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INVERSE COEFFICIENT CONDITIONS FOR ST(K) AND *SP (λ, ρ)*

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Abstract:
A normalized function f analytic in the open unit disc around the origin and non vanishing outside the origin can be expressed in the form $z/g(z)$ where $g(z)$ has Taylor coefficients b_n's. Coefficient conditions in terms of b_n's are derived for functions in the classes ST(K), and SP(λ,ρ) of univalent analytic functions.

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Introduction: Let A_1 be the class of functions f analytic in $U = \{ z\in \mathbf{C}: |z|<1\}$, and normalized by $f(0)=0$, $f'(0)=1$ where $\mathbf{C}$ is the set of complex numbers. An f in A_1 with $f(z)\neq 0$ in the punctured disc $U\backslash\{0\}$, may be expressed as $f(z) = \psi(g) = z / g(z)$ in U,

where $g(z) = 1+\sum_{n=1}^{\infty} b_n z^n$ in U. We call b_n's , the inverse coefficients of f.

Mitrinovic [1],Prawitz[2], Reade et.al [3] ,Silverman and Silvia[6] and Srinivas[7,8] studied these coefficients b_n's.

Mitrinovic [1] obtained estimates for the radius of univalence of certain rational functions. In particular, he found sufficient conditions for functions of the form

$$\frac{z}{1+b_1 z+b_2 z^2+\ldots+b_n z^n},$$

$b_n \neq 0$, to be univalent in the unit disk U.

Prawitz[2] determined the following necessary condition in terms of b_n's of $g(z)$ when

$f=\psi(g)$ is in $\mathbf{S}$, the subclass of A_1 of functions univalent in U:

$$\sum_{n=2}^{\infty}(n-1)|b_n|^{2} \leq 1. \qquad \ldots\ldots\ldots\ldots\ldots\ldots.. \quad (1)$$

Silverman and Silvia [6] found necessary conditions in terms of b_n's for $\psi(g)$ to be starlike of order α with negative Taylor coefficients . They found that for such $\psi(g)$

$$|b_n| \leq \frac{1-\alpha}{n+1-\alpha}, \quad n = 0,1,2,...$$

Reade et al [3] found that, if

$$\sum_{n=2}^{\infty}(n+1-\alpha) \leq \begin{cases} (1-\alpha)-(1-\alpha)|b_1|, 0 \leq \alpha \leq 1/2 \\ (1-\alpha)-\alpha|b_1|, \quad 1/2 \leq \alpha \leq 1. \end{cases} \qquad \ldots\ldots\ldots\ldots.(2)$$

then $\psi(g) \in S^*(\alpha) \equiv \left\{ f \in A_1 : \mathrm{Re}\frac{zf'(z)}{f(z)} > \alpha, z \in U \right\}$.

Let $B(\alpha) \equiv \left\{ f \in A_1 : \mathrm{Re}\frac{f(z)}{z}, z \in U \right\}$, *for* $0 \leq \alpha \leq 1$. Srinivas [7] found sufficient condition on $B(\alpha)$ as

i) *if* $\sum_{n=1}^{\infty}|b_n| \leq 1$, *then* $f \in B(\alpha)$ *for* $0 \leq \alpha \leq 1/2$

ii) if $\sum_{n=1}^{\infty}|b_n| \leq \frac{1-\alpha}{\alpha}$, *then* $f \in B(\alpha)$ *for* $1/2 \leq \alpha \leq 1$. …………(3)

Let K>0 and f be regular and locally univalent in U . Then f is said to be in the class $C(K)$ if and only if

$$\liminf_{|z|\to 1^-} \frac{\mathrm{Re}\left\{1+\frac{zf''(z)}{f'(z)}\right\}}{|zf'(z)|} \geq K.$$

This class was introduced by Wirths [9]. We define an Alexander's type of class related to this class $C(K)$ as

$$\mathrm{ST(K)} \equiv \{zf' : f \in C(K)\}.$$

Let $SP(\lambda,\rho)$ be the class of functions f in A_1 for which

$$\mathrm{Re}\left(\frac{e^{i\lambda}zf'(z)}{f(z)}\right) > \rho\cos\lambda, |\lambda| < \left(\frac{\pi}{2}\right)$$

for all z in U where $0 \leq \rho < 1$. An f in $SP(\lambda,\rho)$ is called λ-spiral of order ρ.

In this paper some necessary conditions on b_n*'s* for some particular form of functions f's in the classes *ST(K)* and $SP(\lambda,\rho)$ are derived.

Section1

First we derive a necessary condition for the class ST(K):

Theorem1. If $\psi(g) = \dfrac{z}{1+\sum_{n=1}^{\infty}|b_n| z^n} \in ST(K)$, $0 < K \le 1$, *and* b_n's are complex then

$$\sum_{n=2}^{\infty}(n-1)|b_n| \le 1-K \quad \text{....(4)}$$

Proof: For $f(z)=\psi(g) = z/g(z) = z/(1+\sum_{n=1}^{\infty}|b_n|z^n), z \in U,$ we have

$$\text{Re}\frac{zf'(z)}{f(z)|f(z)|} = \text{Re}\frac{(g(z)-zg'(z))|g(z)|}{g(z)|z|} > K$$

in U-{0}, by the Alexander type relation between the classes $C(K)$ and $ST(K)$ and the local minimum property due to Wirths [9] for the curvature κ (f ; z). Thus, for $z \in (0,1)$, we have

$$\text{Re}\left[1+\sum_{n=2}^{\infty}(1-n)|b_n|z^n\right] > Kz.$$

By letting z tend to 1^- along the positive reals , we obtain the required inequality (4).

<u>Remark</u>: For $\psi(g) = z/(1+\sum_{n=1}^{\infty}|b_n|z^n) \in \text{ST(K)}$, the inequality (4) is stronger than the inequality (1) of Prawitz [2], where 0<K<1 and b_n's are complex.

<u>Section2</u>

Next a necessary condition is derived for functions of particular form in the class *SP*(*λ,ρ*)**.**

Theorem2. If $\psi(g) = z/(1+\sum_{n=1}^{\infty}|b_n|z^n) \in \text{SP}(\lambda,\rho)$, b_n's are complex , λ is real with $|\lambda| < \pi/2, 0 \le \rho < 1$, then

$$\sum_{n=1}^{\infty}(n+1-\rho)|b_n| \le 1-\rho. \quad \text{............(5)}$$

Proof. For $f(z)=\psi(g) = z/(1+\sum_{n=1}^{\infty}|b_n|z^n) \in \text{SP}(\lambda,\rho)$, we have,

$$\text{Re}\frac{e^{i\lambda}zf'(z)}{f(z)} = \text{Re}\frac{e^{i\lambda}\left[1+\left\{\sum_{n=2}^{\infty}(1-n)|b_n|z^n\right\}\right]}{1+\sum_{n=1}^{\infty}|b_n| z^n} > \rho\cos\lambda$$

in U. Now letting z tend to 1^- along positive reals, the inequality (5) is obtained.

Remark. The inequality (5) is stronger than the inequality (1) of Prawitz [2] for functions $\psi(g) = z/(1+\sum_{n=1}^{\infty}|b_n|z^n) \in \text{SP}(\lambda,\rho)$, when b_n's are complex.

Taking λ=0 and ρ=α in Theorem2 the next result is obtained.

Corollary1 If $\psi(g) = z/(1+\sum_{n=1}^{\infty}|b_n|z^n) \in S^*(\alpha), 0 \le \alpha < 1$ and b_n's are complex , then

$$\sum_{n=1}^{\infty}(n-1+\alpha)|b_n| \le 1-\alpha \qquad \text{………………….(6)}$$

Remarks (i) Necessary condition (6) and sufficient condition (2) suggest that an analytic function $\psi(g) = z/(1+\sum_{n=1}^{\infty}|b_n|z^n)$, $z \in U, b_n$'s are complex , is in the class $S^*(\alpha)$, $1/2 \le \alpha < 1$, if and only if , the inequality (6) holds.

(ii) It can be verified, for functions $\psi(g) = z/(1+\sum_{n=1}^{\infty}|b_n|z^n)$ in $B(\alpha), 1/2 \le \alpha < 1, b_n$'s are complex, that

$$\sum_{n=1}^{\infty}|b_n| \le \frac{1-\alpha}{\alpha}. \qquad \text{………… (7)}$$

Thus, in view of (ii) of (3) a necessary and sufficient condition for the function $\psi(g) = z/(1+\sum_{n=1}^{\infty}|b_n|z^n)$, b_n's are complex to be in $B(\alpha)$, $1/2 \le \alpha < 1$, is that the condition (7) holds.

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