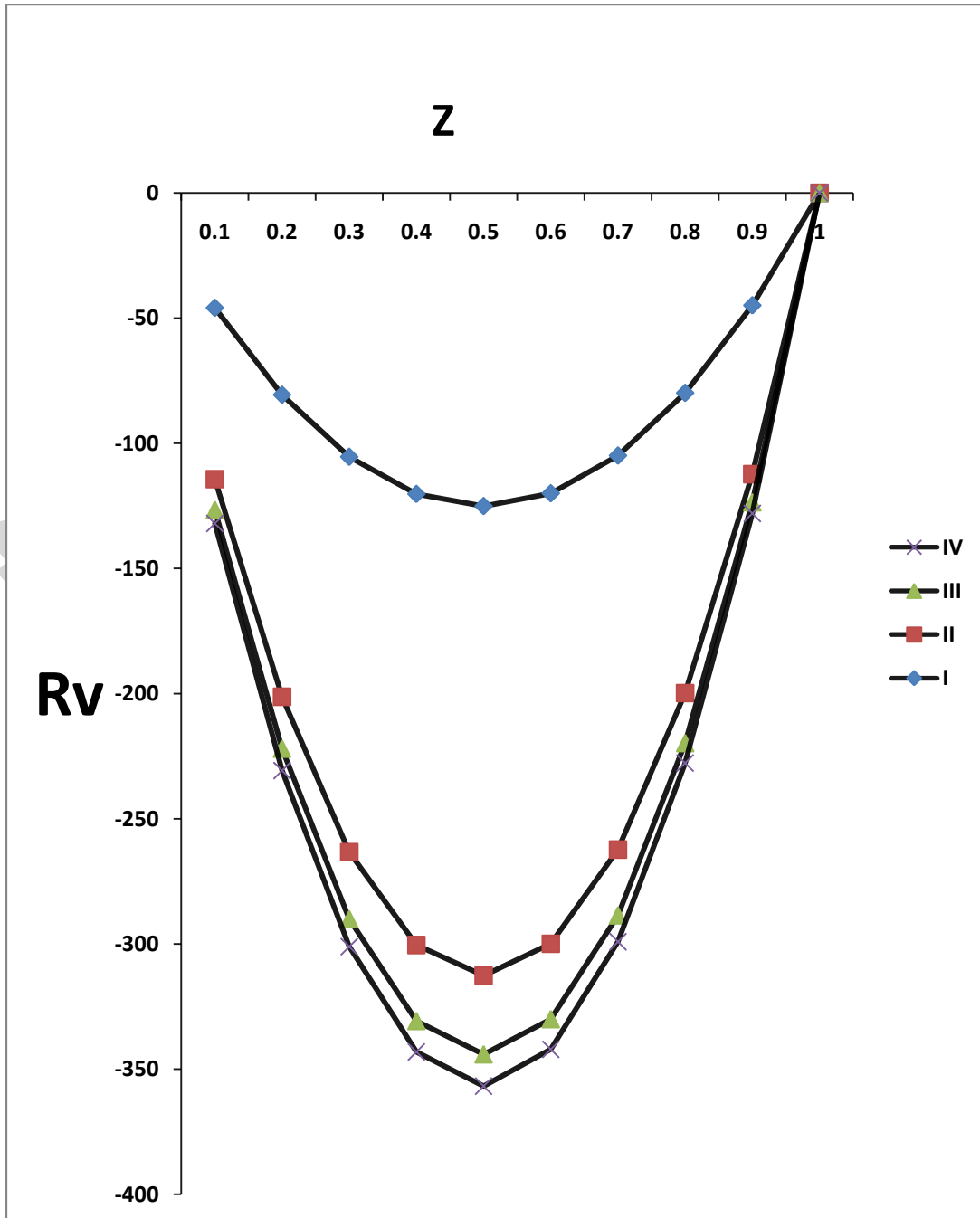
N	I	II	III	IV
	1	2	-0.5	-0.8



**Fig.4 V with w**

**$G = 10^3, Sc = 1.3,$**

<b>N</b>	<b>I</b>	<b>II</b>	<b>III</b>
	<b>2</b>	<b>5</b>	<b>10</b>



**Fig.5 Rv with N**

$G = 10^3, Sc = 1.3, w = 2$

N	I	II	III	IV
	1	2	-0.5	-0.8

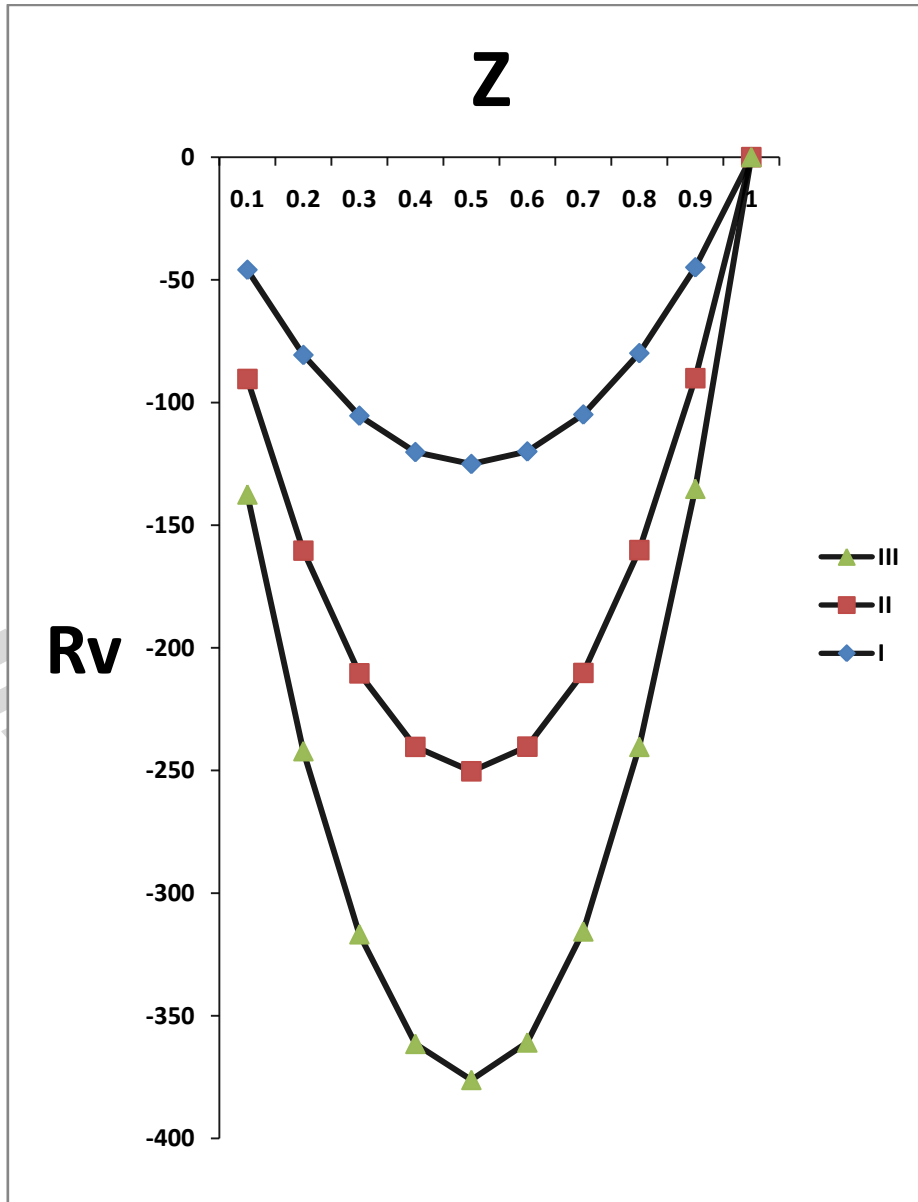


Fig.6 Rv with w

$G = 10^3, Sc = 1.3, N = 1$

W	I	II	III
	2	5	10

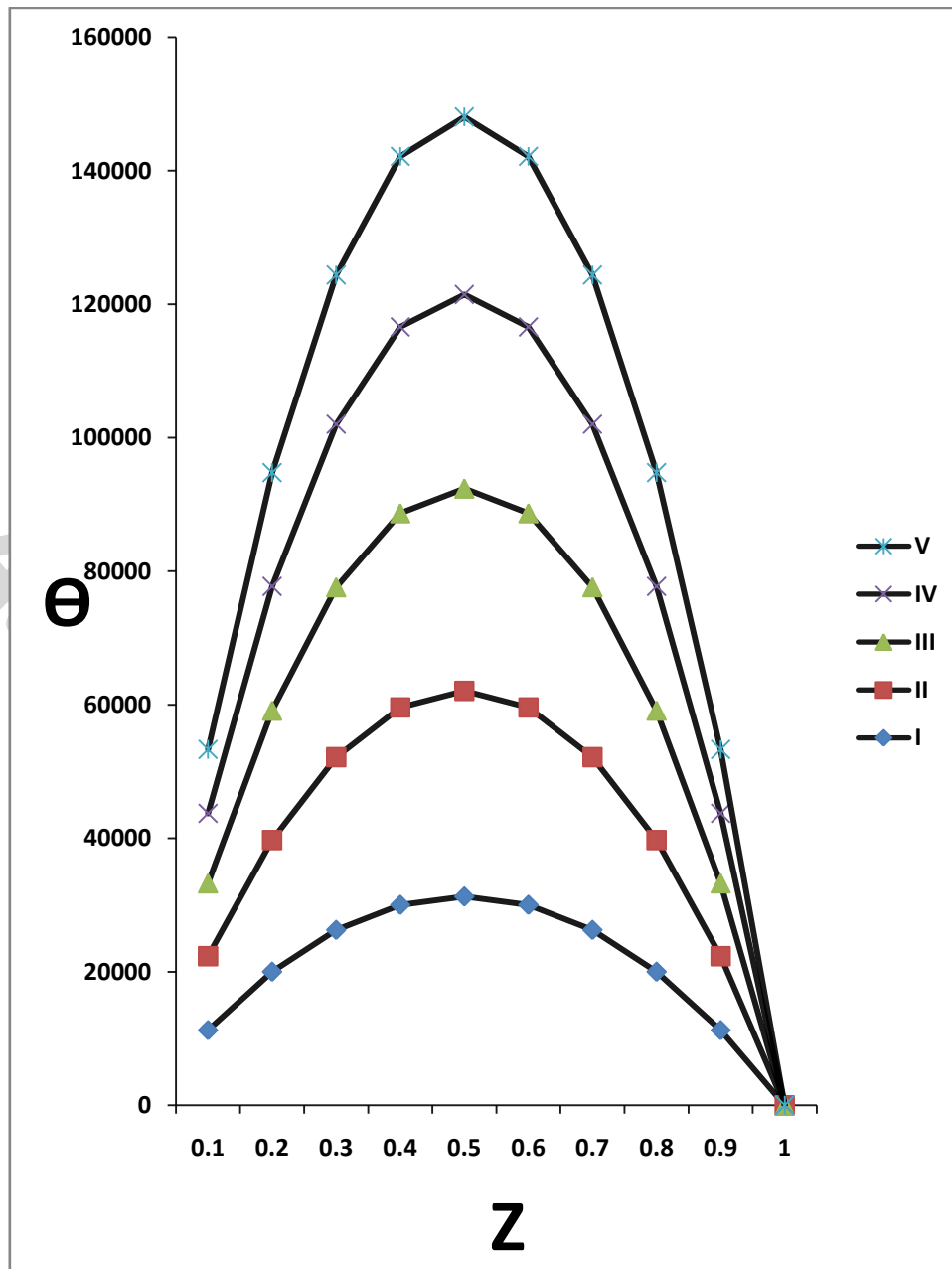


Fig.7  $\theta$  with  $D^{-1}$

$G = 10^3, N=1, Sc = 1.3, w = 2$

$D^{-1}$	I	II	III	IV	V
	$1 \times 10^3$	$3 \times 10^3$	$5 \times 10^3$	$10^4$	$2 \times 10^4$



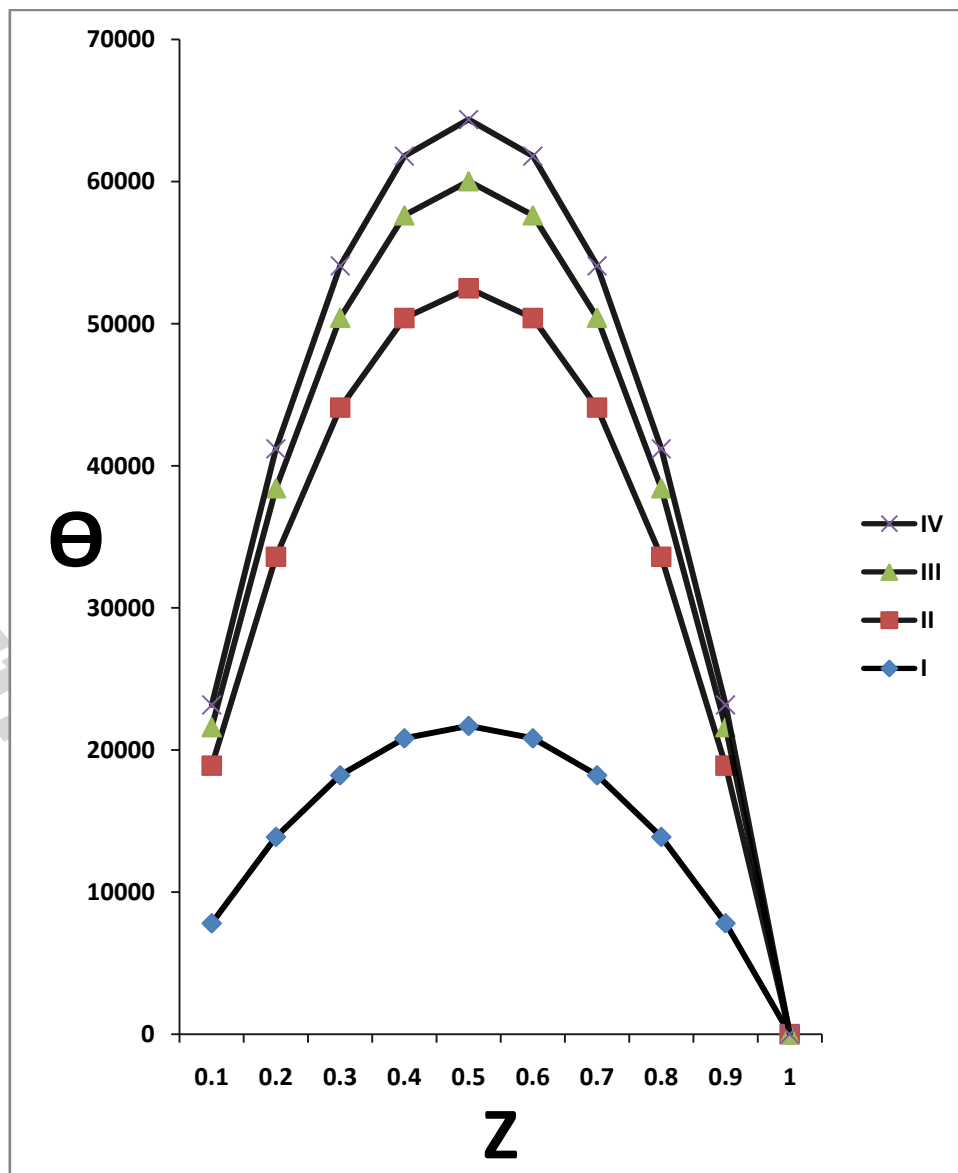


Fig.8  $\theta$  with  $N$

$G = 10^3, D^{-1} = 3 \times 10^3, Sc = 1.3, w = 2$

N	I	II	III	IV
	0.5	1	-0.5	-0.8

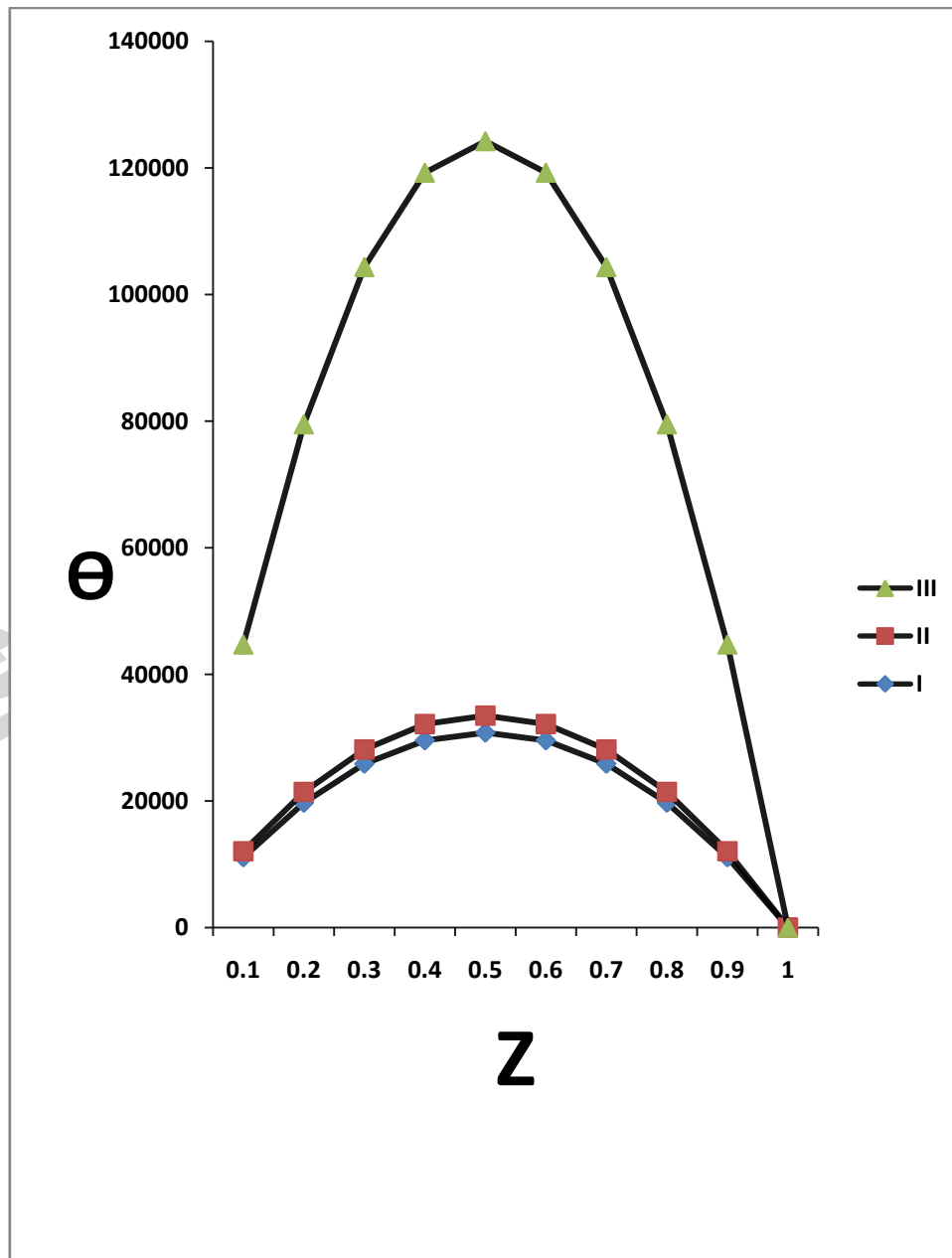
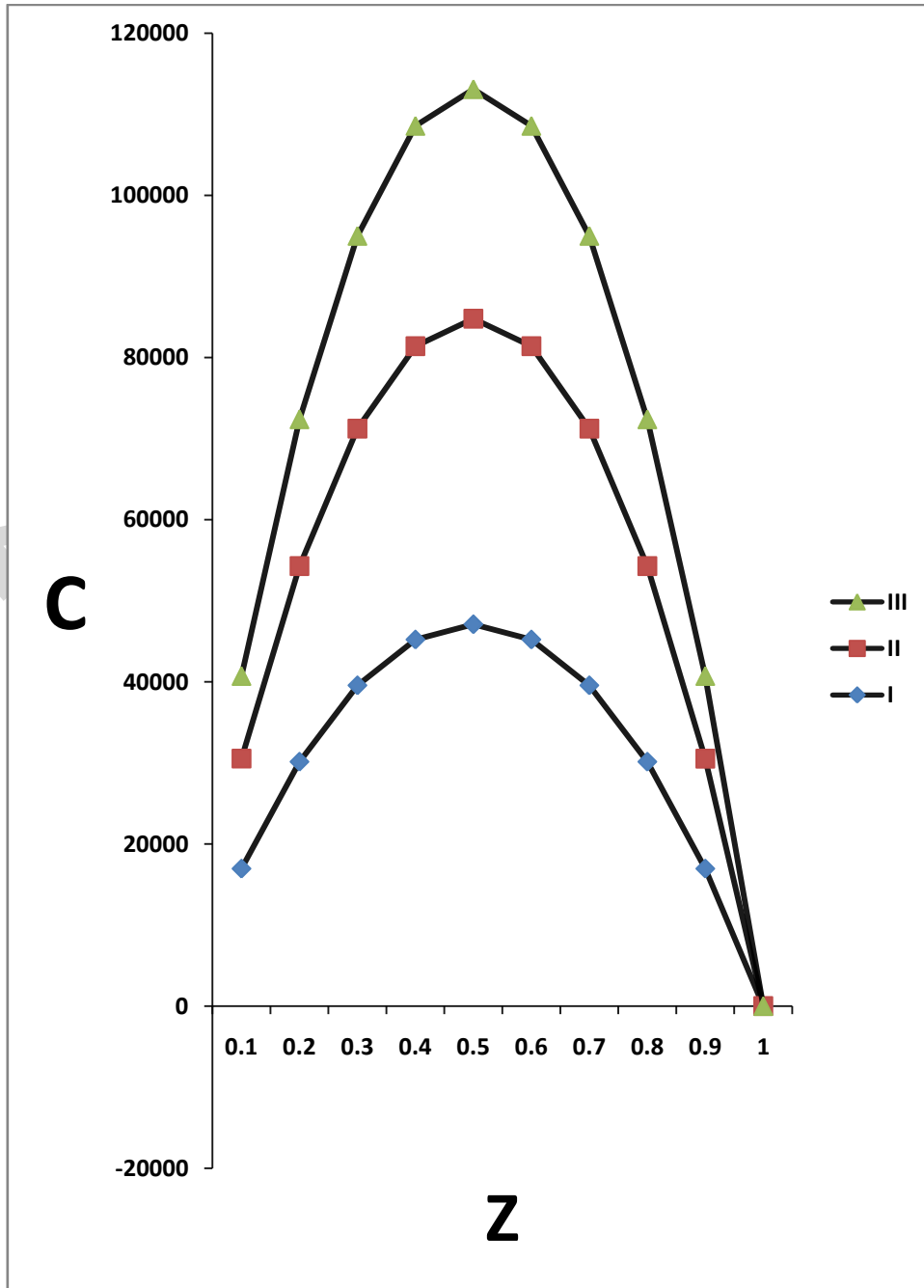


Fig.9  $\theta$  with  $w$

$G = 10^3, D^{-1} = 3 \times 10^3, Sc = 1.3, N = 1$

w	I	II	III
	2	5	8



**Fig.10** *C with N*

$G = 10^3, D^{-1}=3 \times 10^3, Sc = 1.3, w = 2$

N	I	II	III
	1	0.5	0

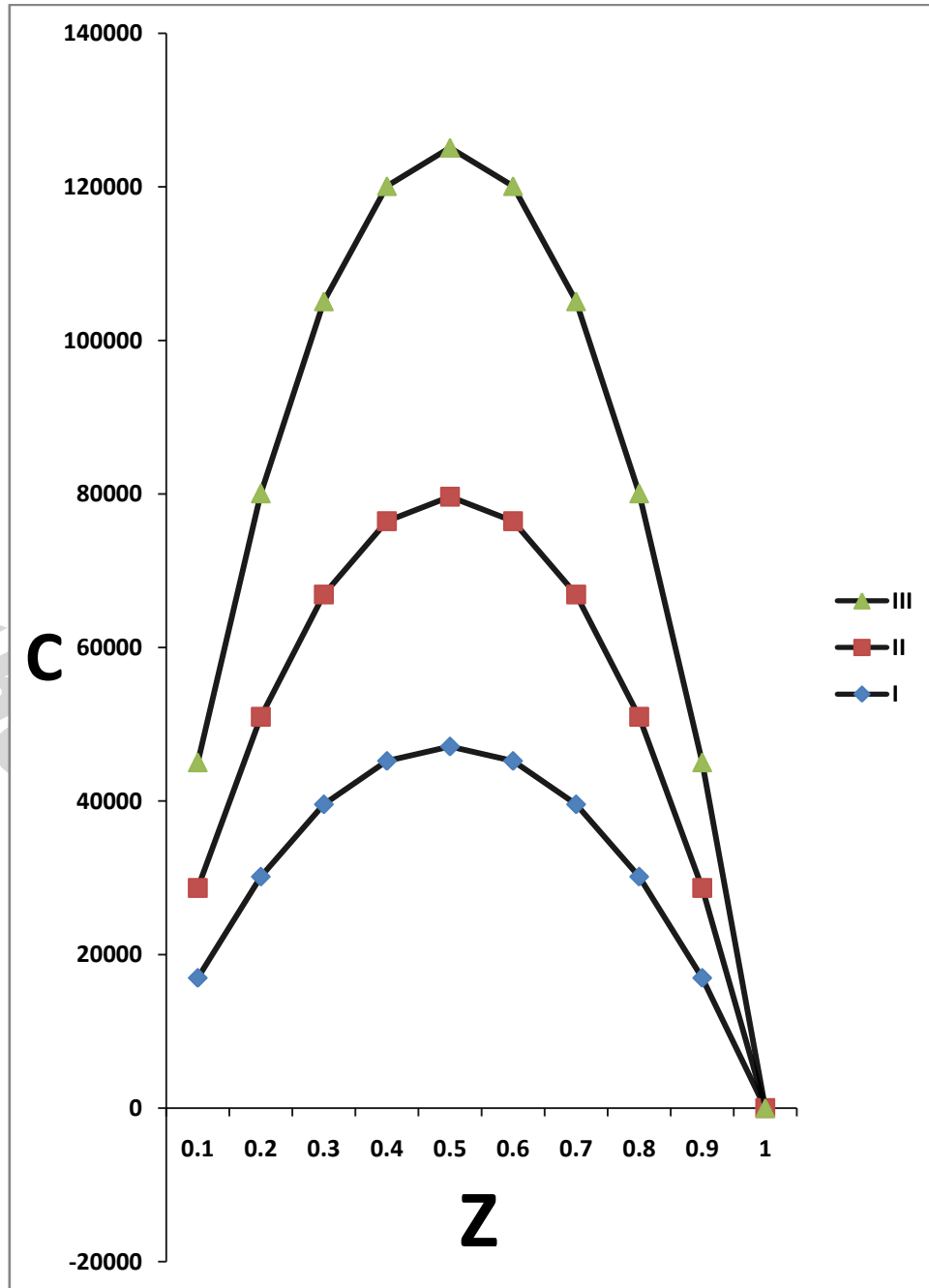


Fig.11 *C with w*

$G = 10^3, D^{-1}=3 \times 10^3, Sc = 1.3, N = 1$

w	I	II	III
	2	5	10

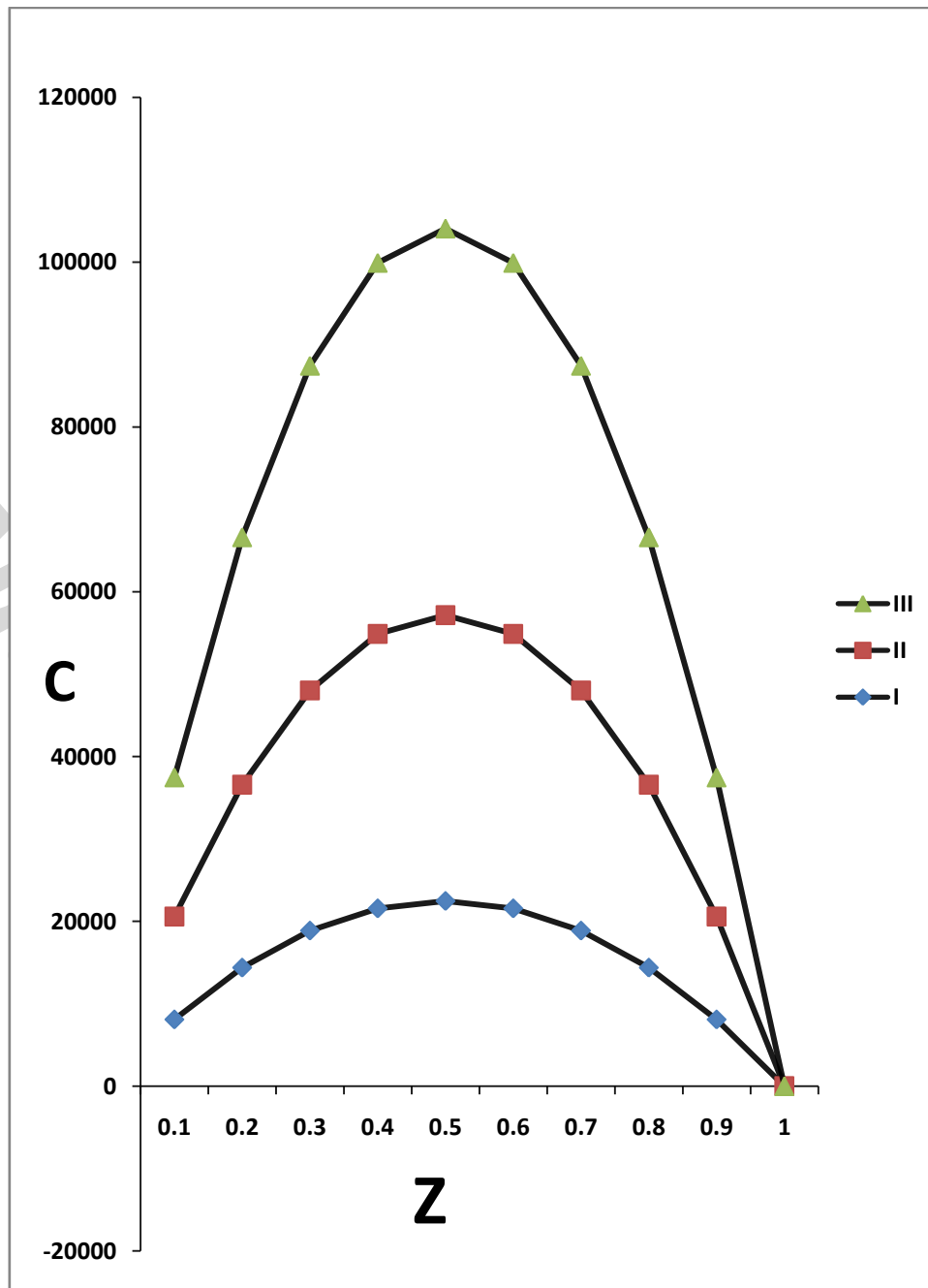


Fig.12 C with Sc

$G = 10^3, D^{-1}=3 \times 10^3, w=2, N = 1$

Sc	I	II	III
	0.6	1.3	2.01