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A TWO SPECIES MONAD AMMENSALISM -GLOBAL STABILITY ANALYSIS

K.V.L.N.ACHARYULU

Faculty of Mathematics
Bapatla Engineering College
Bapatla-522101
Email:kvlna@yahoo.com

ABSTRACT

The present paper intends to test the global stability of a two species Monad Ammensalism which is instituted by Liapunov’s stability criteria. It is extracted by constructing a suitable Liapunov’s function for appraising the global stability of the model in the case of normal steady state.

AMS Classification: 92 D 25, 92 D 40

Key words: Equilibrium states, Stability, Liapunov’s function for global stability

1) Introduction:

K.V.L.N.Acharyulu and and N.Ch.Pattabhi Ramacharyulu [1-5] analyzed the Local stability of an Ammensal- enemy eco-system on the quasi-linear basic balancing equations. Local stability analysis for an Ammensal- enemy eco-system with various resources in different cases has been also fulfilled in the author’s earlier work. Several authors like Lotka[7], Kapur[6] etc. utilized this method in various situations for global stability. The present investigation is mainly focused on the establishment of the global stability of the co-existent equilibrium state of a two species Monad Ammensalism with limited resources by employing a property constructed by Liapunov’s function with Liapunov’s criteria for global stability

2) Basic concepts:

Consider an autonomous system $\frac{dx}{dt} = F_1(x, y)$ and $\frac{dy}{dt} = F_2(x, y)$ (1)

Assume that this system has an isolated initial point taken as (0, 0). Consider a function E(x,y) possessing continuous partial derivatives along with the path of (1).This path is represented by C= [(x (t), y (t)] in the parametric form. E(x,y) can be regarded as a function of ‘t’ along C with rate of change

If the total energy of physical system has a local minimum at a certain equilibrium point then the point is said to be stable .Liapunov generalized this principle by constructing a function $E(N_1, N_2)$ whose rate of change is given by

$$\frac{\partial E}{\partial t} = \frac{\partial E}{\partial N_1}.\frac{\partial N_1}{\partial t} + \frac{\partial E}{\partial N_2}.\frac{\partial N_2}{\partial t} = \frac{\partial E}{\partial N_1}F_1 + \frac{\partial E}{\partial N_2}F_2 \qquad (2)$$

corresponding to the system.

(ii)Theorem (A): If there exists a Liapunov's function E (x,y) for the system (1), then the critical point (0,0) is stable. Further, if this function has additional property that the function (2) is negative definite, then the critical point (0, 0) is asymptotically stable. (3)

The following theorem provides to ascertain the definiteness of a Liapunov's function.

(iii)Theorem (B): The function $E(x,y) = ax^2+bxy+cy^2$ is positive definite if $a>0$ and $b^2-4ac<0$ and negative definite if $a<0$, $b^2-4ac<0$. (4)

Notations adopted

N_1 and N2 are the populations of the Ammensal and enemy species with natural growth rates a_1 and a_2 respectively.

a_{11} = The rate of decrease of the Ammensal due to insufficient food.

a_{12} = The rate of increase of the Ammensal due to inhibition by the enemy.

a_{22} = The rate of decrease of the enemy due to insufficient food.

$K_i = a_i/a_{ii}$ are the carrying capacities of N_i, i = 1, 2

α ,β = Monad type parameters.

The state variables N_1 and N_2 as well as the model parameters a_1, a_2, a_{11}, a_{22}, K_1, and K_2 are assumed to be non-negative constants.

3) Construction of Basic Equations of the model:

The model equations for a two species Ammensal interaction with a monod type-variable coefficient of Ammensalism with limited resources is given by the following system of non-linear ordinary differential equations.

(i) Equation for the growth rate of Ammensal Species (S_1):

$$\frac{dN_1}{dt} = N_1\left[a_{11}\left(K_1 - N_1\right) - F\left(N_2\right)\right] \qquad (5)$$

In equation (5) the function $F(N_2)$ is the characteristic of the Ammensalism N_1 with respect to the enemy N_2 with the properties:

$F(N_1)$ is bounded and $F(N_1) \to$ a constant, as $N_1 \to \infty$.

The Ammensal characteristic model considered is a two parameter model of the monad type :

$$F(N_2) = \frac{\alpha N_2}{\beta + N_2} \qquad (6)$$

Here $\alpha = F(\infty) > 0$ is a parameter characteristic of Ammensalism. Further $\beta(\neq 0)$ is another parameter signifying the strength of Ammensalism: $\beta < 0$ strong Ammensalism, $\beta > 0$ weak Ammensalism and $\beta = 0$, the interaction would be neutral.

(ii) Equation for the growth rate of enemy species (S_2):

$$\frac{dN_2}{dt} = a_2 N_2\left[K_2 - N_2\right] \qquad (7)$$

4) Equilibrium Points and Equilibrium states:

The system has the following four equilibrium states E_1-E_4 resulting from

$$\frac{dN_1}{dt} = 0;\ \frac{dN_2}{dt} = 0 \qquad (8)$$

E_1:	Fully washed out state	:	$\overline{N_1} = 0;\ \overline{N_2} = 0$	(9)
E_2:	The state in which only the enemy survives and the Ammensal is washed out	:	$\overline{N_1} = 0;\ \overline{N_2} = K_2$	(10)
E_3:	The state in which only the Ammensal survives and the enemy is washed out	:	$\overline{N_1} = K_1;\ \overline{N_2} = 0$	(11)

E_4: Co-existent state : $\overline{N_1} = \frac{1}{a_{11}}\left[K_1 a_{11} - \frac{\alpha K_2}{\beta + K_2}\right]$; $\overline{N_2} = K_2$

(Both Ammensal and enemy survive) (12)

5) Global stability of the model by Liapunov's function :

The linearized basic equations are

$$\frac{dU_1}{dt} = -a_{11}\overline{N_1}U_1 - \frac{\alpha}{\beta+\overline{N_2}}\left(1 - \frac{\overline{N_2}}{\beta+\overline{N_2}}\right)\overline{N_1}U_2 \quad (13)$$

$$\frac{dU_2}{dt} = -a_{22}\overline{N_2}U_2 \quad (14)$$

The characteristic equation is

$$\left(\lambda + a_{11}\overline{N_1}\right)\left(\lambda + a_{22}\overline{N_2}\right) = 0 \quad (15)$$

Equation (15) is of the form $\lambda^2 + p\lambda + q = 0$

where

$$p = a_{11}\overline{N_1} + a_{22}\overline{N_2} > 0 \quad (16)$$

$$q = a_{11}a_{22}\overline{N_1}\,\overline{N_2} > 0 \quad (17)$$

$$\left(\because \left(a_1 + a_2 - \frac{\alpha K_2}{\beta+K_2}\right)^2 > 4a_2\left(a_1 - \frac{\alpha K_2}{\beta+K_2}\right)\right)$$

Therefore the conditions for Liapunov's function are satisfied

Now we define

$$E(U_1, U_2) = \tfrac{1}{2}\left(aU_1^2 + 2b\,U_1U_2 + cU_2^2\right) \quad (18)$$

where $a = \dfrac{\left(a_{22}\overline{N_2}\right)^2 + a_{11}a_{22}\overline{N_1}\,\overline{N_2}}{D}$ (19)

$$b = \frac{\frac{\alpha}{\beta+\overline{N_2}}\left(\frac{\overline{N_2}}{\beta+\overline{N_2}} - 1\right)a_{22}\overline{N_1}\,\overline{N_2}}{D} \quad (20)$$

$$c = \frac{\left(a_{11}\overline{N_1}\right)^2 + \left[\frac{\alpha}{\beta+\overline{N_2}}\left(\frac{\overline{N_2}}{\beta+\overline{N_2}} - 1\right)\overline{N_1}\right]^2 + a_{11}a_{22}\overline{N_1}\,\overline{N_2}}{D} \quad (21)$$

$$D = pq = \left(a_{11}\overline{N_1} + a_{22}\overline{N_2}\right)\left(a_{11}a_{22}\overline{N_1}\,\overline{N_2}\right) \quad (22)$$

From equations (16) and (17) it is clear that D > 0 and a > 0

Also

$$D^2(ac - b^2) = D^2\left\{\left[\frac{(a_{22}\overline{N_2})^2 + a_{11}a_{22}\overline{N_1}\,\overline{N_2}}{D}\right]\left[\frac{(a_{11}\overline{N_1})^2 + \left(\frac{\alpha}{\beta+\overline{N_2}}\left(\frac{\overline{N_2}}{\beta+\overline{N_2}} - 1\right)\overline{N_1}\right)^2 + a_{11}a_{22}\overline{N_1}\,\overline{N_2}}{D}\right] - \left[\frac{\frac{\alpha}{\beta+\overline{N_2}}\left(\frac{\overline{N_2}}{\beta+\overline{N_2}} - 1\right)a_{22}\overline{N_1}\,\overline{N_2}}{D}\right]^2\right\} \quad (23)$$

$$= \left(a_{22}^{2}\overline{N_2}^{2} + a_{11}a_{22}\overline{N_1}\,\overline{N_2}\right)\left(a_{11}^{2}\overline{N_1}^{2} + \frac{\alpha^2}{\left(\beta+\overline{N_2}\right)^2}\frac{\beta^2}{\left(\beta+\overline{N_2}\right)^2}\overline{N_1}^{2} + a_{11}a_{22}\overline{N_1}\,\overline{N_2}\right)$$

$$- \left[\frac{\alpha^2}{\left(\beta+\overline{N_2}\right)^2}\frac{\beta^2}{\left(\beta+\overline{N_2}\right)^2}a_{22}^{2}\overline{N_1}^{2}\overline{N_2}^{2}\right] \qquad (24)$$

$$D^2(ac-b^2) = a_{22}^2 a_{11}^2\overline{N_1}^2\overline{N_2}^2 + a_{22}^3 a_{11}\overline{N_1}\,\overline{N_2}^3 + a_{22}a_{11}^3\overline{N_1}^3\,\overline{N_2}$$

$$+ a_{11}a_{22}\frac{\alpha^2\beta^2}{\left(\beta+\overline{N_2}\right)^2}\overline{N_2}\,\overline{N_1}^3 + a_{11}^2a_{22}^2\overline{N_1}^2\overline{N_2}^2 \qquad (25)$$

$\Rightarrow D^2(ac - b^2) > 0$

$\Rightarrow ac - b^2 > 0$ i.e., $b^2 - ac < 0$ (26)

Therefore the function E (U_1, U_2) at (18) is positive definite.

Further $$\frac{\partial E}{\partial U_1}\frac{dU_1}{dt} + \frac{\partial E}{\partial U_2}\frac{dU_2}{dt} = \left(aU_1 + bU_2\right)\left(-a_{11}\overline{N_1}U_1 - \frac{\alpha}{\beta+\overline{N_2}}\left(1-\frac{\overline{N_2}}{\beta+\overline{N_2}}\right)\overline{N_1}U_2\right)$$

$$+ \left(bU_1 + cU_2\right)\left(-a_{22}\overline{N_2}U_2\right) \qquad (27)$$

By substituting values of a, b and c from equations (19), (20) and (21) in (27) we get

$$\frac{\partial E}{\partial U_1}\frac{dU_1}{dt} + \frac{\partial E}{\partial U_2}\frac{dU_2}{dt} = -\left\{\left(\frac{\left(a_{22}\overline{N_2}\right)^2 + a_{11}a_{22}\overline{N_1}\,\overline{N_2}}{D}\right)a_{11}\overline{N_1}\right\}U_1^2$$

$$+\left\{\left(\frac{(a_{22}\overline{N_2})^2 + a_{11}a_{22}\overline{N_1}\,\overline{N_2}}{D}\right)\frac{-\alpha\beta}{(\beta+\overline{N_2})^2}\overline{N_1} - \left(\frac{\frac{\alpha}{\beta+\overline{N_2}}\left(\frac{K_2}{\beta+\overline{N_2}}-1\right)a_{22}\overline{N_1}\,\overline{N_2}}{D}\right)\left(a_{11}\overline{N_1} + a_{22}\overline{N_2}\right)\right\}U_1U_2$$

$$+\left\{\left(\frac{\frac{\alpha}{\beta+\overline{N_2}}\left(1-\frac{\overline{N_2}}{\beta+\overline{N_2}}\right)a_{22}\overline{N_1}\,\overline{N_2}}{D}\right)\frac{\alpha\beta}{(\beta+\overline{N_2})^2}\overline{N_1} - \left(\frac{(a_{11}\overline{N_1})^2 + \left(\frac{\alpha}{\beta+\overline{N_2}}\left(\frac{\overline{N_2}}{\beta+\overline{N_2}}-1\right)\overline{N_1}\right)^2 + a_{11}a_{22}\overline{N_1}\,\overline{N_2}}{D}\right)a_{22}\overline{N_2}\right\}u_2^2 \qquad (28)$$

$$= -\left\{\left(\frac{a_{22}^2\overline{N_2}^2 + a_{11}a_{22}\overline{N_1}\,\overline{N_2}}{D}\right)a_{11}\overline{N_1}\right\}U_1^2$$

}

$$+\left\{\left(\frac{a_{22}^2\overline{N_2}^2 + a_{11}a_{22}\overline{N_1}\,\overline{N_2}}{D}\right)\frac{-\alpha\beta}{(\beta+\overline{N_2})^2}\overline{N_1} + \left(\frac{\frac{\alpha\beta}{(\beta+\overline{N_2})^2}a_{22}\overline{N_1}\,\overline{N_2}}{D}\right)\left(a_{11}\overline{N_1} + a_{22}\overline{N_2}\right)\right\}U_1U_2$$

$$+\left\{\left(\frac{\frac{\alpha\beta}{(\beta+\overline{N_2})^2}a_{22}\overline{N_1}\,\overline{N_2}}{D}\right)\frac{\alpha\beta}{(\beta+\overline{N_2})^2}\overline{N_1}-\left(\frac{a_{11}^2\overline{N_1}^2+\frac{\alpha^2\beta^2}{(\beta+\overline{N_2})^4}\overline{N_1}^2+a_{11}a_{22}\overline{N_1}\,\overline{N_2}}{D}\right)a_{22}\overline{N_2}\right\}U_2^2$$

$$=-\left\{\left(\frac{a_{11}a_{22}^2\overline{N_1}\,\overline{N_2}^2+a_{11}^2a_{22}\overline{N_1}^2\,\overline{N_2}}{D}\right)\right\}U_1^2$$

$$+\left\{\left(\frac{-a_{22}^2\overline{N_1}\,\overline{N_2}^2\alpha\beta-a_{11}a_{22}\overline{N_1}^2\,\overline{N_2}\alpha\beta}{\left(\beta+\overline{N_2}\right)^2D}\right)+\left(\frac{a_{11}a_{22}\overline{N_1}^2\,\overline{N_2}\alpha\beta+a_{22}^2\overline{N_1}\,\overline{N_2}^2\alpha\beta}{\left(\beta+\overline{N_2}\right)^2D}\right)\right\}U_1U_2$$

$$+\left\{\left(\frac{a_{22}\overline{N_1}^2\,\overline{N_2}\alpha^2\beta^2}{(\beta+\overline{N_2})^4D}\right)-\left(\frac{a_{11}^2a_{22}\overline{N_1}^2\,\overline{N_2}+\frac{\alpha^2\beta^2}{(\beta+\overline{N_2})^4}+a_{22}\overline{N_1}^2\,\overline{N_2}+a_{11}a_{22}^2\overline{N_1}\,\overline{N_2}^2}{D}\right)\right\}U_2^2$$

$$=-\left(\frac{a_{11}a_{22}\overline{N_1}\,\overline{N_2}(a_{11}\overline{N_1}+a_{22}\overline{N_2})}{D}\right)U_1^2$$

$$+\left(\frac{-a_{22}^2\overline{N_1}\,\overline{N_2}^2\alpha\beta-a_{11}a_{22}\overline{N_1}^2\,\overline{N_2}\alpha\beta+a_{11}a_{22}\overline{N_1}^2\,\overline{N_2}\alpha\beta+a_{22}^2\overline{N_1}\,\overline{N_2}^2\alpha\beta}{\left(\beta+\overline{N_2}\right)^2D}\right)U_1U_2$$

$$+\left(\frac{a_{22}\overline{N_1}^2\,\overline{N_2}\alpha^2\beta^2}{(\beta+\overline{N_2})^4D}-\frac{a_{11}^2a_{22}\overline{N_1}^2\,\overline{N_2}}{D}-\frac{\alpha^2\beta^2a_{22}\overline{N_1}^2\,\overline{N_2}}{(\beta+\overline{N_2})^4D}-\frac{a_{11}a_{22}^2\overline{N_1}\,\overline{N_2}^2}{D}\right)U_2^2$$

$$=-\left(\frac{a_{11}a_{22}\overline{N_1}\,\overline{N_2}(a_{11}\overline{N_1}+a_{22}\overline{N_2})}{D}\right)U_1^2+\left(\frac{-a_{11}^2a_{22}\overline{N_1}\,\overline{N_2}-a_{11}a_{22}^2\overline{N_1}\,\overline{N_2}^2)}{D}\right)U_2^2$$

$$=-\left(\frac{a_{11}a_{22}\overline{N_1}\,\overline{N_2}\left(a_{11}\overline{N_1}+a_{22}\overline{N_2}\right)}{D}\right)U_1^2-\left(\frac{a_{11}a_{22}\overline{N_1}\,\overline{N_2}\left(a_{11}\overline{N_1}+a_{22}\overline{N_2}\right)}{D}\right)U_2^2$$

$$=-\frac{D}{D}U_1^2-\frac{D}{D}U_2^2 \qquad = -(U_1^2+U_2^2) \tag{29}$$

$$\therefore \frac{\partial E}{\partial u_1}\frac{dU_1}{dt}+\frac{\partial E}{\partial u_2}\frac{dU_2}{dt}=-(U_1^2+U_2^2) \tag{30}$$

which is clearly negative definite.

So E (U_1,U_2) is a Liapunov's function for the linear system.
Next we prove that E (U_1, U_2) is also a Liapunov's function for the non-Linear system also.
If f_1 and $f_2$are two functions in N_1 and N_2 defined by

$$f_1(N_1, N_2) = N_1\left[a_1-a_{11}N_1-\frac{\alpha N_2}{\beta+N_2}\right] \tag{31}$$

$$f_2(N_1, N_2) = N_2\left[a_2-a_{22}N_2\right] \tag{32}$$

we now have to show that $\frac{\partial E}{\partial U_1}f_1+\frac{\partial E}{\partial U_2}f_2$ is negative definite.

Putting $N_1 = \overline{N_1} + U_1$ and $N_2 = \overline{N_2} + U_2$ in (5) and (7) we get

$$\frac{du_1}{dt} = \left(\overline{N_1}+U_1\right)\left(\left(a_1-a_{11}(\overline{N_1}+U_1\right)-\frac{\alpha(\overline{N}_2+U_2)}{\beta+(\overline{N}_2+U_2)}\right)$$

$$=\left(\overline{N_1}+U_1\right)a_1-a_{11}(\overline{N}_1+U_1)^2-\frac{\alpha}{\beta+\overline{N_2}}\left[1+\frac{U_2}{\beta+N_2}\right]^{-1}(\overline{N}_2+U_2)(\overline{N}_1+U_1)$$

$$= a_1\overline{N_1}+a_1U_1-a_{11}(\overline{N_1}^2+2\overline{N_1}U_1+U_1^2)-\frac{\alpha}{\beta+\overline{N_2}}\left(1-\frac{U_2}{\beta+\overline{N_2}}\right)\left(\overline{N}_1\overline{N_2}+\overline{N}_1U_2+\overline{N_2}U_1+U_1U_2\right)$$

$$=-a_{11}\overline{N_1}U_1+\left(\frac{\alpha\overline{N_1}\,\overline{N_2}}{\left(\beta+\overline{N_2}\right)^2}-\frac{\alpha\overline{N_1}}{\beta+\overline{N_2}}\right)U_2-a_{11}U_1^2+\frac{\alpha\overline{N_1}U_2^2}{\left(\beta+\overline{N_2}\right)^2}+\left(\frac{\alpha\overline{N_2}}{\left(\beta+\overline{N_2}\right)^2}-\frac{\alpha}{\beta+\overline{N_2}}\right)U_1U_2+\frac{\alpha}{\left(\beta+\overline{N_2}\right)^2}U_1U_2^2$$

$$\Rightarrow f_1(U_1, U_2) = \frac{du_1}{dt}=-a_{11}\overline{N_1}U_1+\left(\frac{\alpha\overline{N_1}\,\overline{N_2}}{\left(\beta+\overline{N_2}\right)^2}-\frac{\alpha\overline{N_1}}{\beta+\overline{N_2}}\right)U_2+F(U_1,U_2) \quad (33)$$

where $F(U_1, U_2) = -a_{11}U_1^2+\frac{\alpha\overline{N_1}}{\beta+\overline{N_2}}U_2^2+\left(\frac{\alpha\overline{N_2}}{\left(\beta+\overline{N_2}\right)^2}-\frac{\alpha}{\beta+\overline{N_2}}\right)U_1U_2+\frac{\alpha}{\left(\beta+\overline{N_2}\right)^2}U_1U_2^2$

Also $\frac{du_2}{dt}=\left(\overline{N_2}+U_2\right)\left(a_2-a_{22}\left(\overline{N_2}+U_2\right)\right)$

$$\Rightarrow f_2(U_1,U_2)=\frac{dU_2}{dt}=-a_{22}\overline{N}_2U_2+G(U_1,U_2) \quad (34)$$

where $G(U_1,Uu_2) = -a_{22}U_2^2$

From (18) $\frac{\partial E}{du_1} = aU_1 + bU_2$ (35)

$\frac{\partial E}{du_2} = bU_1 + cU_2$ (36)

Now $\frac{\partial E}{dU_1}f_1+\frac{\partial E}{\partial U_2}f_2=\left(aU_1+bU_2\right)\left[-a_{11}\overline{N_1}U_1+\left(\frac{\alpha\overline{N_1}\,\overline{N_2}}{\left(\beta+\overline{N_2}\right)^2}-\frac{\alpha\overline{N_1}}{\beta+\overline{N_2}}\right)U_2+F(U_1,U_2)\right]$

$(b\,U_1 + cUu_2)[-a_{22}\overline{N_2}\,Uu_2 + G(U_1, U_2)]$ (37)

$$\frac{\partial E}{dU_1}f_1+\frac{\partial E}{\partial U_2}f_2=\left\{\left(aU_1+bU_2\right)\left[-a_{11}\overline{N_1}U_1+\frac{\alpha}{\beta+\overline{N_2}}\left(\frac{\overline{N_2}}{\beta+\overline{N_2}}-1\right)\overline{N_1}U_2\right]\right.$$

$$\left.+\left(bU_1+cU_2\right)\left(-a_{22}\overline{N_2}U_2\right)\right\} + [(aU_1 + bU_2)\,F(U_1, U_2) + (bU_1+cU_2)\,G(U_1, U_2)] \quad (38)$$

From (29)

$\frac{\partial E}{dU_1}f_1+\frac{\partial E}{\partial U_2}f_2 = -(U_1^2 + U_2^2) + (aU_1+bU_2)\,F(U_1, U_2) + (bU_1+cU_2)\,G(U_1,U_2)$ (39)

By introducing polar co-ordinates $U_1 = r\cos\theta$, $U_2 = r\sin\theta$ we can write the equation(39) as

$\frac{\partial E}{dU_1}f_1+\frac{\partial E}{\partial U_2}f_2 = -(r^2) + r\{[a\cos\theta + b\sin\theta]\,F(U_1, U_2) + [b\cos\theta + c\sin\theta]\,G(U_1, U_2)\}$ (40)

Let us denote largest of the numbers |a|, |b|, |c| by M.

Our assumptions imply that $|F(U_1, U_2)| < \frac{r}{6M}$ and $|G(U_1, U_2)| < \frac{r}{6M}$ for all sufficiently small $r > 0$.

So $\frac{\partial E}{dU_1}f_1+\frac{\partial E}{\partial U_2}f_2<-r^2+\frac{4Kr^2}{6M}=-\frac{r^2}{3}<0$ (41)

Thus the function $E(U_1,U_2)$ is positive definite with the condition that

$\frac{\partial E}{dU_1} f_1 + \frac{\partial E}{\partial U_2} f_2$ is negative definite

$\therefore$ The equilibrium state $\mathbf{E_4}$ is **"asymptotically stable"** .

Conclusion: The Global stability of a mathematical model of two species Monad Ammensalism with limited resources in the co-existent equilibrium state is explained and It is observed that the normal state is asymptotically stable.

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