ISSN 0976 – 1411

Available online at www.internationalejournals.com

International eJournals

International eJournal of Mathematics and Engineering
2 (2015) Vol. 6, Issue 1, pp 15 – 21

INTERNATIONAL eJOURNAL OF MATHEMATICS AND ENGINEERING
www.InternationaleJournals.com

Homotopy Analysis to Flow of Walter's Liquid B over an Exponentially Stretching Sheet

Hymavathi T[1], Mallipriya V[2] and Vijay Kumar P[3]

[1,2,3]:Department of Mathematics, Adikavi Nannaya University, Rajahmundry, A.P., India.
E-Mail: talla.hymavathianur@gmail.com

Abstract:

In this paper, we analyzed the behaviour of boundary layer flow of Walter's liquid B over an exponentially stretching sheet. The governing boundary layer equations are transferred into ordinary differential equations using a suitable similarity transformation and then solved using an analytical technique known as homotopy analysis method (HAM), which provides us an aptable way to control and adjust the convergence region using a non-zero auxiliary parameter $\hbar$. Variation of visco-elastic parameter on velocity is analyzed through graphs.

Key Words: Walter's Liquid-B, Stretching Sheet, Boundary Layer, Similarity Transformation, HAM.

1. Introduction:

The non-Newtonian fluids have gained the attention of many researchers due to their various applications in Scientific and Engineering fields. Most of the fluids which are used in industries are non-Newtonian and particularly visco-elastic in nature. There are many visco-elastic fluids that cannot be characterized by Maxwell's or Oldroyd's constitutive relations. One such fluid is Walters' (model B) visco-elastic fluid which is used in chemical technology and industry.

The work on boundary layer flow past a moving surface, initiated by Sakiadis [1], has motivated many researchers [2-3] to carry out their research on two-dimensional boundary-layer flows. Rajagopal et al. [4] obtained similarity solutions numerically for the boundary layer equations by taking small elastic parameter k_1. Abel et al. [5] studied the problem of visco-elastic fluid flow over a stretching sheet. Recently, Hymavathi [6] carried out the numerical study of boundary layer flow of Walter's liquid B over exponentially stretching sheet using quasi-linearization method.

Here, we present the analytical solution of visco-elastic fluid flow over an exponentially stretching sheet [7] using HAM [8, 9].

2. Mathematical formulation:

Consider the steady two dimensional boundary layer flow of a viscous and incompressible fluid past a stretching sheet coinciding with the plane $y = 0$. For formulating the problem consider the following assumptions:

(i) The boundary sheet is assumed to be moving axially with a velocity of exponential order in the axial direction and generating the boundary layer type of flow.

(ii) A steady two-dimensional laminar flow of an incompressible, electrically conducting visco-elastic liquid (Walters' liquid B model) due to an exponentially stretching sheet is considered.

Then the boundary layer equations are [4]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \gamma\frac{\partial^2 u}{\partial y^2} - k_0\left(u\frac{\partial^3 u}{\partial x\partial y^2} + v\frac{\partial^3 u}{\partial y^3} - \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}\right), \tag{2}$$

where u and v are the velocity components in x and y directions respectively, γ is the kinematics coefficient of viscosity, k_0 is the elastic parameter. Other quantities have their usual meanings [5].

The appropriate boundary conditions are:

$$\begin{aligned} u &= U_w(x) = U_0 e^{\frac{x}{l}} \quad && at \quad y = 0, \\ v &= 0 && at \quad y = 0, \\ u &= 0 && as \quad y \to \infty. \end{aligned} \tag{3}$$

Where U_0 is a constant and l is the reference length.

The velocity component u and v in terms of stream function $\psi(x,y)$ can be written as

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}.$$

Introducing a similarity variable η, such that

$$\eta = y\sqrt{\frac{U_0}{2l\gamma}}e^{\frac{x}{2l}}, \qquad \psi(x,y) = \sqrt{2\gamma l U_0}\, f(\eta) e^{\frac{x}{2l}}. \tag{4}$$

In terms of the above variables, equation (2) takes the form:

$$2f'^2 - ff'' = f''' - k_1\left(3f'f''' - \frac{1}{2}ff'''' - \frac{3}{2}f''^2\right), \tag{5}$$

where $k_1 = \frac{k_0 U_w}{\gamma}$, is the dimensionless visco-elastic parameter.

The boundary conditions are:

$$\begin{aligned} f &= 0, \qquad f' = 1 \quad at \quad \eta = 0, \\ f' &= 0 \qquad\qquad\quad as \quad \eta \to \infty. \end{aligned} \tag{6}$$

3. HAM solution:

For finding the solution of (5) using HAM, initial approximation and linear operator are chosen as

$$f_0(\eta) = 1 - e^{-\eta},$$
$$L(f) = f''' - f',$$

with property

$$L(C_1 + C_2 e^{\eta} + C_3 e^{-\eta}) = 0,$$

where C_1, C_2 and C_3 are the arbitrary constants determined from the boundary conditions.

Let $p \in [0,1]$ be the embedding parameter and $\hbar$ indicates the non-zero auxiliary parameter. Then the following equations are contributed:

Zeroth-order deformation equations:

$$(1-p)L\big(f(\eta;p) - f_0(\eta)\big) = p\hbar\, N[f(\eta;p)], \tag{7}$$

$$f(0;p) = 0, \quad f'(0;p) = 1, \quad f'(\infty;p) = 0, \tag{8}$$

$$N[f(\eta;p)]=\frac{\partial^3 f(\eta;p)}{\partial\eta^3}-2\left(\frac{\partial f(\eta;p)}{\partial\eta}\right)^2+f(\eta;p)\frac{\partial^2 f(\eta;p)}{\partial\eta^2}-k_1\left(3\frac{\partial f(\eta;p)}{\partial\eta}\frac{\partial^3 f(\eta;p)}{\partial\eta^3}\right.$$
$$\left.-\frac{1}{2}f(\eta;p)\frac{\partial^4 f(\eta;p)}{\partial\eta^4}-\frac{3}{2}\left(\frac{\partial^2 f(\eta;p)}{\partial\eta^2}\right)^2\right). \quad (9)$$

For $p=0\, and\; p=1$

$$f(\eta;0)=f_0(\eta) \qquad f(\eta;1)=f(\eta). \quad (10)$$

When p increases from 0 to 1 then $f(\eta;p)$ varies from $f_0(\eta)\; to\; f(\eta)$. Using Taylor's theorem w.r.to p, we have

$$f(\eta;p)=f_0(\eta)+\sum_{m=1}^{\infty} f_m(\eta)p^m, \quad (11)$$

where $$f_m(\eta)=\frac{1}{m!}\frac{\partial^m f(\eta;p)}{\partial p^m}\bigg|_{p=0}. \quad (12)$$

Let $\hbar$ be chosen in such a way that the series (11) is convergent at $p=1$, therefore, through (11) the following equations are obtained

$$f(\eta)=f_0(\eta)+\sum_{m=1}^{\infty} f_m(\eta). \quad (13)$$

mth-order deformation equations:

Differentiating equation (7) m times w.r.to p, setting $p=0$ and finally dividing with $m!$, we get,

$$L(f_m(\eta)-\chi_m f_{m-1}(\eta))=\hbar R_m^f(\eta), \quad (14)$$

$$f_m(0)=0,\quad f_m'(0)=0,\quad f_m'(\infty)=0, \quad (15)$$

$$R_m^f(\eta)=f_{m-1}'''-2\sum_{i=0}^{m-1} f_{m-1-i}' f_i' +\sum_{i=0}^{m-1} f_{m-1-i} f_i'' -k_1\left(3\sum_{i=0}^{m-1} f_{m-1-i}' f_i''' -\frac{1}{2}\sum_{i=0}^{m-1} f_{m-1-i} f_i'''' -\frac{3}{2}\sum_{i=0}^{m-1} f_{m-1-i}'' f_i''\right), \quad (16)$$

$$\chi_m=\begin{cases}0, & m\le 1\\ 1, & m>1.\end{cases} \quad (17)$$

The symbolic software MATHEMATICA is employed to solve the mth order deformation equation (14) with the conditions (15).

4. Convergence of HAM:

As given by Liao [10], the non-zero auxiliary parameter $\hbar$ in series (13) helps us to control and adjust the convergence of series solutions. For admissible values of $\hbar$, we plotted the $\hbar$-curve of $f''(0)$. Figure 1 shows that the admissible values of $\hbar$ are $-1.2 \le \hbar \le 0.0$ respectively. The series converge in the whole region of η when $\hbar = -0.87$. The convergence of homotopy solution for different approximations is given in Table 1.

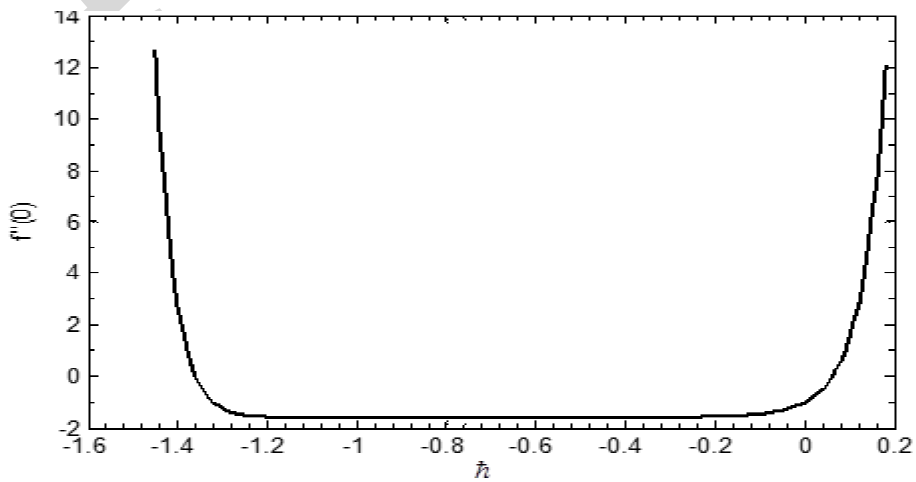

Figure 1: $\hbar$-curve for $f(\eta)$ at 20th order approximation when $k_1 = 0.1$.

Table 1: Convergence of HAM solution for different orders of approximations when $k_1 = 0.1$.

Order	$-f''(0)$
1	1.34075
5	1.38594
10	1.386
15	1.386
20	1.386
25	1.386

5. Results and Discussion:

The obtained ordinary differential equations are solved analytically using HAM.
The effect of visco-elastic parameter k_1 on velocity profiles $f(\eta)$ and $f'(\eta)$ is given in Figure 2 and Figure 3. From the figures it is clear that the effect of visco-elastic parameter k_1 is to decrease the velocity throughout the boundary layer flow field.

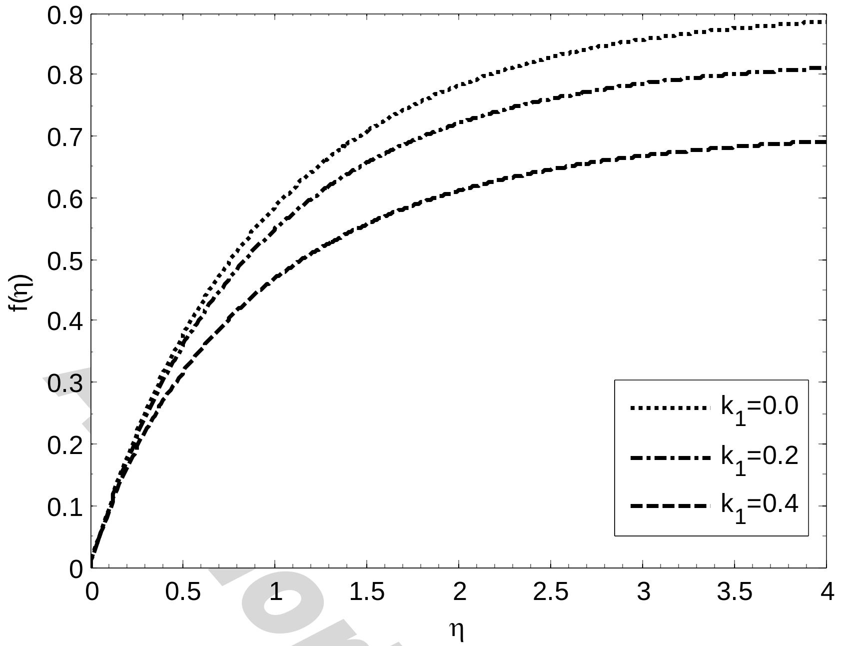

Figure 2: vertical velocity profiles for different values of visco-elastic parameter.

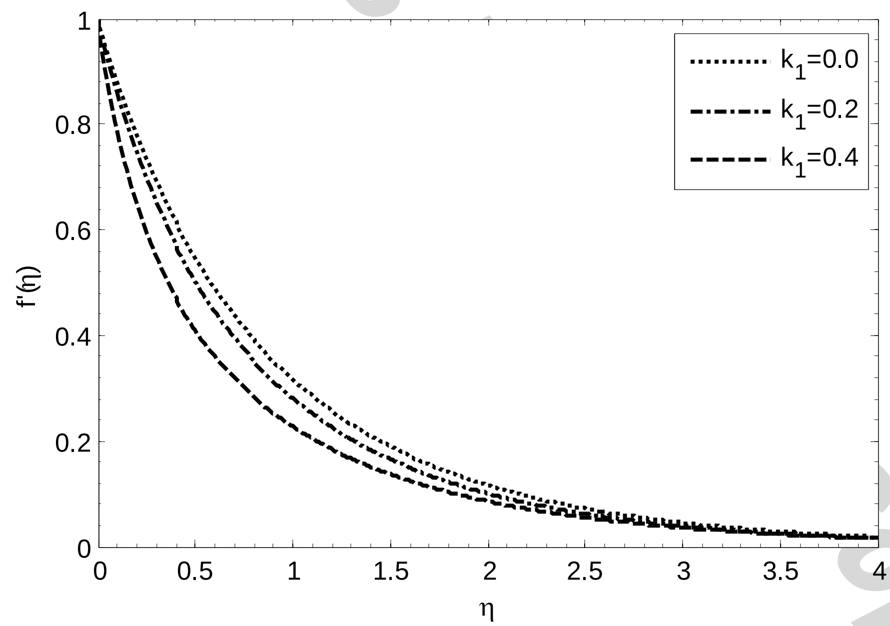

Figure 3: horizontal velocity profiles for different values of visco-elastic parameter.

6. CONCLUSION

In this paper, HAM has been applied successfully to find the characteristics boundary layer flow of Walter's liquid B over an exponentially stretching sheet. It is evidently seen that HAM is a very powerful and efficient technique in finding analytical solutions for wide classes of linear and non-linear problems.

REFERENCES

[1] B. C. Sakiadis, Boundary layer behavior on continuous solid surfaces, *American Institute of Chemical Engineers*, 7, 26-28, 1961.

[2] L. J. Crane, Flow past a stretching plate, *Zeitschriftf¨urAngewandteMathematikund Physik*, 21, 645–647, 1970.

[3] P. S. Gupta and A. S. Gupta, Heat and mass transfer on a stretchingsheet withsuction or blowing,*The Canadian Journal of Chemical* Engineering, 55, 744-746, 1977.

[4] K.R. Rajagopal, Y.T. Na and A.S. Gupta, Flow of a visco-elastic fluid over a stretching sheet, *RheologicaActa*, 23, 213–215, 1984.

[5] M. Subhas Abel, A. Joshi and R.A. Santh, Heat transfer in MHD visco-elastic flow over a stretching surface, *Zeitschriftf¨urAngewandteMathematikundPhysik*, 81, 691– 698, 2001.

[6] T. Hymavathi, Numerical study of flow of Walter's liquid Bover an exponentially stretching sheet, *International Journal of Scientific and Research Publications*, 3, 1-7, 2013.

[7] Sujitkumarkhan, Boundary layer visco-elastic fluid flow over an exponentially stretching sheet, *International Journal of Applied Mechanics and Engineering*, 11, 1321–335, 2006.

[8] T. Hayat and M. Sajid, Analytic solution for axisymmetric flow and heat transfer of a second grade fluid past a stretching sheet, *International Journal of Heat andMass Transfer*, 50, 75–84, 2007.

[9] S. J. Liao, *The proposed homotopy analysis technique for the solution of non-linear problems [Ph.D. thesis]*, Shanghai Jiao Tong University, 1992.

[10] S. J. Liao, An analytic solution of unsteady boundary-layer flows caused by an impulsively stretching plate, *Communicationsin Nonlinear Science and Numerical Simulation*, 11, 326–339, 2006.