ISSN 0976 – 1411

Available online at www.internationalejournals.com

International eJournals

International eJournal of Mathematics and Engineering
4 (2015) Vol. 6, Issue 2, pp 32 – 47

INTERNATIONAL eJOURNAL OF MATHEMATICS AND ENGINEERING

www.InternationaleJournals.com

MHD FLOW OF A NON-NEWTONIAN FLUID THROUGH POROUS MEDIUM WITH HEAT TRANSFER USING SLIP BOUNDARY CONDITION

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ABSTRACT

The aim of present study is to study the unsteady flow of second order non-Newtonian fluid through porous medium contained in a planar channel subject to the periodic pressure gradient under the influence of transverse magnetic field together with heat transfer. Analytic solution for the velocity of the fluid and shear stress are obtained. The results of various situations are deduced. The effects of various physical parameters are discussed and graphically represented.

1. INTRODUCTION

The study of flow through porous medium assumed importance because of the interesting applications in the diverse fields of science, Engineering and Technology. The practical applications are in the percolation of water through soil, extraction and filteration of oils from wells, the drainage of water, irrigation and sanitary engineering and also in the inter disciplinary fields such as biomedical engineering, the lung alveolar is an example that finds applications in an animal body. The classical Darcy's law Musakat [1, 2], states that the pressure gradient pushes the fluid against the body forces exerted by the medium which can be expressed as $\vec{V} = -\left(\frac{k}{\mu}\right)\nabla P$ with usual notation.

The flow of a non-Newtonian fluid has wide range of applications in the industrial, Technological and scientific field. The fluid flow through the porous duct is an example, which find its application to the technical problems like flow through the packed beds, environmental pollution and blood flow through cardiovascular system. The non-Newtonian fluids such as molten plastic pump, emulsions etc. find important application in petroleum and chemical process. These materials are of electrically conducting nature. Hence MHD flow of non-Newtonian fluid is physically more relevant. Several investigations were made earlier, Nguyen and Chandra [3] have studied applying hodographs transformations to obtain the solution of non-Newtonian MHD flow. Exact solutions of non-Newtonian MHD flow are obtained by Thakur and Singh [4].

The problem of oscillating flow is an application that finds important in various fields of Engineering and Technology. Frater [5] discussed the flow of an elastico-various liquid between torsionally oscillating discs. Johri [6] discussed the problem of elastico-viscous flow induced by circular oscillations of two infinite parallel discs. Later, Johri [7] studied the problem of unsteady slow flow between two infinite parallel discs under elliptic harmonic oscillations of the discs. The problem of oscillating inlet flow of a viscous liquid through circular discs studied by Bulent and Chieh [8]. Narasimha Charyulu and Pattabhai Rama Charyulu [9] studied the oscillating inlet flow of a second order simple fluid between two parallel plates. Narasimha Charyulu *et al*. studied the problem of unsteady flow oscillating non-Newtonian fluid through porous medium [10-12].

The Navier found a boundary condition of fluid slip at solid surface such that $u = h\left(\frac{\partial u}{\partial y}\right)$ where h is the slip coefficient and u the velocity along x-axis. If $h=0$ the condition becomes no slip at the boundary.

Soltani and Yilmazar [13] founded a slip velocity and slip layer thickness in flow of concentrated suspensions. Rim [14] founded an unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction. Yu and Ameel [15] investigated a slip low heat transfer in rectangular micro channels. Derek *et al*. [16] founded an apparent fluid slip at hydrophobic microchannel walls. Khaled and Vafai [17] have been discussed the effect of slip condition on stokes and coutte flows due to an oscillating wall. Malcinde and Mhone [18] investigated a heat transfer to MHD oscillatory flow in a channel filled with porous medium. Kumar *et al*. [19] investigated a viscous flow coaxial cylinder in the presence of magnetic field. Mahmood and Ali [20] founded the effect of slip condition on unsteady MHD oscillatory flow of a viscous fluid in a planar channel. Mishra *et al*. [21] investigated a flow and heat transfer of a MHD viscoelastic fluid in a channel with stretching walls.

In the present problem the flow of a second order non-Newtonian fluid through a planar channel filled with porous medium is studied under the influence transverse magnetic field and heat transfer with slip boundary condition. The effect of various physical parameters such as permeability parameter, magnetic parameter etc is discussed at length. The graphical representations of the results are given.

2. FORMULATION OF THE PROBLEM :

The flow of an incompressible non-Newtonian fluid of second order is considered through planar channel with saturated porous medium in the presence of transverse magnetic field.

The co-ordinate system *0(x, y, z)* chosen such that x-axis being parallel to the length of the channel and y-axis perpendicular to the length of the channel. A transverse magnetic field of strength β_o is applied perpendicular to the flow of the fluid. The width of the channel is taken to be 'a'. The velocity of the fluid is taken to be $\vec{v}(u,0,0)$ which satisfies the equation of continuity

$$\nabla.\vec{v} = 0 \qquad \ldots \qquad (2.1)$$

The governing equations of motion of the fluid will be

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \left(\nu + \beta_1 \frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial y^2} - \frac{\nu}{K}u - \frac{\sigma B_0^2 u}{\rho} + g\beta(T - T_o) \qquad \ldots \qquad (2.2)$$

Heat equation is

$$\frac{\partial T}{\partial t} = \frac{Ko}{\rho C_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p}\frac{\partial q}{\partial y} \qquad \ldots \qquad (2.3)$$

Boundary conditions are given by

$$\left.\begin{array}{ll} u = h\dfrac{\partial u}{\partial y}, \; T = T_W & at \quad y = 0 \\ u = 0, \quad T = T_O & at \quad y = a \end{array}\right\} \qquad \ldots \qquad (2.4)$$

Where u is the velocity, ρ is the fluid density, p is the pressure, t is the time, ν the kinematic viscosity coefficient, k the porous medium permeability coefficient, σ the conductivity of the fluid, B_o the electro magnetic induction, g the acceleration due to gravity, β the coefficient of volume expansion due to temperature, T the fluid temperature, K_o the thermal conductivity, C_p the specific heat of constant pressure and q is the radiative heat flux, β_1 is non-Newtonian parameter.

Consider the fluid which is optically thin with a relatively low density and its radiative heat flux is given by

$$\frac{\partial q}{\partial y} = 4\alpha^2 (T - T_O) \qquad \ldots \qquad (2.5)$$

Where α is the mean radiation absorption coefficient. Here define dimension less quantities as under

$$x^* = \frac{x}{a},\ y^* = \frac{y}{a},\ u^* = \frac{u}{v},\ \theta = \frac{T - T_O}{T_W - T_O},\ t^* = \frac{tv}{a}$$

$$p^* = \frac{ap}{\nu\rho\nu},\ N^2 = \frac{4\alpha^2 a^2}{K_O},\ G_r^* = \frac{g\beta(T_W - T_O)}{\nu\nu} a^2$$

(Grashoff number)

$$P_e = \frac{\nu a\rho C_P}{K_O} \text{ (Peclet number)}, \quad R_e = \frac{\nu a}{\nu} \text{ (Reynold's number)}$$

$$M = \sqrt{\frac{a^2 \sigma B_0^2}{\rho \nu}} \text{ (Hartmann number)}$$

where ν is the mean of flow velocity, N radiation parameter T fluid temperature, T_o fluid temperature of the boundary y = 0 T_w fluid temperature at the boundary y = a, θ is non dimensional temperature.

Employing the above non dimensional quantities and removing (∗), the Governing equation and heat equation become.

$$R_e \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \left(1 + s.\frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial y^2} - \left(\eta^2 + M^2\right)u + G_r\theta \qquad \ldots \qquad (2.6)$$

Where $\eta^2 = \frac{a^2}{K}, \quad s = \frac{\beta v}{a\nu}$

$$P_e \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} - N^2\theta \qquad \ldots \qquad (2.7)$$

Non-dimensional boundary conditions are

$$u = h\frac{\partial u}{\partial y},\ \theta = 1\ \ at\ \ y = 0 \qquad \ldots \qquad (2.8)$$

$$u = 0,\ \theta = 0\ \ at\ \ y = 1 \qquad \ldots \qquad (2.9)$$

We assume $-\frac{\partial P}{\partial x} = \lambda e^{\text{int}}$, where λ is constant and n is the frequency of oscillations.

3. SOLUTION OF THE PROBLEM :

To solve the equations (2.6), (2.7) with the boundary conditions (2.8) and (2.9) we apply perturbation technique. The velocity and temperature and assumed to be

$$u(y,t) = u_0(y) + \in u_1(y)\, e^{\text{int}} + 0(\in^2) \qquad \ldots \qquad (3.1)$$

$$\theta(y,t) = \theta_0(y) + \in \theta_1(y)\, e^{\text{int}} + 0(\in^2) \qquad \ldots \qquad (3.2)$$

Substituting the above expansions for u, θ in the equations. (2.6) & (2.7) respectively and collecting the like terms we get.

$$u_0^{\|} - l_1^2\, u_0 = -G_r\theta_0 \qquad \ldots \qquad (3.3)$$

$$u_1^{\|} - l_2^2\, u_1 = -\frac{\lambda}{\in(1+isn)} - \frac{G_r\theta_1}{(1+isn)} \qquad \ldots \qquad (3.4)$$

$$\theta_0^{\|} + N^2\,\theta_0 = 0 \qquad \ldots \qquad (3.5)$$

$$\theta_1^{\|} + l_3^2\,\theta_1 = 0 \qquad \ldots \qquad (3.6)$$

where $l_1 = \sqrt{\eta^2 + M^2}$ $\quad l_2 = \sqrt{\frac{\eta^2 + M^2 + inR_e}{1+isn}}$ $\quad l_3 = \sqrt{N^2 + iP_e n}$

The corresponding boundary conditions are

$$\left.\begin{aligned} &u = h\frac{\partial u}{\partial y},\ \theta = 1\ \ at\ \ y = 0 \\ &u = 0,\qquad \theta = 0\ \ at\ \ y = 1 \end{aligned}\right\} \qquad \ldots \qquad (3.7)$$

On solving equations (3.3) – (3.6) and using boundary conditions we get.

$$\theta_0(y) = 0 \qquad \ldots \qquad (3.8)$$

$$\theta_1(y) = \frac{\sin\, l_3 y}{\in \sin l_3} \qquad \ldots \qquad (3.9)$$

$$u_0(y) = 0 \qquad \ldots \qquad (3.10)$$

and

$$u_1(y) = \frac{C_1}{\in}\cos h\, l_2 y + \frac{C_2}{\in}\sin h\, l_2 y + \frac{\lambda}{\in l_2^2(1+isn)} + \frac{G_r}{\in (1+isn)\left(l_2^2 + l_3^2\right)}\cdot\frac{\sin l_3 y}{\sin l_3} \qquad (3.11)$$

where

$$C_1 = -\frac{\lambda}{(1+isn)\; l_2^2} + h\left\{C_2 l_2 + \frac{l_3.G_r}{(1+isn)\left(l_2^2 + l_3^2\right)\sin l_3}\right\}$$

$$C_2 = \frac{1}{\sin h\, l_2 + hl_2\, \cos h\, l_2}\left[\left(\cos hl_2 - 1\right)\frac{\lambda}{l_2^2(1+isn)} + \frac{G_r}{(1+isn)\left(l_2^2 + l_3^2\right)}\left\{1 + h\, l_3\, \frac{\cos h\, l_2}{\sin h\, l_2}\right\}\right]$$

Therefore

$$u(y,t) = \left\{C_1 \cos h\, l_2 y + C_2 \sin h\, l_2 y + \frac{\lambda}{l_2^2(1+isn)} + \frac{G_r}{(1+isn)\left(l_2^2 + l_3^2\right)}\frac{\sin l_3 y}{\sin l_3}\right\}e^{\text{int}} \qquad (3.12)$$

and $\theta(y,t) = \left(\dfrac{\sin l_3 y}{\sin l_3}\right)e^{\text{int}} \qquad \ldots \qquad (3.13)$

The shear stress is given by

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = \left\{l_2 C_2 + \frac{l_3 G_r}{(1+isn)\left(l_2^2 + l_3^2\right)\sin l_3}\right\}e^{\text{int}} \qquad \ldots \qquad (3.14)$$

Case 1 : Flow of Non-Newtonian fluid through porous medium when the magnetic field is absent. i.e. M=0

$$u(y,t) = \left\{C_1 \cos h\, l_2 y + C_2 \sin h\, l_2 y + \frac{\lambda}{l_2^2(1+isn)} + \frac{G_r}{(1+isn)\left(l_2^2 + l_3^2\right)}\frac{\sin l_3 y}{\sin l_3}\right\}e^{\text{int}}$$

and $\theta(y,t) = \left(\dfrac{\sin l_3 y}{\sin l_3}\right)e^{\text{int}}$

The shearing stress is given by

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = \left\{ l_2 C_2 + \frac{l_3 G_r}{(1+isn)\left(l_2^2 + l_3^2\right)\sin l_3} \right\} e^{\text{int}}$$

Where $l_2 = \sqrt{\dfrac{\eta^2 + inR_e}{1+isn}}$ $\quad l_3 = \sqrt{N^2 + iP_e n}$

Case 2 : Flow of Non-Newtonian fluid through clear region under magnetic field. i.e., η = 0.

$$u(y,t) = \left\{ C_1 \cos h\, l_2 y + C_2 \sin h\, l_2 y + \frac{\lambda}{l_2^2 (1+isn)} + \frac{G_r}{(1+isn)\left(l_2^2 + l_3^2\right)} \frac{\sin l_3 y}{\sin l_3} \right\} e^{\text{int}}$$

and $\theta(y,t) = \left(\dfrac{\sin l_3 y}{\sin l_3}\right) e^{\text{int}}$

The shearing stress is given by

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = \left\{ l_2 C_2 + \frac{l_3 G_r}{(1+isn)\left(l_2^2 + l_3^2\right)\sin l_3} \right\} e^{\text{int}}$$

Where $l_2 = \sqrt{\dfrac{M^2 + inR_e}{1+isn}}$ $\quad l_3 = \sqrt{N^2 + iP_e n}$

Case 3 : Flow of Newtonian fluid through porous medium under magnetic field

$$u(y,t) = \left\{ C_1 \cos h\, l_2 y + C_2 \sin h\, l_2 y + \frac{\lambda}{l_2^2} + \frac{G_r}{\left(l_2^2 + l_3^2\right)} \frac{\sin l_3 y}{\sin l_3} \right\} e^{\text{int}}$$

and $\theta(y,t) = \left(\dfrac{\sin l_3 y}{\sin l_3}\right) e^{\text{int}}$

The shearing stress is given by

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = \left\{ l_2 C_2 + \frac{l_3 G_r}{\left(l_2^2 + l_3^2\right)\sin l_3} \right\} e^{\text{int}}$$

where $l_2 = \sqrt{\eta^2 + M^2 + inR_e}$ $\quad l_3 = \sqrt{N^2 + iP_e n}$

Case 4 : Flow of Newtonian fluid through porous medium when magnetic field absent s = 0, M = 0.

$$u(y,t)=\left\{C_1\cos h\, l_2 y + C_2 \sin h\, l_2 y + \frac{\lambda}{l_2^2} + \frac{G_r}{\left(l_2^2+l_3^2\right)}\frac{\sin l_3 y}{\sin l_3}\right\}e^{\text{int}}$$

and $\theta(y,t)=\left(\frac{\sin l_3 y}{\sin l_3}\right)e^{\text{int}}$

The shearing stress is given by

$$\tau=\left(\frac{\partial u}{\partial y}\right)_{y=0}=\left\{l_2 C_2 + \frac{l_3 G_r}{\left(l_2^2+l_3^2\right)\sin l_3}\right\}e^{\text{int}}$$

where $l_2=\sqrt{\eta^2+inR_e}$ $\quad l_3=\sqrt{N^2+iP_e n}$

Case 5 : Flow of Newtonian fluid through clear region when magnetic field absent. S = 0, M = 0, η = 0.

$$u(y,t)=\left\{C_1\cos h\, l_2 y + C_2 \sin h\, l_2 y + \frac{\lambda}{l_2^2} + \frac{G_r}{\left(l_2^2+l_3^2\right)}\frac{\sin l_3 y}{\sin l_3}\right\}e^{\text{int}}$$

and $\theta(y,t)=\left(\frac{\sin l_3 y}{\sin l_3}\right)e^{\text{int}}$

The shearing stress is given by

$$\tau=\left(\frac{\partial u}{\partial y}\right)_{y=0}=\left\{l_2 C_2 + \frac{l_3 G_r}{\left(l_2^2+l_3^2\right)\sin l_3}\right\}e^{\text{int}}$$

where $l_2=\sqrt{inR_e}$ $\quad l_3=\sqrt{N^2+iP_e n}$

Case 6 : Flow of non-Newtonian fluid through porous medium under magnetic field when the pressure gradient is a constant. i.e. n = 0.

$$u(y,t)=\left\{C_1\cos h\,l_2 y + C_2 \sin h\,l_2 y + \frac{\lambda}{l_2^2} + \frac{G_r}{l_2^2+N^2}\frac{\sin Ny}{\sin N}\right\}$$

and $\theta(y,t)=\left(\frac{\sin Ny}{\sin N}\right)$

The shearing stress is given by

$$\tau=\left(\frac{\partial u}{\partial y}\right)_{y=0}=\left\{l_2 C_2+\frac{N\,G_r}{\left(l_2^2+N^2\right)\sin N}\right\}$$

where $l_2=\sqrt{\eta^2+M^2}$

RESULTS AND DISCUSSION

The unsteady flow of second order non-Newtonian fluid is investigated, through porous medium using slip boundary condition with periodic pressure gradient under transverse magnetic field with heat transfer. Expressions for temperature and velocity and deduced for different situations of the flow of the fluid.

From Fig. 1, it is observed that the velocity of the fluid is decreasing corresponding to increase in the Hartmann number.

From Fig. 2 it can be noticed that increase in the permeability parameter decreases the velocity of the fluid. A similar effect will be observed in the case of Reynold’s number, increase in the Reynold’s number shows a decrease in the velocity field from the Fig. 3.

From Fig. 4 the effect of peclet number and from Fig. 5 the effect of Radiation parameter shows as the parameters are increasing there is decrese in the velocity field.

From Fig. 6 we observe that the influence of non-Newtonian parameter on the flow field. As the non-Newtonian parameter increases the velocity field decreases.

From Fig. 7 & Fig. 8 it is observed that increasing Radiation parameter increases the temperature parameter θ, but increasing peclet number decrease the temperature parameter.

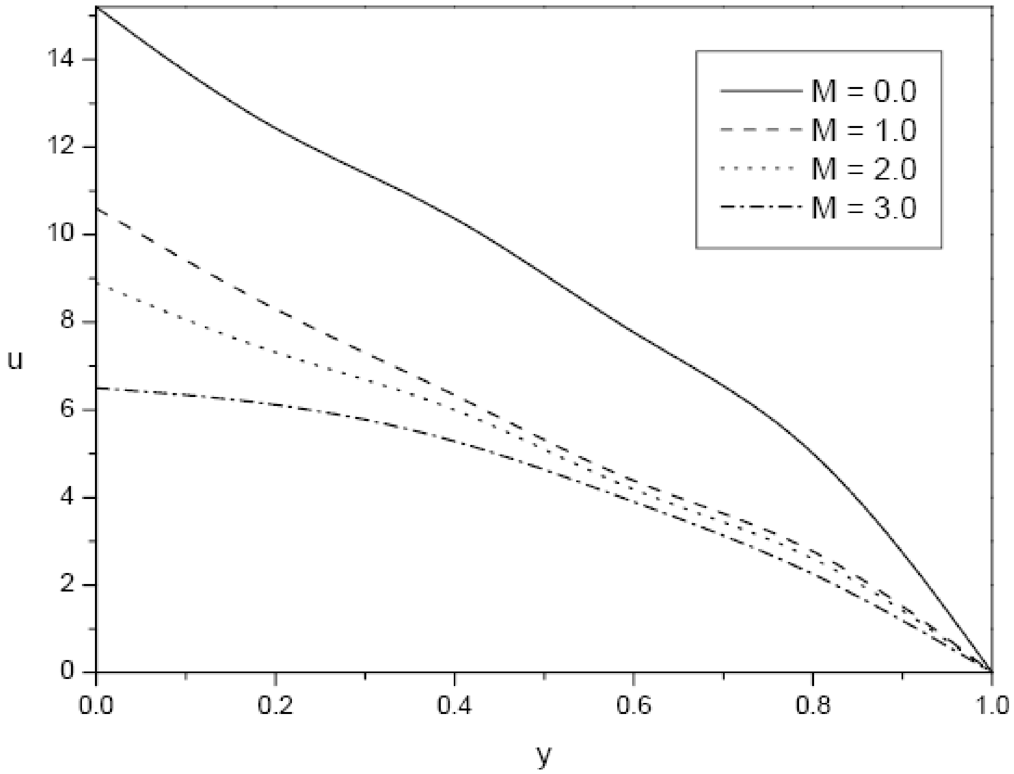

Fig. 1 : Variation of velocity with magnetic parameter M

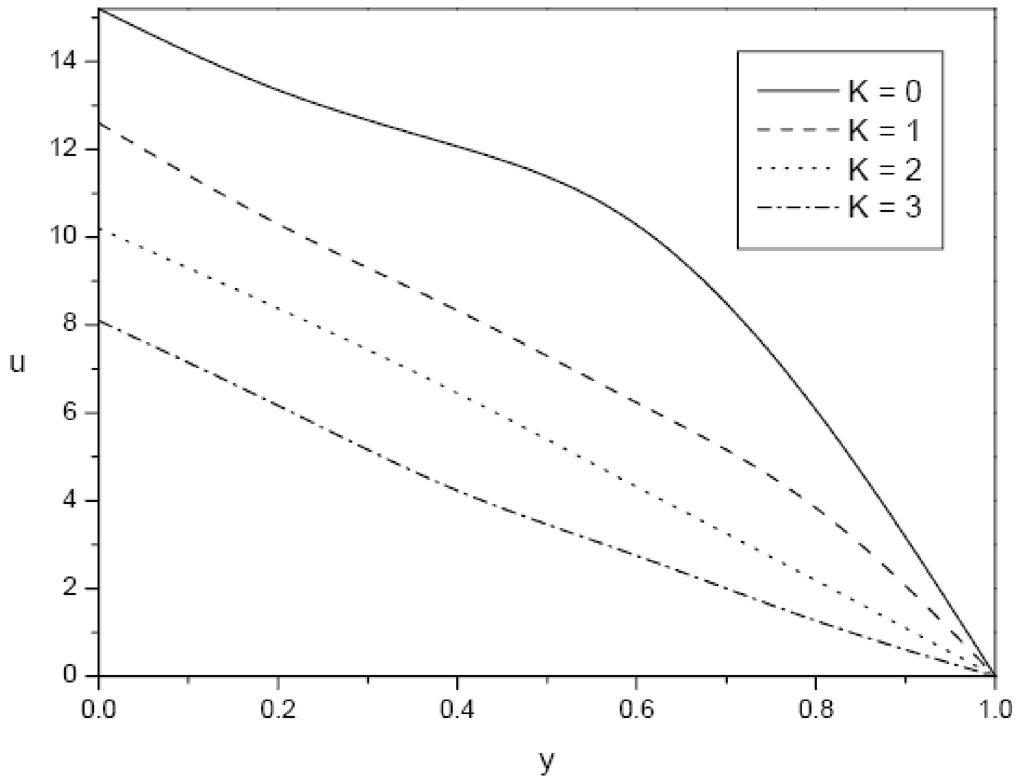

Fig. 2 : Variation of velocity with permeability parameter K

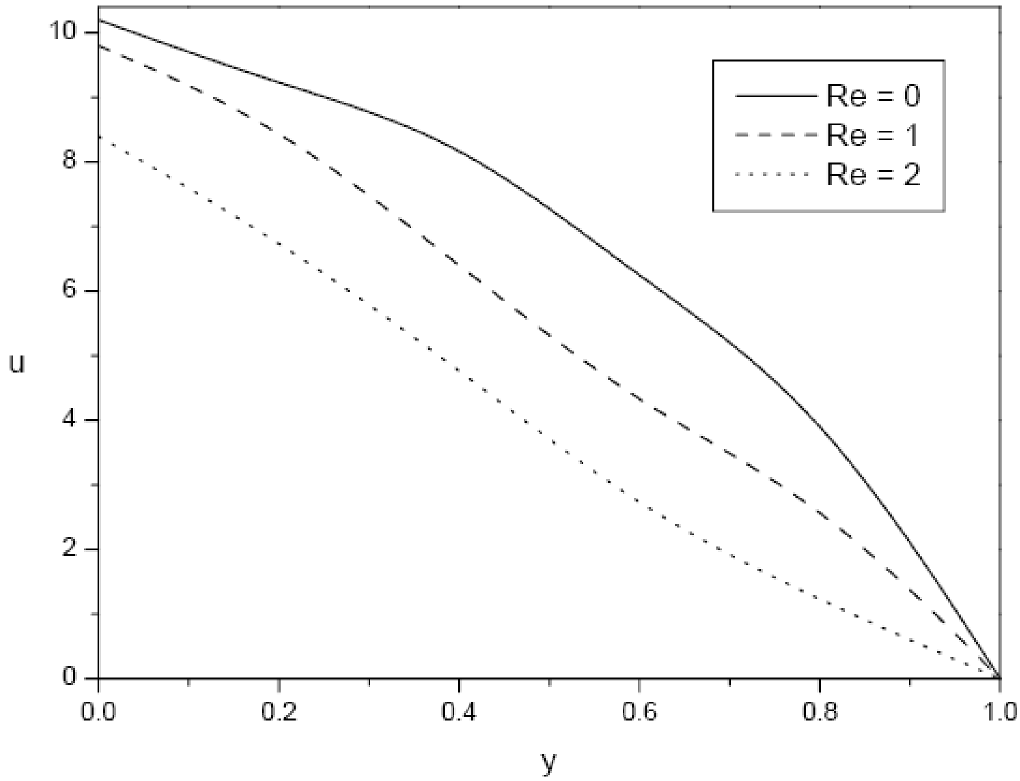

Fig. 3 : Variation of velocity with Reynold's number R_e

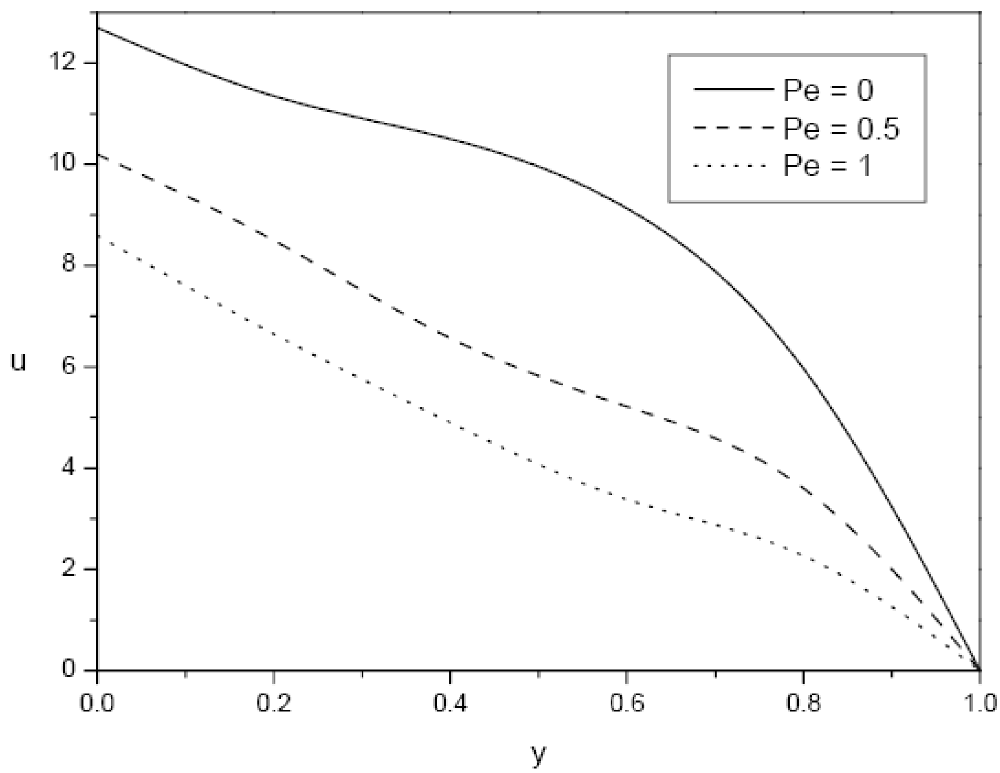

Fig. 4 : Variation of velocity with peclet parameter N

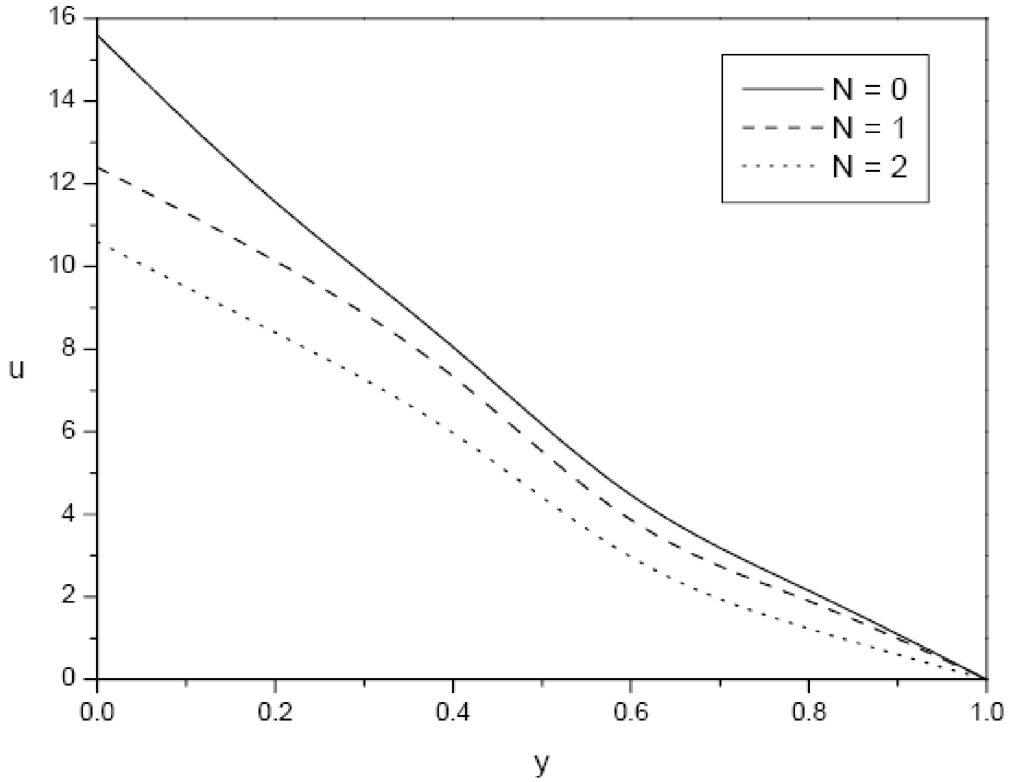

Fig. 5 : Variation of velocity with radiation parameter N

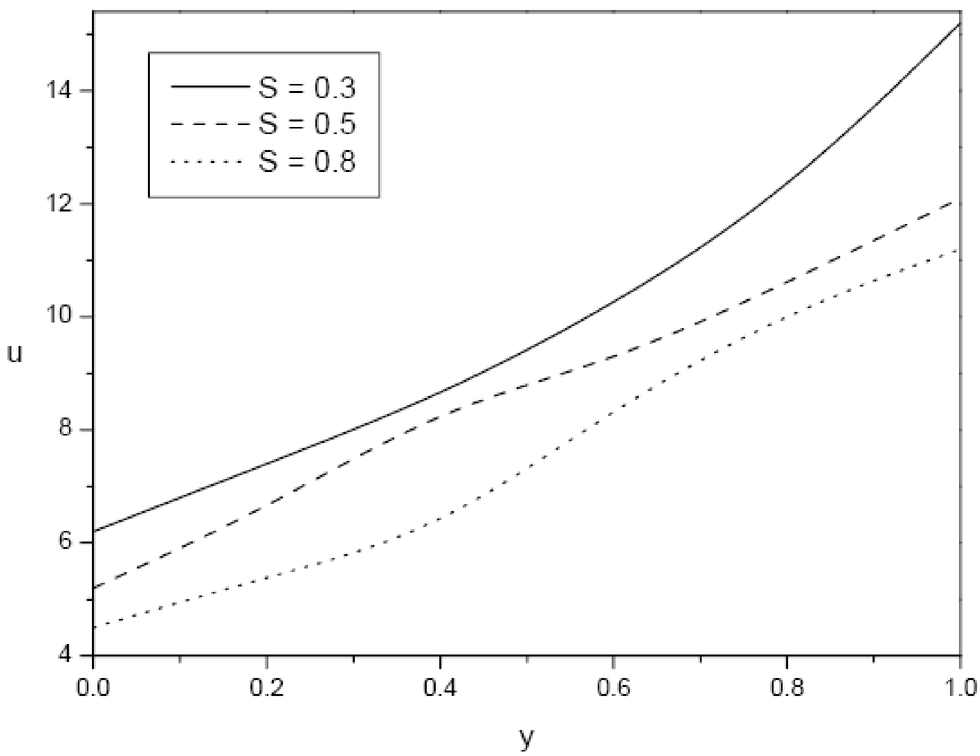

Fig. 6 : Variation of velocity with Non-Newtonian parameter

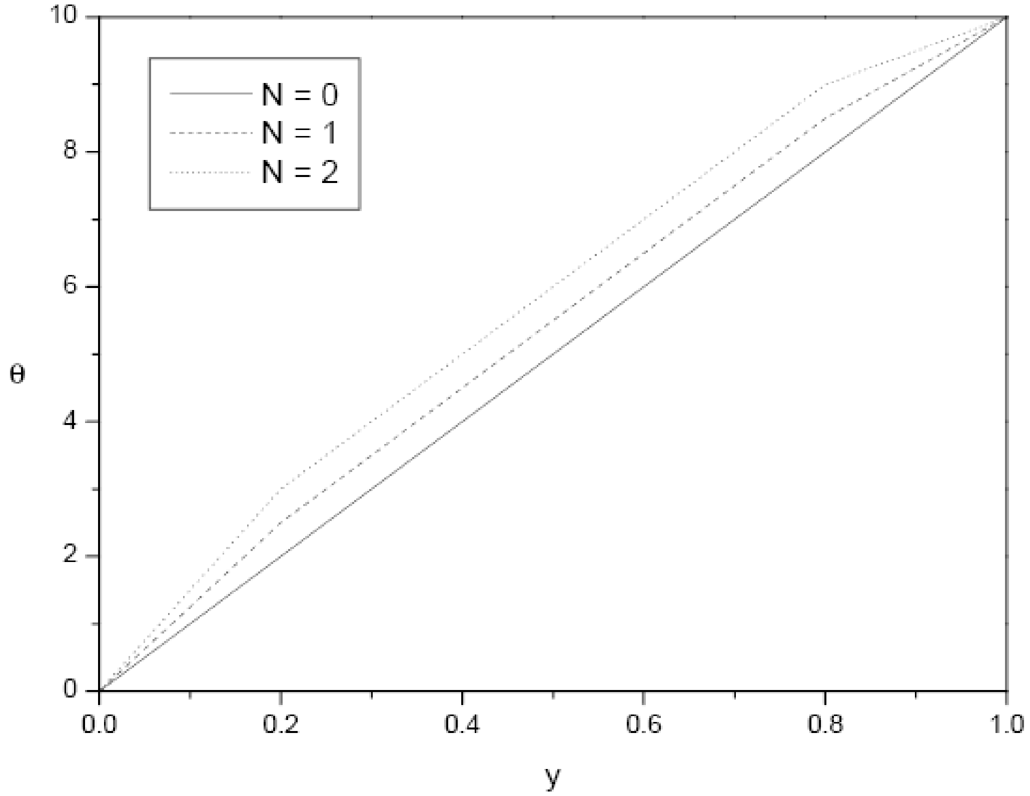

Fig. 7 : Variation of temperature with radiation parameter

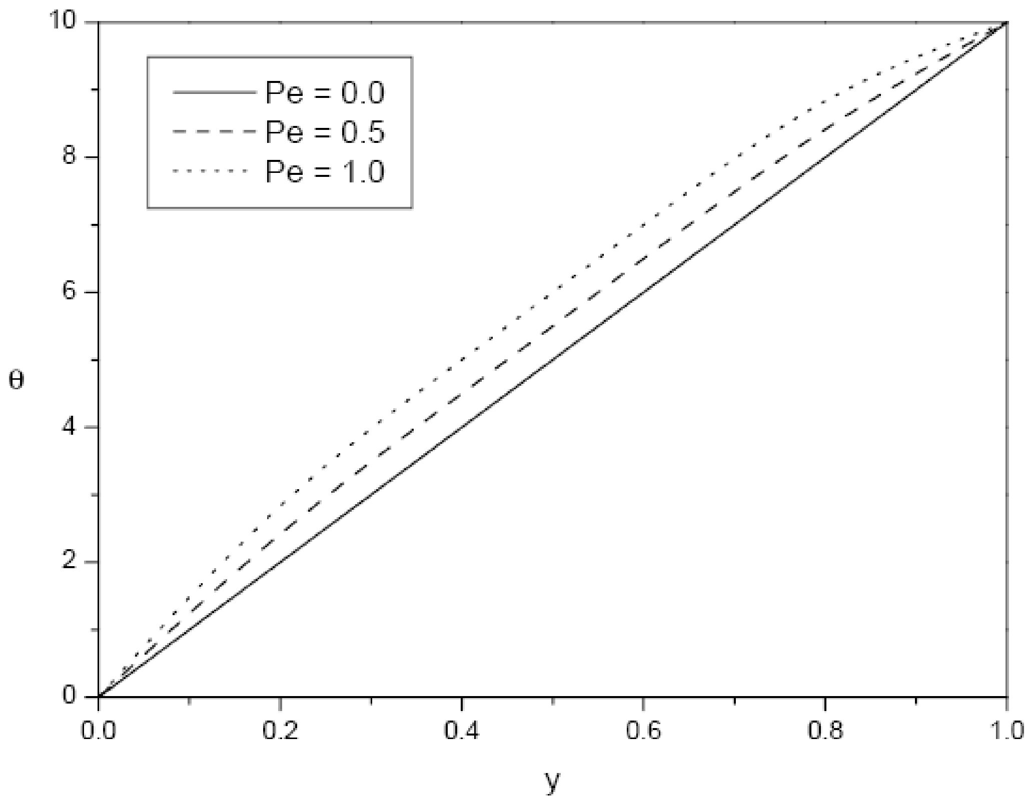

Fig. 8 : Variation of temperature with peclet number

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