ISSN 0976 – 1411

Available online at www.internationaleJournals.com

International eJournals

International eJournal of Mathematics and Engineering
4 (2017) Vol. 8, Issue 2, pp 30 – 35

INTERNATIONAL eJOURNAL OF MATHEMATICS AND ENGINEERING

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Pumping a Viscous Fluid with Convective Boundary Conditions in a Porous Asymmetric Channel

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ABSTRACT

The proposed study is to model the pumping of a viscous fluid in a porous asymmetric channel. Convective boundary conditions are used in this problem. Analytical solutions are obtained on solving the governing equations. Effect of various parameter on flow characteristics is studied with the help of graphs.

Keywords: Convective boundary conditions; porous channel;

1.INTRODUCTION

Pumping fluids is a mechanism which is very important in the context of physiological fluid flow. This phenomenon is known as “peristalsis”. Probably, the recent progress in the application of heat (hyperthermia), radiation (laser therapy) and coldness (cryosurgery) acts as a means to destroy undesirable tissues including cancer. It has stimulated much interest in the mathematical modeling in order to describe the properties of tissues. Additionally, Radiofrequency therapy plays an important role to cure the diseases that even includes; tissue coagulation, liver cancer, lung cancer and reflux of stomach acid. Based on the progress, the researchers have been made in order to analyze the peristaltic flow with the transfer of heats, along with the simultaneous effects of heat and mass transfer that has a key role to extract the geothermal energy, underground disposal of nuclear waste, spreading of chemical pollutants in a saturated soil and the migration of moisture in fibrous insulation.

Lathem(1996) is the one who initiated theoretical and experimental investigation of this phenomenon. Later, several researchers studied the peristaltic flow of various fluid models considering different geometrices.

2. MATHEMATICAL ANALYSIS

Consider the peristaltic transfer of an incompressible viscous fluid in a porous asymmetric channel with a flexible porous wall of width 2d. The asymmetric channel is generated by the propagation of waves on the porous walls moving with the constant speed c, along with different phases and amplitudes

$$h_1 = \bar{d}_1 + \bar{a}_1 \cos\left[\frac{2\pi}{\lambda}(\bar{X} - c\bar{t})\right] \text{(upper wall)} \quad (1)$$

$$h_2 = -\bar{d}_2 - \bar{a}_2 \cos\left[\frac{2\pi}{\lambda}(\bar{X} - c\bar{t}) + \phi\right] \text{(lower wall)} \quad (2)$$

where $\bar{a}_1$ and $\bar{a}_2$ are amplitudes of the waves, λ is the wave length and c is the wave speed, $\phi(0 \le \phi \le \pi)$ is the phase difference, $\bar{t}$ is the time. $\bar{X}$ and $\bar{Y}$ are the rectangular coordinates with $\bar{X}$ measured along the axis of the channel and $\bar{Y}$ perpendicular to $\bar{X}$.

The laws of momentum, mass and energy yield

$$\frac{\partial \bar{U}}{\partial \bar{X}} + \frac{\partial \bar{V}}{\partial \bar{Y}} = 0 \quad (3)$$

$$\rho\left(\frac{\partial}{\partial \bar{t}} + \bar{U}\frac{\partial}{\partial \bar{X}} + \bar{V}\frac{\partial}{\partial \bar{Y}}\right)\bar{U} = -\frac{\partial \bar{P}}{\partial \bar{X}} + \mu\left[\frac{\partial^2 \bar{U}}{\partial \bar{X}^2} + \frac{\partial^2 \bar{U}}{\partial \bar{Y}^2}\right] - \sigma B_0^2 \bar{U} \quad (4)$$

$$\rho\left(\frac{\partial}{\partial \bar{t}} + \bar{U}\frac{\partial}{\partial \bar{X}} + \bar{V}\frac{\partial}{\partial \bar{Y}}\right)\bar{V} = -\frac{\partial \bar{P}}{\partial \bar{Y}} + \mu\left[\frac{\partial^2 \bar{V}}{\partial \bar{X}^2} + \frac{\partial^2 \bar{V}}{\partial \bar{Y}^2}\right] \quad (5)$$

$$\rho C_p\left(\frac{\partial}{\partial \bar{t}} + \bar{U}\frac{\partial}{\partial \bar{X}} + \bar{V}\frac{\partial}{\partial \bar{Y}}\right)T = K\left[\frac{\partial^2 T}{\partial \bar{X}^2} + \frac{\partial^2 T}{\partial \bar{Y}^2}\right] + \mu\left[2\left(\left(\frac{\partial \bar{U}}{\partial \bar{X}}\right)^2 + \left(\frac{\partial \bar{U}}{\partial \bar{Y}}\right)^2\right) + \left(\frac{\partial \bar{U}}{\partial \bar{Y}} + \frac{\partial \bar{U}}{\partial \bar{X}}\right)^2\right] + \sigma B_0^2 \bar{U} \quad (6)$$

where $\bar{U}$ and $\bar{V}$ are the velocity component in $\bar{X}$ and $\bar{Y}$ directions, respectively; P is the pressure; μ the dynamic viscosity; ρ is the density of fluid ; σ is the electrical conductivity; K is the thermal conductivity and C_p is the specific heat.

We introduce a wave/moving frame $(\bar{x}, \bar{y})$ moving with velocity c away from the fixed frame $(\bar{X}, \bar{Y})$ by the transformation

$$\bar{x} = \bar{X} - c\bar{t},\ \bar{y} = \bar{Y},\ \bar{u} = \bar{U} - c, \text{and } \bar{p}(\bar{x}, \bar{y}) = \bar{P}(\bar{X}, \bar{Y}, t) \quad (7)$$

Now, we also define the following non-dimensional quantities

$$x = \frac{\bar{x}}{\lambda},\quad y = \frac{\bar{y}}{d_1},\quad u = \frac{\bar{u}}{c},\quad v = \frac{\bar{v}}{c\delta},$$

$$\delta = \frac{d_1}{\lambda},\ h_1 = \frac{\bar{H}_1}{d_1},\ h_2 = \frac{\bar{H}_2}{d_1},\ d = \frac{d_2}{d_1},\ a = \frac{a_1}{d_1},\ b = \frac{b_1}{d_1},$$

$$p = \frac{d_1^2}{c\lambda\mu},\quad \upsilon = \frac{\mu}{\rho},\quad \mathrm{Re} = \frac{\rho c d_1}{\mu},\quad t = \frac{c\bar{t}}{\lambda},$$

$$M = \left(\frac{\sigma}{\mu}\right)^{1/2} B_0 d_1,\ \theta = \frac{T - T_1}{T_1 - T_0},\ Br = \Pr E,$$

$$E = \frac{c^2}{C_p(T_1 - T_0)},\ \Pr = \frac{\mu C_p}{K},\ u = \frac{\partial \psi}{\partial y},\ v = -\frac{\partial \psi}{\partial x}$$

After using long wave length and low Reynolds number eqs. (3) to (6) reduces to

$$\frac{dp}{dx} = \frac{\partial^3 \psi}{\partial y^3} - \left(\frac{1}{Da} + M^2\right)\left(\frac{\partial \psi}{\partial y} + 1\right) \quad (8)$$

$$\frac{\partial p}{\partial y} = 0 \quad (9)$$

where Eq.(9) indicates that p≠p(y), ν denotes the kinematic viscosity, M is the Hartman number, Re is the Reynolds number, Br is the Brinkman number, E is the Eckert number, Pr is the Prandtl number, δ is the wave number, θ is the dimensionless temperature and Da Darcy number.

Elimination of pressure from Eqs.(8) and (9) yield

$$\frac{\partial}{\partial y}\left[\frac{\partial^3 \psi}{\partial y^3} - \left(\frac{1}{Da} + M^2\right)\left(\frac{\partial \psi}{\partial y} + 1\right)\right] = 0 \quad (10)$$

Define η and F as the dimensionless mean flows in the laboratory and wave frames as

$$\eta = \frac{\overline{Q}}{cd_1}, \quad F = \frac{q}{cd_1} \qquad (11)$$

where Q=q+c(d_1-d_2), then

η =F+1-d
(12)

$$F = \int_{h_1}^{h_2} \frac{\partial \psi}{\partial y} dy \qquad (13)$$

The convective boundary condition for the temperature can be expressed as

$$-K(\partial T/\partial Y) = l(T - T_w) \qquad (14)$$

where K is the thermal conductivity, l is the wall heat transfer coefficient and T_w is the temperature of the wall . Physically, this condition states that the rate of heat transfer from the wall to the fluid (or vice versa) is proportional to the difference of their temperature. Having in mind the asymmetric nature of considered channel, we choose l_1 and l_2 as the heat transfer coefficients of upper and lower walls, respectively. Hence, the dimensionless boundary conditions are

$$\begin{cases} \psi = \frac{F}{2}, & \frac{\partial \psi}{\partial y} = -1, \quad \frac{\partial \theta}{\partial y} + Bi_1\theta = 0, at\ y = h_1 \\ \psi = -\frac{F}{2}, & \frac{\partial \psi}{\partial y} = -1, \quad \frac{\partial \theta}{\partial y} - Bi_2(\theta - 1) = 0, at\ y = h_2 \end{cases}$$

(15)

$$\begin{cases} \{h_1(x) = 1 + a\cos(2\pi x), \quad h_2(x) = -d - b\cos(2\pi x + \phi) \\ Bi_1 = \frac{l_1 d_1}{K}, Bi_2 = \frac{l_2 d_1}{K} \end{cases} \qquad (16)$$

where Bi_1 and Bi_2 are the Biot-numbers for the upper and lower walls.

3. SOLUTION OF THE PROBLEM

The solution of Eq. (10) subjected to the conditions Eq. (15) are given by

$$\frac{\partial}{\partial y}\left[\frac{\partial^3 \psi}{\partial y^3} - \beta\left(\frac{\partial \psi}{\partial y} + 1\right)\right] = 0$$

$$\frac{\partial^4 \psi}{\partial y^4} - \frac{\partial}{\partial y}\left[\beta \frac{\partial \psi}{\partial y} + \beta\right] = 0$$

$$\frac{\partial^4 \psi}{\partial y^4} - \beta \frac{\partial^2 \psi}{\partial y^2} = 0 \qquad (17)$$

where $\beta = \sqrt{\left(\frac{1}{Da} + M^2\right)}$

$$\therefore\ \psi = c_1 + c_2 y + c_3 e^{\beta y} + c_4 e^{-\beta y} \qquad (18)$$

$$c_1 = -\frac{(h_1 + h_2)\left(2e^{h_1\beta} - 2e^{h_2\beta} + F_0\beta e^{h_1\beta} + F_0\beta e^{h_2\beta}\right)}{2\left(-2e^{h_1\beta} + 2e^{h_2\beta} + h_1\beta e^{h_1\beta} + h_1\beta e^{h_2\beta} - h_2\beta e^{h_1\beta} - h_2\beta e^{h_2\beta}\right)}$$

$$c_2 = \frac{\left(2e^{h_1\beta} - 2e^{h_2\beta} + F_0\beta e^{h_1\beta} + F_0\beta e^{h_2\beta}\right)}{\left(-2e^{h_1\beta} + 2e^{h_2\beta} + h_1\beta e^{h_1\beta} + h_1\beta e^{h_2\beta} - h_2\beta e^{h_1\beta} - h_2\beta e^{h_2\beta}\right)}$$

$$c_3 = -\frac{(F_0 + h_1 - h_2)}{\left(-2e^{h_1\beta} + 2e^{h_2\beta} + h_1\beta e^{h_1\beta} + h_1\beta e^{h_2\beta} - h_2\beta e^{h_1\beta} - h_2\beta e^{h_2\beta}\right)}$$

$$c_4 = \frac{(F_0 + h_1 - h_2)\left(e^{h_1\beta + h_2\beta}\right)}{(-2e^{h_1\beta} + 2e^{h_2\beta} + h_1\beta e^{h_1\beta} + h_1\beta e^{h_2\beta} - h_2\beta e^{h_1\beta} - h_2\beta e^{h_2\beta})}$$

The axial pressure gradiant at zeroth order is

$$\frac{\partial P}{\partial x} = \beta^3 c_3 e^{\beta y} - \beta^3 c_4 e^{-\beta y} - \beta c_2 - \beta^2 c_3 e^{\beta y} + \beta^2 c_4 e^{-\beta y} - \beta$$

(19)

For one wavelength the integration of (19) gives, the non-dimensional pressure rise over one wavelength for the velocity is

$$\Delta P_\lambda = \int_0^1 \frac{\partial P}{\partial x} dx$$

4. RESULTS AND DISCUSSION

The section discussed till now deals with the graphical and numerical results of the present problem. The expression for pressure rise and pressure gradient is calculated numerically with a mathematical software name by “Mathematica”. Figure (1), (2) , (3) & (4) shows the variation of pressure gradient with respect to the space variable 'x' for different values of Upper wave amplitude 'a', Lower wave amplitude 'b' channel width 'd', Darcy number 'Da'. It is thereby observed that, the axial pressure gradient (dp/dx) increases with an increase in upper wave amplitude 'a' and the axial pressure gradient (dp/dx) decreases with a rise in Darcy number (Da).

Figure (4), (5), (6) & (8) shows the variation of axial velocity 'u' with 'y' for different values of Upper wave amplitude 'a', Lower wave amplitude 'b', Darcy number 'Da' and Hartmann number (M). The graphical representation also shows that , in the pumping region the η increases with a rise in Upper wave amplitude 'a', Lower wave amplitude 'b', Darcy number (Da) and Hartmann number (M)

Figure (9) &(10) the variation of pressure rise with η for different values of Darcy number (Da) and Hartmann number (M). It concludes that, the velocity traces a parabolic trajectory with maximum value at the center of channel.

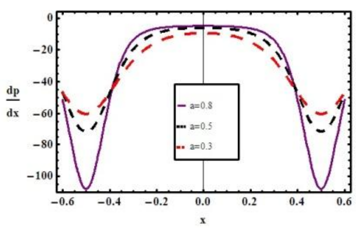

Fig. 1 Pressure gradient dp/dx for different values of a

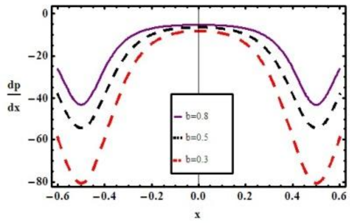

Fig. 2 Pressure gradient dp/dx for different values of b

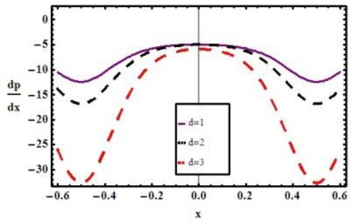

Fig. 3 Pressure gradient dp/dx for different values of d

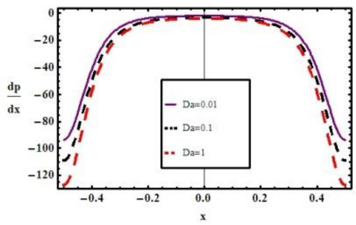

Fig. 4 Pressure gradient dp/dx for different values of Da

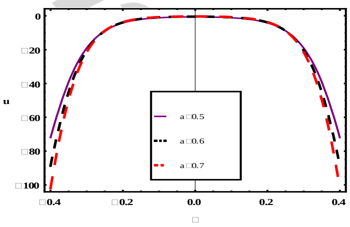

Fig. 5 Velocity profiles for different values of a

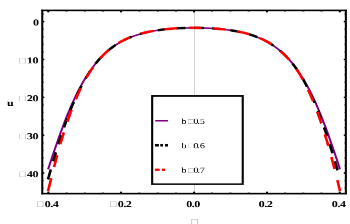

Fig. 6 Velocity profiles for different values of b

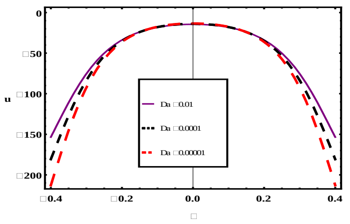

Fig. 7 Velocity profiles for different values of Da

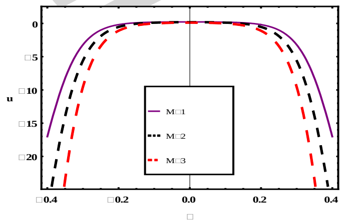

Fig.8 Velocity profiles for different values of M

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