ISSN 0976 – 1411

Available online at www.internationaleJournals.com

International eJournals

International eJournal of Mathematics and Engineering 88 (2010) 837 - 845

INTERNATIONAL eJOURNAL OF MATHEMATICS AND ENGINEERING

www.InternationaleJournals.com

MHD EFFECTS ON MOVING VERTICAL PLATE IN THE PRESENCE OF CHEMICAL REACTION AND THERMAL RADIATION

R.Muthucumaraswamy [a], S.Meenakshisundaram [b]

[a] Department of Applied Mathematics Sri Venkateswara College of Engineering Sriperumbudur 602 105, India, E-mail : msamy@svce.ac.in

[b] Research Scholar, Department of Computer Applications, Bharath University 173, Agaram Road, Selaiyur, Chennai 600 073, India.

Abstract

An exact solution of thermal radiation and chemical reaction on unsteady free convective flow over a moving vertical plate with mass transfer in the presence of magnetic field is studied. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. The temperature is raised linearly with time and concentration level near the plate are raised uniformly. The dimensionless governing equations are solved using the Laplace transform technique. The velocity, temperature and concentration are studied for different parameters like the magnetic field parameter, radiation parameter, chemical reaction parameter and time. It is observed that the velocity decreases with increasing magnetic field parameter or radiation parameter.

Key Words: gray, radiation, magnetic field, chemical reaction, vertical plate.
Mathematics Subject Classification: 76R10

1. Introduction

Magnetoconvection plays an important role in various industrial applications. Examples include magnetic control of molten iron flow in the steel industry, liquid metal cooling in nuclear reactors and magnetic suppression of molten semi-conducting materials. It is of importance in connection with many engineering problems, such as sustained plasma confinement for controlled thermonuclear fusion and electromagnetic casting of metals. The effects of transversely applied magnetic field, on the flow of an electrically conducting fluid past an impulsively started infinite isothermal vertical plate was studied by Soundalgekar *et al* [8]. MHD effects on impulsively started vertical infinite plate with variable temperature in the presence of transverse magnetic field were studied by Soundalgekar *et al* [9]. The dimensionless governing equations were solved using Laplace transform technique.

Radiative convective flows are encountered in countless industrial and environment processes e.g. heating and cooling chambers, fossil fuel combustion energy processes, evaporation from large open water reservoirs, astrophysical flows, solar power technology and space vehicle re-entry. Radiative heat and mass transfer play an important role in manufacturing industries for the design of reliable equipment. England and Emery[5] have studied the thermal radiation effects of a optically thin gray gas bounded by a stationary vertical plate. Soundalgekar and Takhar[7] have considered the radiative free convective flow of an optically thin gray-gas past a semi-infinite vertical plate. Radiation effect on mixed convection along a isothermal vertical plate were studied by Hossain and Takhar[6]. In all above studies, the stationary vertical plate is considered. Das *et al*[3] have analyzed radiation effects on flow past an impulsively started infinite isothermal vertical plate. The governing equations were solved by the Laplace transform technique.

Diffusion rates can be altered tremendously by chemical reactions. The Effect of a chemical reaction depend whether the reaction is homogeneous or heterogeneous. This depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems, the reaction is heterogeneous, if it takes place at an interface and homogeneous, if it takes place in solution. Chambre and Young [2] have analyzed a first order chemical reaction in the neighborhood of a horizontal plate. APelblat [1] studied analytical solution for mass with a chemical reaction of first order. Das *et al.* [4] have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. The dimensionless governing equations were solved by the usual Laplace-transform technique.

It is proposed to study first order chemical reaction and thermal radiation effects on flow past an impulsively started infinite vertical plate with variable temperature and uniform mass diffusion in the presence of transverse applied magnetic field. The governing equations are solved by the Laplace-transform technique. The effect of velocity, temperature and concentration for different magnetic field parameter, chemical reaction parameter, radiation parameter and time are studied graphically.

2. Mathematical Analysis

Thermal radiation effects on unsteady MHD flow past an impulsively started infinite vertical plate with variable temperature and uniform mass diffusion, in the presence of chemical reaction of first order. The x-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to the plate. Initially, the plate and fluid are at the same temperature and concentration. At time $t' > 0$, the plate is given an impulsive motion in the vertical direction against gravitational field with constant velocity u_0 in a fluid, in the presence of thermal radiation. At the same time, the plate temperature is raised linearly with time and the level of concentration near the plate are raised to C'_w. A transverse magnetic field of uniform strength B_0 is assumed to be applied normal to the plate. The induced magnetic field and viscous dissipation is assumed to be negligible. It is also assumed that there exists a homogeneous first order chemical reaction between the fluid and species concentration. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. Then by usual Boussinesqs' approximation, the unsteady flow is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u}{\partial y'^2} - \frac{\sigma B_0^2}{\rho}u \tag{1}$$

$$\rho\, C_p \frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} - K_l\, C' \tag{3}$$

In most cases of chemical reactions, the reaction rate depends on the concentration of the species itself. A reaction is said to be of the order n, if the reaction rate if proportional to the n^{th} power of the concentration. In particular, a reaction is said to be first order, if the rate of reaction is directly proportional to concentration itself .

with the following initial and boundary conditions:

$$\begin{aligned} t' \le 0: &\quad u = 0, \quad T = T_\infty, &&\quad C' = C'_\infty \quad for\ all \quad y \\ t' > 0: &\quad u = u_0, \quad T = T_\infty + (T_w - T_\infty)\, A\, t', &&\quad C' = C'_w \quad at \quad y = 0 \\ &\quad u = 0, \quad T \to T_\infty, &&\quad C' \to C'_\infty \quad as \quad y \to \infty \end{aligned} \qquad (4)$$

where $A = \dfrac{u_0^2}{\nu}$.

The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y} = -4a^* \,\sigma\left(T_\infty^4 - T^4\right) \qquad (5)$$

It is assume that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T^4 \cong 4T_\infty^3\, T - 3T_\infty^4 \qquad (6)$$

By using equations (5) and (6), equation (2) reduces to

$$\rho\, C_p \,\frac{\partial T}{\partial t'} = k\,\frac{\partial^2 T}{\partial y^2} + 16a^*\sigma T_\infty^3\,(T_\infty - T) \qquad (7)$$

On introducing the following non-dimensional quantities:

$$\begin{aligned} &U = \frac{u}{u_0}, t = \frac{t' u_0^2}{\nu}, Y = \frac{y u_0}{\nu}, \theta = \frac{T - T_\infty}{T_w - T_\infty}, \\ &Gr = \frac{g\beta\nu(T_w - T_\infty)}{u_0^3}, C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, Gm = \frac{\nu g\beta^*(C'_w - C'_\infty)}{u_0^3}, \\ &Pr = \frac{\mu C_p}{k}, Sc = \frac{\nu}{D}, M = \frac{\sigma B_0^2 \nu}{\rho u_0^2}, R = \frac{16a^*\nu^2\sigma T_\infty^3}{k u_0^2}, K = \frac{\nu K_l}{u_0^2} \end{aligned} \qquad (8)$$

in equations (1) to (4), leads to

$$\frac{\partial U}{\partial t} = Gr\,\theta + Gc\,C + \frac{\partial^2 U}{\partial Y^2} - M\,U \qquad (9)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\,\frac{\partial^2 \theta}{\partial Y^2} - \frac{R}{Pr}\,\theta \qquad (10)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\,\frac{\partial^2 C}{\partial Y^2} - K\,C \qquad (11)$$

The initial and boundary conditions in dimensionless form are as follows:

$$\begin{aligned} &U = 0, \quad \theta = 0, \quad C = 0 \quad for\ all \quad Y, t \le 0 \\ t > 0: \;&U = 1, \quad \theta = t, \quad C = 1 \quad at \quad Y = 0 \\ &U = 0, \quad \theta \to 0, \quad C \to 0 \quad as \quad Y \to \infty \end{aligned} \qquad (12)$$

All the physical variables are defined in the nomenclature. The solutions are obtained for hydromagnetic flow field in the presence of thermal radiation.

The equations (9) to (11), subject to the boundary conditions (12), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$U = \left(\frac{1}{2} + d(1+bt) + e\right)\left[\begin{array}{l}\exp(2\eta\sqrt{Mt})\, erfc(\eta + \sqrt{Mt}) \\ + \exp(-2\eta\sqrt{Mt}) erfc(\eta - \sqrt{Mt})\end{array}\right]$$

$$-\frac{db\eta\sqrt{t}}{\sqrt{M}}\left[\exp(-2\eta\sqrt{Mt})\, erfc(\eta - \sqrt{Mt}) - \exp(2\eta\sqrt{Mt})\, erfc(\eta + \sqrt{Mt})\right]$$

$$-d\exp(bt)\left[\begin{array}{l}\exp(2\eta\sqrt{(M+b)t})\, erfc(\eta + \sqrt{(M+b)t}) \\ + \exp(-2\eta\sqrt{(M+b)t})\, erfc(\eta - \sqrt{(M+b)t})\end{array}\right]$$

$$-e\exp(ct)\left[\begin{array}{l}\exp(2\eta\sqrt{(M+c)t})\, erfc(\eta + \sqrt{(M+c)t}) \\ + \exp(-2\eta\sqrt{(M+c)t})\, erfc(\eta - \sqrt{(M+c)t})\end{array}\right]$$

$$-d(1+bt)\left[\begin{array}{l}\exp(2\eta\sqrt{Rt})\, erfc(\eta\sqrt{Pr} + \sqrt{at}) \\ + \exp(-2\eta\sqrt{Rt})\, erfc(\eta\sqrt{Pr} - \sqrt{at})\end{array}\right]$$

$$+d\left[\begin{array}{l}\exp(2\eta\sqrt{ctSc}) erfc(\eta\sqrt{Sc} + \sqrt{ct}) \\ + \exp(-2\eta\sqrt{ctSc})\, erfc(\eta\sqrt{Sc} - \sqrt{ct})\end{array}\right]$$

$$+d\exp(bt)\left[\begin{array}{l}\exp(2\eta\sqrt{Pr(a+b)t})\, erfc(\eta\sqrt{Pr} + \sqrt{(a+b)t}) \\ + \exp(-2\eta\sqrt{Pr(a+b)t}) erfc(\eta\sqrt{Pr} - \sqrt{(a+b)t})\end{array}\right]$$

$$-e\left[\exp(2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc} + \sqrt{Kt}) + \exp(-2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc} - \sqrt{Kt})\right]$$

$$+\frac{db\eta\sqrt{t\,Pr}}{\sqrt{R}}\left[\exp(-2\eta\sqrt{Rt})\, erfc(\eta\sqrt{Pr} - \sqrt{at}) - \exp(2\eta\sqrt{Rt})\, erfc(\eta\sqrt{Pr} + \sqrt{at})\right]$$

$$+e\exp(ct)\left[\exp(2\eta\sqrt{Sc(K+c)t})\, erfc(\eta\sqrt{Sc} + \sqrt{(K+c)t})\right.$$

$$\left. + \exp(-2\eta\sqrt{Sc(K+c)t}) erfc(\eta\sqrt{Sc} - \sqrt{(K+c)t})\right] \tag{13}$$

$$\theta = \frac{t}{2}\left[\exp(2\eta\sqrt{Rt})\, erfc(\eta\sqrt{Pr} + \sqrt{at}) + \exp(-2\eta\sqrt{Rt})\, erfc(\eta\sqrt{Pr} - \sqrt{at})\right]$$

$$-\frac{\eta Pr\sqrt{t}}{2\sqrt{R}}\left[\exp(-2\eta\sqrt{Rt})\, erfc(\eta\sqrt{Pr} - \sqrt{at}) - \exp(2\eta\sqrt{Rt}) erfc(\eta\sqrt{Pr} + \sqrt{at})\right] \tag{14}$$

$$C = \frac{1}{2}\left[\exp(2\eta\sqrt{KtSc})\ erfc(\eta\sqrt{Sc} + \sqrt{Kt}) + \exp(-2\eta\sqrt{KtSc})\ erfc(\eta\sqrt{Sc} - \sqrt{Kt})\right] \qquad (15)$$

where, $a = \frac{R}{Pr}, b = \frac{M-R}{Pr-1}, c = \frac{M-KSc}{Sc-1}, d = \frac{Gr}{2b^2(1-Pr)}, e = \frac{Gc}{2c(1-Sc)}$ and $\eta = \frac{Y}{2\sqrt{t}}$.

3. Discussion of Results

The numerical values of the velocity, temperature and wall concentration are computed for different parameters like magnetic field parameter, radiation parameter, Schmidt number, chemical reaction parameter, time and $Pr = 0.71$. The purpose of the calculations given here is to study the effects of the parameters M, K, R, t and Sc upon the nature of the flow and transport.

The temperature profiles are calculated for different values of thermal radiation parameter($R = 5,10,15,20$) from Equation (14) and these are shown in Figure 1. for air (Pr =0.71) at time $t = 0.2$. The effect of thermal radiation parameter is important in temperature profiles. It is observed that the temperature increases with decreasing radiation parameter.

The effect of concentration for different time($t = 0.2,0.4,0.6,1), Sc = 0.6$ and $K = 2$ are shown in Figure 2. In this case, the concentration increases with increasing time t. Figure 3 demonstrates the effect of the concentration profiles for different values of the chemical reaction parameter ($K = 2,5,10), Sc = 0.6$ and time $t = 2$. It is observed that the concentration increases with decreasing chemical reaction parameter.

The effect of velocity profiles for different time ($t = 0.4,0.6,0.8,1), R = 10, K = M = Gr = Gc = 2, Pr = 0.71, Sc = 0.6$ are shown in Figure 4. In this case, the velocity increases gradually with increasing values of time t. Figure 5 illustrates the effect of the velocity for different values of the reaction parameter ($K = 2,10,25,50), M = 0.2, R = 10, Gr = 2 = Gc, Sc = 0.6, Pr = 0.71$ and $t = 2$. The trend shows that the velocity increases with decreasing chemical reaction parameter. The velocity profiles for different magnetic field parameter($M = 0.2,0.6,0,9), Gr = Gc = 2, R = 10, Sc = 0.6, Pr = 0.71$ and $t = 2$ are presented in Figure 6. It is clear that the velocity increases with decreasing magnetic field parameter.

4. Concluding Remarks

Theoretical solution of thermal radiation effects on flow past an impulsively started infinite vertical plate with variable temperature and uniform mass diffusion in the presence of transverse applied magnetic field is considered. It is also assumed that there exists a homogeneous first order chemical reaction between the fluid and species concentration. The dimensionless governing equations are solved by the usual Laplace-transform technique. The velocity, temperature and wall concentration are studied for different physical parameters are studied graphically. The conclusions of the study are as follows:

- The velocity decreases with increasing Radiation parameter or magnetic field parameter.
- The temperature increases with decreasing values of thermal radiation parameter.
- Velocity increases with decreasing values of the chemical reaction parameter but the trend is just reversed with respect to time t.

References

1. A.Apelblat, Mass transfer with a chemical reaction of the first order:Analytical solutions, The Chem. Engg. J. 19 (1980),19-37.

2. P.L.Chambre and J.D.Young, On the diffusion of a chemically reactive species in a laminar boundary layer flow, The Physics of Fluids 1(1958),48-54.

3. U.N.Das, R.K.Deka and V.M.Soundalgekar, Radiation effects on flow past an impulsively started vertical infinite plate, J.Theo.Mech. 1 (1996),111-115.

4. U.N.Das, R.K.Deka and V.M.Soundalgekar, Effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction, Forschung im Ingenieurwesen 60 (1994), 284-287.

5. W.G.England and A.F.Emery, Thermal radiation effects on the laminar free convection boundary layer of an absorbing gas, J. of Heat Transfer 91 (1969) 37-44.

6. M.A.Hossain and H.S.Takhar, Radiation effect on mixed convection along a vertical plate with uniform surface temperature, Heat and Mass Transfer 31 (1996), 243-248.

7. V.M.Soundalgekar and H.S.Takhar, Radiation effects on free convection flow past a semi-infinite vertical plate, Modeling, Measurement and Control B51 (1993), 31-40.

8. V.M.Soundalgekar, S.K.Gupta and N.S.Birajdar, Effects of Mass transfer and free convection currents on MHD Stokes problem for a vertical plate, Nuclear Engg. Des. 53 (1979), 339-346.

9. V.M.Soundalgekar, M.R.Patil and M.D.Jahagirdar, MHD Stokes problem for a vertical plate with variable temperature, Nuclear Engg. Des. 64 (1981), 39-42.

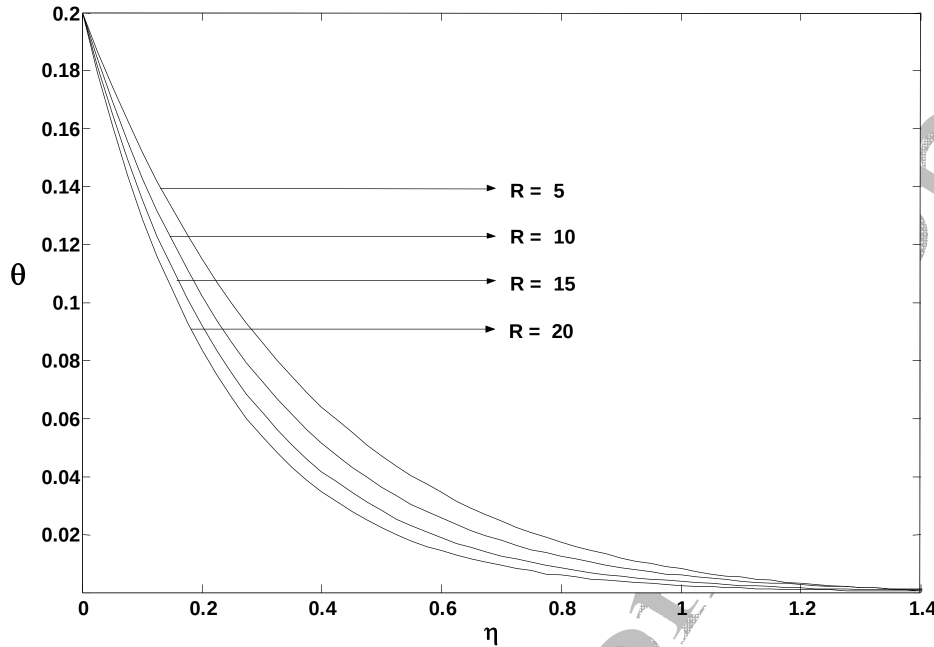

Figure 1. Temperature profiles for different values of R

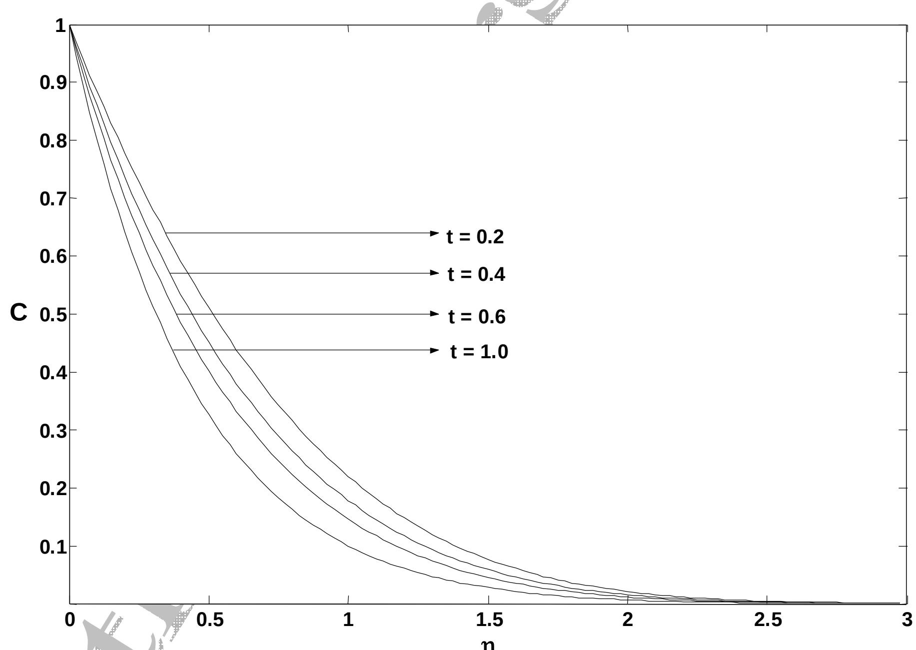

Figure 2. Concentration profiles for different values of t

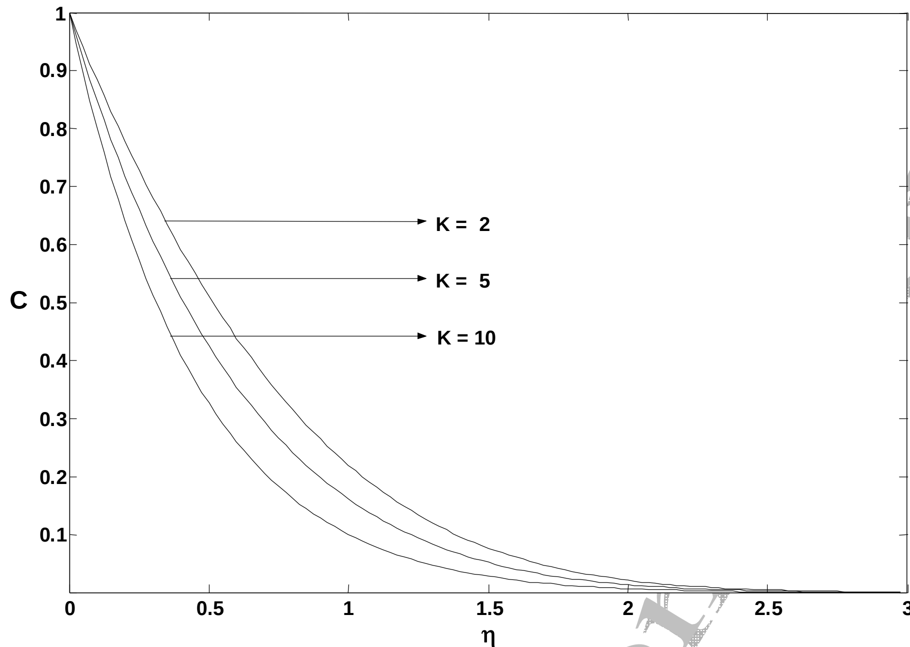

Figure 3. Concentration profiles for different values of K

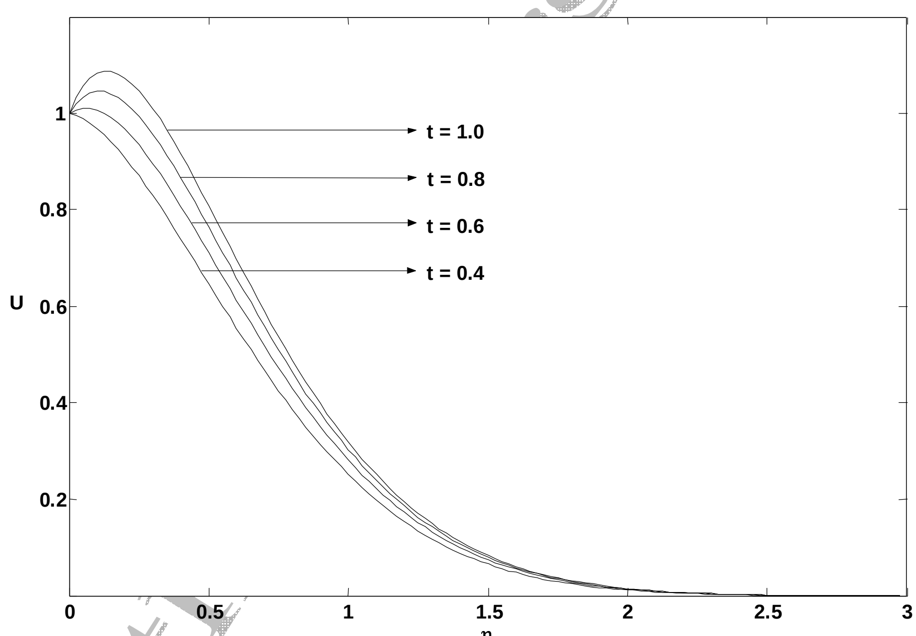

Figure 4. Velocity profiles for different values of t

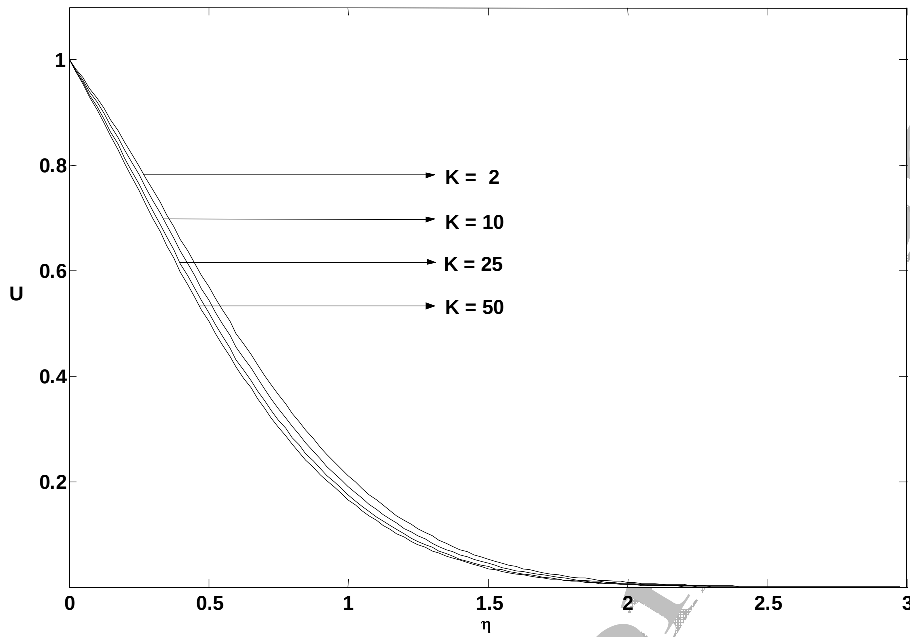

Figure 5. Velocity profiles for different values of K

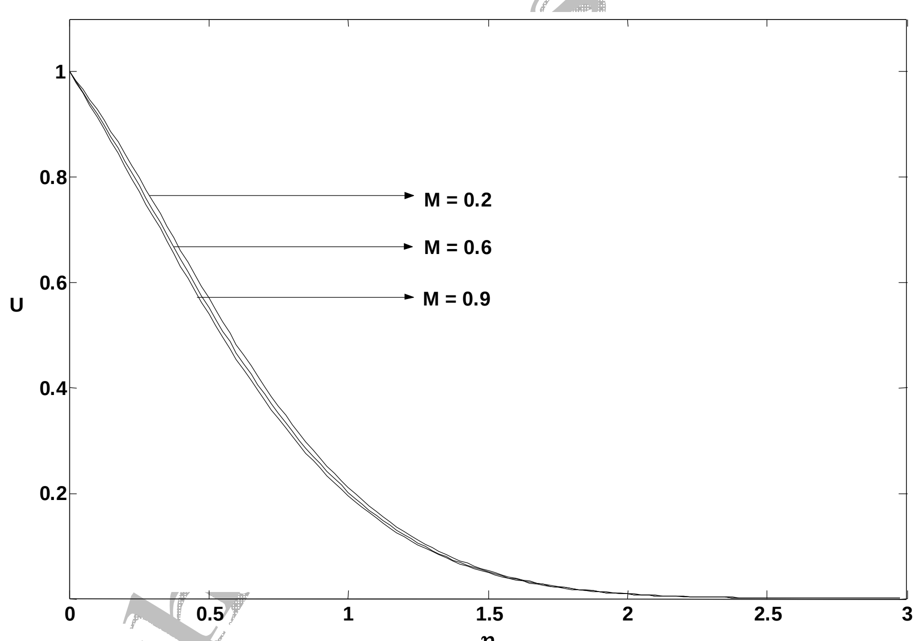

Figure 6. Velocity profiles for different values of M