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A Fluid Flow and heat transfer problem in porous media with a uniformly distributed constant heat source in the flow region.

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Abstract

This paper deals with a steady forced convective flow of a viscous fluid of finite depth in a porous medium over a fixed horizontal, impermeable bottom with a uniformly distributed constant heat source in the flow region. Exact solutions of Momentum and Energy equations are obtained when the temperatures on the fixed bottom and on the free surface are prescribed. Flow rate, Mean velocity, Temperature, Mean Temperature in the flow region and the heat transfer rate on the boundaries have been obtained. The cases of large and small values of porosity coefficient have been obtained as limiting cases.

KeyWords: Porous Medium, Velocity, Flow rate, Temperature, Mean temperature, Nusselt number, Porosity parameter.

Introduction:

Forced convective flows through porous and non porous channels of a variety of geometries was examined by Raghava charyulu [1]in 1984 and G.V.Satya narayana Raju [2] in the year 1989. Forced convective flow of a viscous liquid of finite depth in a porous medium over a fixed horizontal impermeable plate was studied by K.Moinuddin and Prof. N.Ch.Pattabhiramacharyulu [3] in the year 2012.Jaweed Ameen[4] studied, Some investigations on fluid flows through generalized porous media in 2009.

This paper deals with the steady forced convective flow of a viscous liquid of viscosity μ and of finite depth H through a porous medium of permeability coefficient ‘k*’ over a fixed impermeable horizontal bottom with a uniformly distributed constant heat source in the flow region.The flow is generated by a constant pressure gradient parallel to the fixed bottom plate . The momentum equation considered is the generalized Darcy’s law proposed by Yama Moto and Iwamura[5] which takes into account the convective acceleration and the Newtonian viscous stresses in addition to the classical Darcy force.

The basic equations of momentum and energy are solved to give exact expressions for velocity and temperature distributions. Employing, the flow rate , mean velocity, mean temperature, and the nusselt numbers at the fluid boundaries have been obtained and illustrated graphically.

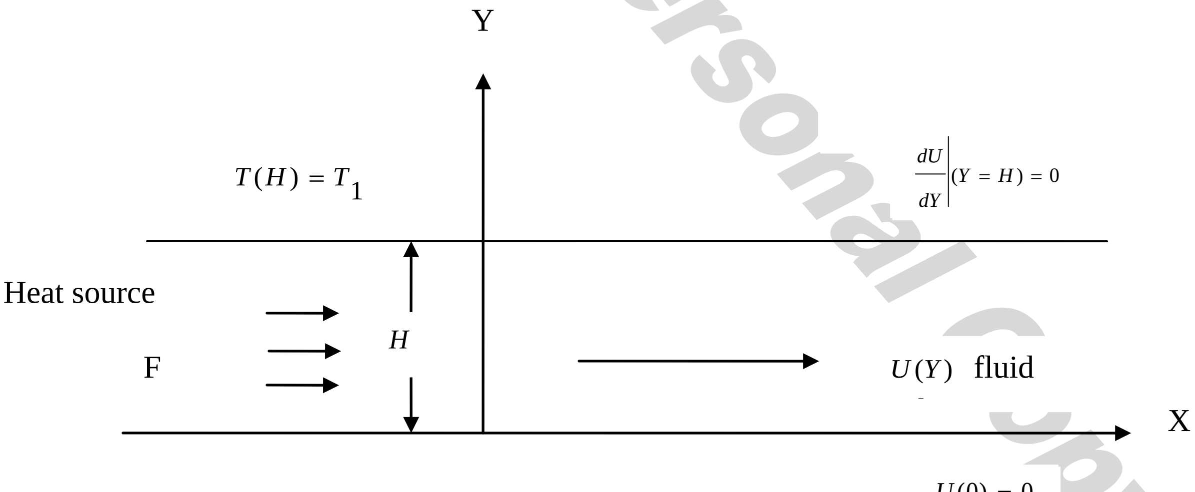
The cases of 1. high porosity 2. low porosity are also discussed.

Mathematical Formulation

Consider the steady forced convective flow of a viscous fluid through a porous medium of viscosity coefficient μ and of finite depth (H) over a fixed horizontal impermeable bottom. The flow is generated by a constant pressure gradient parallel to the plate. Further the bottom is kept at a constant temperature T_0 and the free surface is exposed to atmospheric temperature T_1 with a uniformly distributed constant heat source F in the flow region.

With reference to a rectangular Cartesian co-ordinates system with the origin 'O' on the bottom, X-axis in the flow direction (that is parallel to the applied pressure gradient).The Y-axis vertically upwards, the bottom is represented as Y=0 and the free surface as Y=H.

Let the flow be characterized by a velocity U = (U(Y), 0, 0). This choice of velocity evidently satisfies the continuity equation $\nabla.U = 0$. Further let T(Y) denotes the temperature distribution and 'F' a constant heat source distributed uniformly.



Flow Configuration

Basic Equations:

Let the convective flow be characterized by the velocity field U = (U(Y), 0, 0) and the temperature T(Y). This choice of the velocity satisfies the continuity equation.

$$\nabla.U = 0 \quad \text{----- (1)}$$

The Momentum Equation is

$$-\frac{\partial P}{\partial X} + \mu \frac{d^2U}{dY^2} - \mu \frac{U}{k^*} = 0 \quad \text{---- (2)}$$

and the Energy Equation is

$$\rho c U \frac{\partial T}{\partial X} = K \frac{d^2T}{dY^2} + \mu \left(\frac{dU}{dY}\right)^2 + F \quad \text{----- (3)}$$

where F is a constant heat source distributed uniformly in the flow region. In the above equations ρ is the fluid density, k^* the coefficient of porosity of the medium, c is the specific heat ,K the thermal conductivity of the fluid and P the fluid pressure.

Boundary Conditions:
Since the bottom is fixed,

$$U(0) = 0 \qquad \text{---- (4a)}$$

At the free surface shear stress vanishes

$$\mu \frac{dU}{dY} = 0 \text{ at } Y=H. \qquad \text{----(4b)}$$

Also $T(0) = T_0$ -----(5a) and $T(H) = T_1$ ----(5b)

where T_0 is the bottom temperature and T_1 is the atmosphere temperature.
In terms of the non-dimensional variables defined by

$$Y=ay;\ X=ax;\ H=ah\ ;\ U = \frac{\mu u}{\rho a^2}\ ;\ P = \frac{\mu^2 p}{\rho a^2};\ \ T = T_0 + (T_1 - T_0)\theta;\ \Pr = \frac{\mu c}{k};\ k^* = \frac{a^2}{\alpha^2};$$

$$E = \frac{\mu^3}{\rho^2 a^2 K(T_1 - T_0)};\ -\frac{\partial P}{\partial X} = \frac{\mu^2 c_1}{\rho a^3}(c_1 = -\frac{\partial p}{\partial x}) \text{ and } \frac{\partial T}{\partial X} = \frac{(T_1 - T_0)}{a}c_2 \text{ where } c_2 = \frac{\partial \theta}{\partial x}$$

and

$$f = \frac{a^2 F}{K(T_1 - T_0)} \qquad \text{----(6)}$$

the basic field equations are rewritten as :

<u>Momentum Equation:</u>

$$\frac{d^2 u}{dy^2} - \alpha^2 u = -c_1 \qquad \text{---- (7)}$$

<u>Energy Equation</u>:

$$\frac{d^2\theta}{dy^2} = P_r c_2 u - E(\frac{du}{dy})^2 - f \qquad \text{---- (8)}$$

together with the boundary conditions

For velocity $u(0) = 0$ and $\frac{du}{dy} = 0$ at y=h ---- (9)

and for the temperature $\theta(0) = 0$ and $\theta(h) = 1$ ---- (10)

The momentum equation together with the related boundary conditions (9) yields the velocity distribution:

$$u(y) = \frac{c_1}{\alpha^2}(1 - \frac{\cosh\alpha(h-y)}{\cosh\alpha h}) \qquad \text{---- (11)}$$

The energy equation satisfying the boundary conditions yields the temperature distribution:

$$\theta(y) = \frac{y}{h} + \frac{P_r c_1 c_2}{\alpha^2}\left[\frac{(h-y)}{h\alpha^2} - \frac{y(h-y)}{2} + \frac{1}{\alpha^2 \cosh \alpha h}\left(\frac{y}{h} - \cosh \alpha(h-y)\right)\right] +$$

$$\frac{Ec_1^2}{2\alpha^2 \cosh^2 \alpha h}\left[\frac{(h-y)\cosh(2\alpha h)}{4\alpha^2 h} - \frac{y(h-y)}{2} + \frac{1}{4\alpha^2}\left(\frac{y}{h} - \cosh 2\alpha(h-y)\right)\right]$$

$$+ \frac{fy(h-y)}{2}$$

----(12)

The mean velocity in the non-dimensional form is

$$\frac{1}{h}\int_0^h u(y)dy = \frac{c_1}{h\alpha^2}\left(h - \frac{\tanh \alpha h}{\alpha}\right) \qquad \text{----(13)}$$

Further the mean temperature in non-dimensional form is given by

$$\bar{\theta} = \frac{1}{h}\int_0^h \theta dy$$

$$= \frac{1}{2} + \frac{P_r c_1 c_2}{\alpha^2}\left(\frac{-h^2}{12} + \frac{1}{2\alpha^2} + \frac{1}{2\alpha^2 \cosh \alpha h} - \frac{\tanh \alpha h}{h\alpha^3}\right) -$$

$$\frac{Ec_1^2}{2\alpha^2 \cosh^2 \alpha h}\left(\frac{-1}{8\alpha^2} + \frac{h^2}{12} - \frac{\cosh 2\alpha h}{8\alpha^2} + \frac{\sinh 2\alpha h}{8h\alpha^3}\right) + \frac{fh^2}{12}$$

----(14)

Heat transfer coefficient (Nusselt Number):

On the bottom:

$$\frac{d\theta}{dy}\Big|_{y=0} = \frac{1}{h} + \frac{P_r c_1 c_2}{\alpha^2}\left(\frac{\tanh \alpha h}{\alpha} - \frac{h}{2} + \frac{1}{h\alpha^2 \cosh \alpha h} - \frac{1}{h\alpha^2}\right) -$$

---(15)

$$\frac{Ec_1^2}{2\alpha^2 \cosh^2 \alpha h}\left(\frac{\sinh 2\alpha h}{-2\alpha} - \frac{1}{4h\alpha^2} + \frac{h}{2} + \frac{\cosh 2\alpha h}{4h\alpha^2}\right) + \frac{fh}{2}$$

On the free surface :

$$\frac{d\theta}{dy}\Big|_{y=h} = \frac{1}{h} + \frac{P_r c_1 c_2}{\alpha^2}\left(\frac{h}{2} + \frac{1}{h\alpha^2 \cosh \alpha h} - \frac{1}{h\alpha^2}\right) - \frac{Ec_1^2}{2\alpha^2 \cosh^2 \alpha h}\left(-\frac{h}{2} - \frac{1}{4h\alpha^2} + \frac{\cosh 2\alpha h}{4h\alpha^2}\right) - \frac{fh}{2} \qquad \text{----(16)}$$

Case 1. Fluid flow in a medium with high porosity i.e flow for small values of α or large values of the porosity coefficient k*

Neglecting terms of α higher than $O(\alpha^2)$ we get

Velocity: $$u(y) = c_1\left\{\frac{(2hy - y^2)}{2} - \frac{\alpha^2}{24}\left(8h^3 y - 4hy^3 + y^4\right)\right\} \qquad \text{----(17)}$$

Mean velocity $$\bar{u} = \frac{1}{h}\int_0^h u\,dy = \frac{c_1 h^2}{15}(5 - 2\alpha^2 h^2) \qquad \text{---(18)}$$

Temperature :

$$\theta(y) = \frac{y}{h} + \frac{P_r c_1 c_2}{720}\left[(120hy^3 - 90h^3 y - 30y^4) - \alpha^2(h^6 - 36h^5 y + 40h^3 y^3 - 6hy^5 + y^6)\right] + \frac{Ec_1^2}{8640}\left[720(3h^3 y - 6h^2 y^2 + 4hy^3 - y^4) - \alpha^2\begin{pmatrix}791h^6 - 576h^5 y + 2160h^4 y^2 + \\ 960h^3 y^3 + 720h^2 y^4 - 576hy^5 + 96y^6\end{pmatrix}\right] + \frac{fy(h-y)}{2} \qquad \text{---(19)}$$

Mean temperature:

$$\bar{\theta} = \frac{1}{h}\int_0^h \theta\,dy = \frac{1}{2} - \frac{p_r c_1 c_2 h^4}{5040}(147 + 575\alpha^2 h^2) + \frac{Ec_1^2 h^4}{210960}(5274 + 907\alpha^2 h^2) + \frac{fh^2}{12} \qquad \text{----(20)}$$

Nusselt Number:
On the bottom:

$$\frac{d\theta}{dy}\Big|_{y=0} = \frac{1}{h} + \frac{P_r c_1 c_2 h^3}{40}(2\alpha^2 h^2 - 5) + \frac{Ec_1^2 h^3}{4320}(1080 - 1223\alpha^2 h^2) + \frac{fh}{2} \qquad \text{---(21)}$$

On the top:

$$\frac{d\theta}{dy}\Big|_{y=h} = \frac{1}{h} + \frac{P_r c_1 c_2 h^3}{24}(5 - 2\alpha^2 h^2) - \frac{Ec_1^2 h^3}{180}(15 - 13\alpha^2 h^2) - \frac{fh}{2} \qquad \text{----(22)}$$

Case 2. For large values of α i.e for low porosity

For large α $\sinh \alpha h \approx \frac{e^{\alpha h}}{2}$; $\cosh \alpha h \approx \frac{e^{\alpha h}}{2}$; $\tanh \alpha h \approx 1$ and neglecting the terms of $O(\frac{1}{\alpha^3})$ we get

Velocity: $u(y) = \frac{c_1}{\alpha^2}\left(1 - e^{-\alpha y}\right)$ ---- (23)

Mean velocity: $\bar{u} = \frac{c_1}{\alpha^2}$ ---- (24)

Temperature:

$$\theta(y) = \frac{y}{h} - \frac{p_r c_1 c_2 (h-y)y}{2\alpha^2} + \frac{fy(h-y)}{2} \quad \text{----(25)}$$

Mean temperature:

$$\bar{\theta} = \frac{1}{h}\int_0^h \theta dy = \frac{1}{2} - \frac{p_r c_1 c_2 h^2}{12\alpha^2} + \frac{fh^2}{12} \quad \text{----(26)}$$

Nusselt Number:

On the bottom:

$$\frac{d\theta}{dy}\Big|_{y=0} = \frac{1}{h} - \frac{P_r c_1 c_2 h}{2\alpha^2} + \frac{fh}{2} \quad \text{---(27)}$$

On the top

$$\frac{d\theta}{dy}\Big|_{y=h} = \frac{1}{h} + \frac{P_r c_1 c_2}{2\alpha^2} - \frac{fh}{2} \quad \text{---(28)}$$

Results and Discussions

It is noticed that the velocity profiles are more steep for large values of α that is the velocity of the fluid decreases with the increase in the value of α (Fig 1). From (Fig.2), in the case of small α, it is noticed that the velocity of the flow region decreases with the increase in the values of α .For the case of large α, (Fig.3), illustrates that the velocity decreases with the increase in the values of α and tends to zero for the values of α bigger than 100.

It is evident from the (Fig.4), that for the increasing values of the pressure gradient c1 the mean velocity increases and appears to be decreasing with the increase in the values of α. (Fig.5) (for the case of small α), illustrates that the mean velocity is high for the higher values of the pressure gradient c_1 at the bottom plate and decreases gradually with the increase in the values of α. In the case of large α, an increase in the values of α decreases the mean velocity of the flow region (Fig.6).

It is observed from (Fig.7), that the temperature of fluid flow slightly decreases with increasing porosity parameter α when the heat source f=10 and from (Fig.8), it is clear that temperature remains unaltered for the same porosity parameters when f=100. In the case of small α (Fig.9), illustrates that the temperature slightly decreases with the increasing values of the porosity parameter α when f=10. (Fig.10), illustrates that the temperature of the fluid increases with the increasing smaller values of α and remains unaltered for the larger values of the porosity parameter α in the case of large α.

(Fig.11), illustrates that the mean temperature decreases with the increasing values of the prandtl number ‘p’. In the case of small α it is evident that the mean temperature decreases with the increasing values of the prandtl number ‘p’ (Fig.12). For large α’s Mean temperature decreases with the increasing values of the prandtl number ‘p’ for smaller values of the porosity parameter α (Fig.13).

The rate of heat transfer decreases as the porosity parameter α and prandtl number ‘p’ Increases (Fig.14).In the case of small α the heat transfer rate on the bottom plate decreases for increasing values of the prandtl number ‘p’ (Fig.15).For large α’s rate of heat transfer decreases with the increase in the values of the prandtlNumber ‘p’ on the bottom plate (Fig.16).

Heat transfer rate increases with the increase in the values of ‘p’ and α and the presence of heat source increases the heat transfer rate(Fig.17).For small α’s heat transfer rate on the top plate increases with the increasing values of the prandtl number ‘p’(Fig.18). Rate of heat transfer increases with the increase in the values of the prandtlnumber ‘p’ and remains unchanged for large α’s (Fig.19).

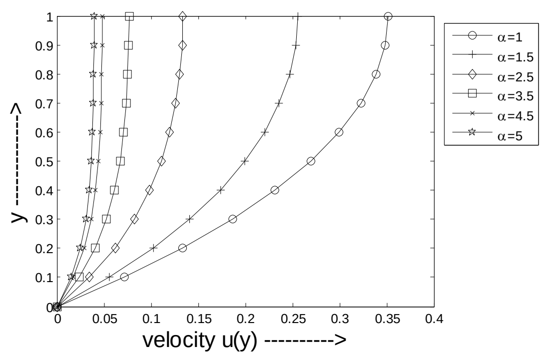


Fig.1 velocity profile for c1=1 and h=1

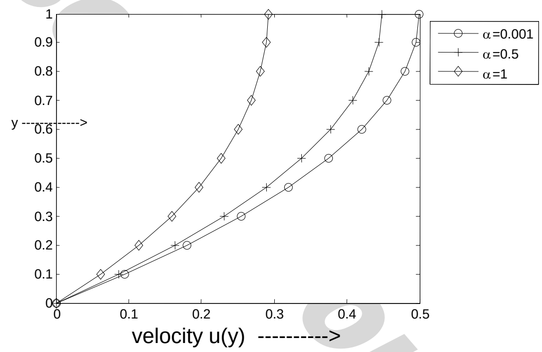


Fig.2.velocity profile for small α,c1=1 & h=1

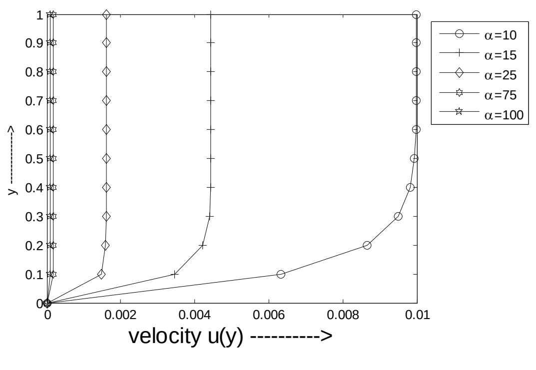


Fig.3. velocity profile for large α ,c1=1 & h=1

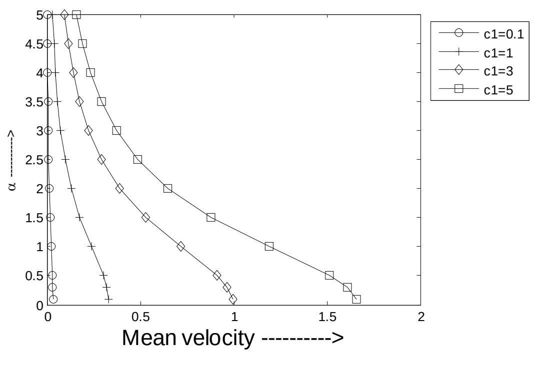


Fig.4 mean velocity for h=1

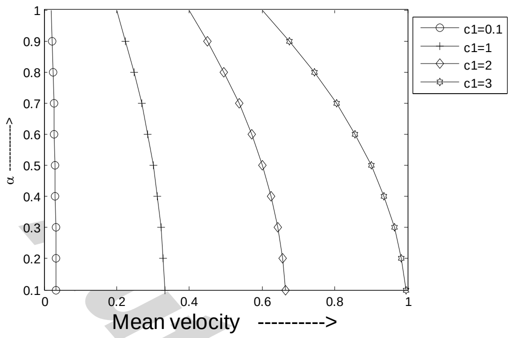


Fig.5. Mean velocity for small α & h=1

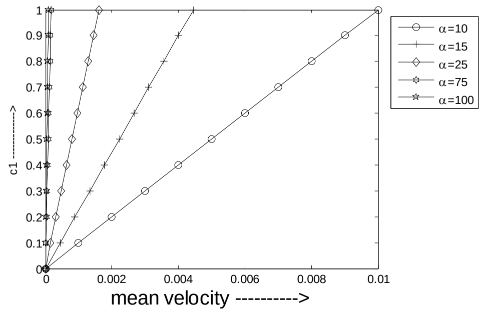


Fig.6. mean velocity for large α

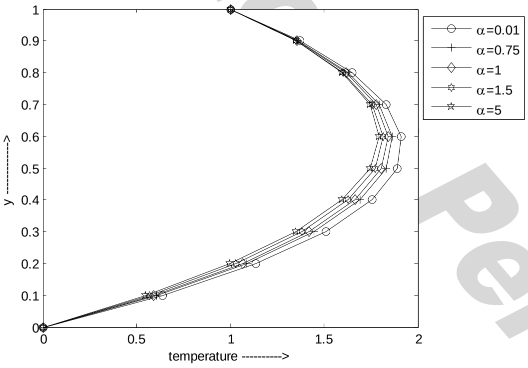


fig.7 Temperature distribution for h=1,E=5,c1=1,c2=1,p=1,f=10

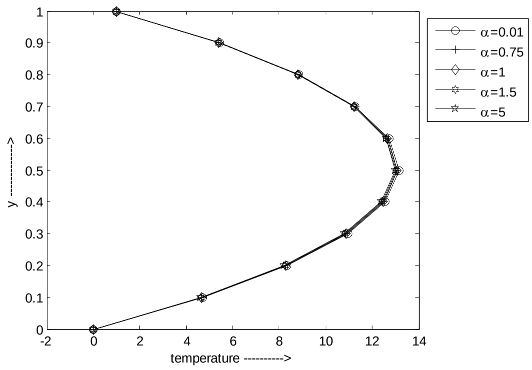


fig.8 Temperature distribution for h=1,E=5,c1=1,c2=1,p=1,f=100

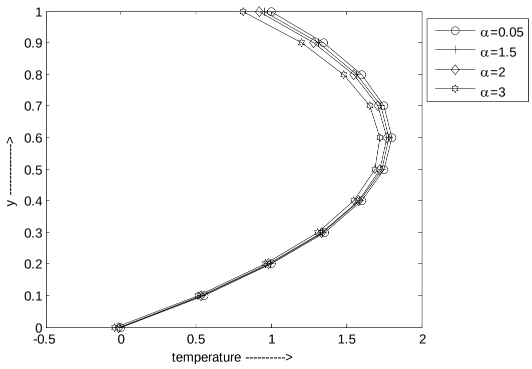


fig.9 Temperature distribution for small α & for h=1,E=5,c1=.1,c2=.5,p=.1,f=10

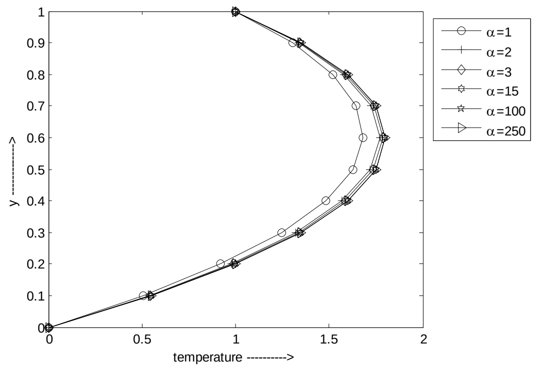


fig.10 Temperature distribution for large α & for h=1,E=5,c1=1,c2=1,p=1,f=10

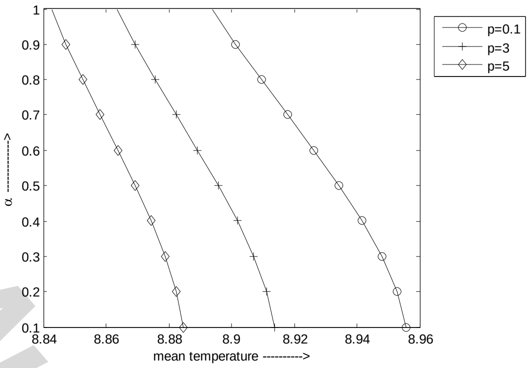


fig.11 Mean Temperature for h=1,E=5,c1=1,c2=.5,f=100

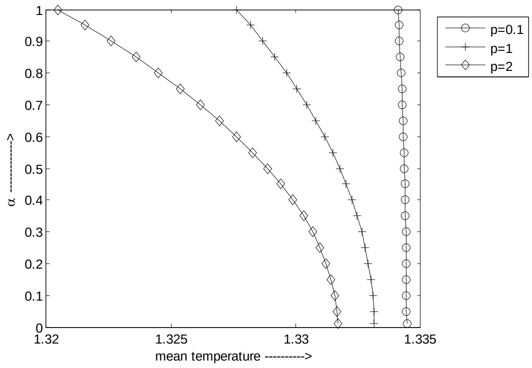


fig.12 Mean Temperature for h=1,E=5,c1=.1,c2=.5,f=10

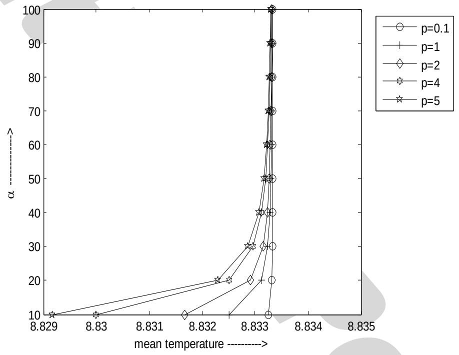


fig.13 mean temperature distribution for large α & for h=1,E=5,c1=1,c2=1,f=100

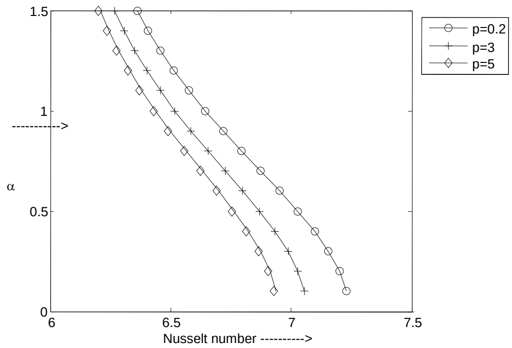


fig 14 Nusselt number on the bottom for h=1,E=5,c1=1,c2=.5,f=10

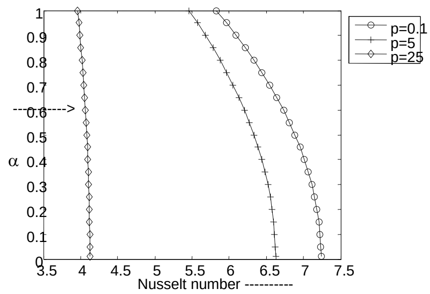


fig 15 Nusselt number on the bottom for h=1,E=5,c1=1,c2=1,f=10

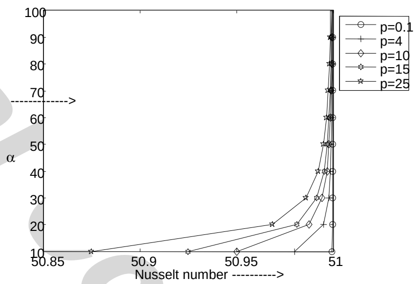


fig.16 Nusselt number on the bottom for h=1,E=5,c1=1,c2=1,f=100

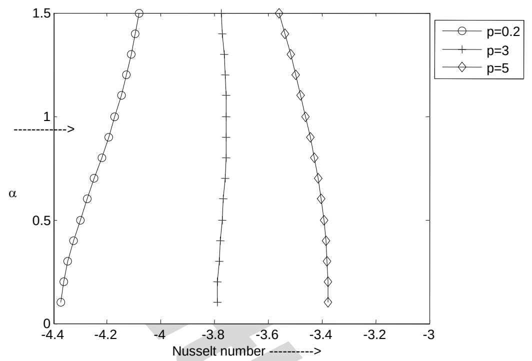


fig.17 Nusselt number on the top for h=1,E=5,c1=1,c2=1,f=10

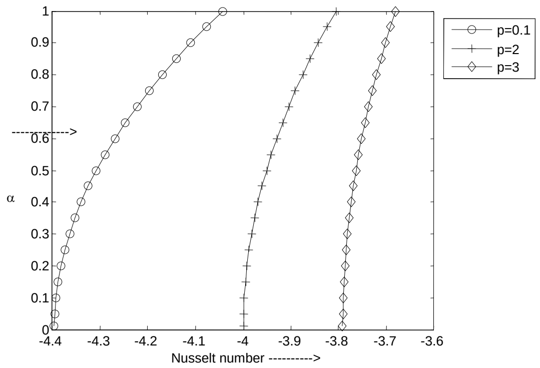


fig.18 Nusselt number on the top for h=1,E=5,c1=1,c2=1,f=10

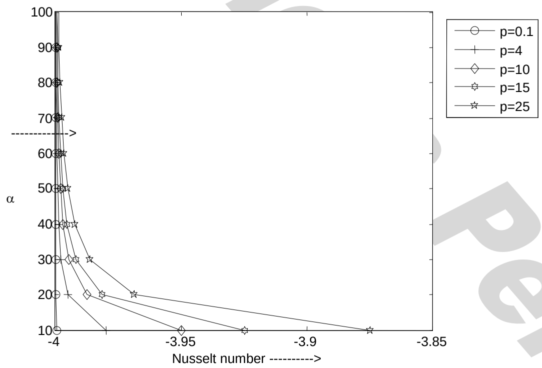


fig.19 Nusselt number on the top for h=1,E=5,c1=1,c2=1,f=10

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