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THERMAL RADIATION AND MHD EFFECTS ON ISOTHERMAL VERTICAL OSCILLATING PLATE WITH UNIFORM MASS DIFFUSION

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Abstract

Thermal radiation effects on unsteady free convective flow of a viscous incompressible flow past an infinite isothermal vertical oscillating plate with uniform mass diffusion in the presence magnetic field has been considered. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. The plate temperature is raised to T_w and the concentration level near the plate is also raised to C'_w. An exact solution to the dimensionless governing equations has been obtained by the Laplace transform method, when the plate is oscillating harmonically in its own plane. The effects of velocity, temperature and concentration are studied for different parameters like magnetic field parameter, phase angle, Schmidt number, thermal Grashof number, mass Grashof number and time. It is observed that the velocity increases with decreasing magnetic field parameter or radiation parameter. It is also observed that the velocity increases with decreasing phase angle ωt.

2000 Mathematics Subject classification : 80A20

Key words: radiation, isothermal, oscillating, vertical plate, magnetic field.

1. Introduction

Radiative heat and mass transfer play an important role in manufacturing industries for the design of fins, steel rolling, nuclear power plants, gas turbines and various propulsion device for aircraft, missiles, satellites and space vehicles are examples of such engineering applications. If the temperature of the surrounding fluid is rather high, radiation effects play an important role and this situation does exist in space technology. In such cases, one has to take into account the effect of thermal radiation and mass diffusion.

England and Emery[2] have studied the thermal radiation effects of a optically thin gray gas bounded by a stationary vertical plate. Radiation effect on mixed convection along a isothermal vertical plate were studied by Hossain and Takhar[3]. The governing equations were solved analytically. Das {\em et al}[1] have analyzed radiation effects on flow past an impulsively started infinite isothermal vertical plate.

Magnetoconvection plays an important role in agriculture,petroleum industries, geophysics and in astrophysics. Important applications in the study of geological formations, in exploration and thermal recovery of oil,and in the assessment of aquifers,geothermal reservoirs and underground nuclear waste storage sites. MHD flow has application in metrology, solar physics and in motion of earths core. Also it has applications in the field of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics. The effects of transversely applied magnetic field, on the flow of an electrically conducting fluid past an impulsively started infinite isothermal vertical plate was studied by Soundalgekar *et al* [7]. MHD effects on impulsively started vertical infinite plate with variable temperature in the presence of transverse magnetic field were studied by Soundalgekar *et al* [8]. The dimensionless governing equations were solved using Laplace transform technique.

The flow of a viscous, incompressible fluid past an infinite isothermal vertical plate, oscillating in its own plane, was solved by Soundalgekar [4]. The effect on the flow past a vertical oscillating plate due to a combination of concentration and temperature differences was studied extensively by Soundalgekar and Akolkar [5]. The effect of mass transfer on the flow past an infinite vertical oscillating plate in the presence of constant heat flux has been studied by Soundalgekar et al. [6].

However the combined study of MHD and thermal radiation effects on infinite oscillating isothermal vertical plate with uniform mass diffusion is not studied in the literature. It is proposed to study the unsteady flow past infinite isothermal vertical oscillating plate, in the presence of magnetic field and thermal radiation. The dimensionless governing equations are tackled using the Laplace transform technique. The solutions are in terms of exponential and complementary error function.

2. Basic Equations and Analysis

Here the unsteady flow of a viscous incompressible fluid which is initially at rest and surrounds an infinite vertical plate with temperature T_∞ and concentration C'_∞ . Here, the x-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to the plate. Initially, it is assumed that the plate and the fluid are of the same temperature and concentration. At time $t' > 0$, the plate starts oscillating in its own plane with frequency ω' and the temperature of the plate is raised to T_w and the concentration level near the plate are also raised to C'_w. The plate is also subjected to a uniform magnetic field of strength B_0. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. It is assumed that the effect of viscous dissipation is negligible in the energy equation. Then by usual Boussinesq's approximation, the unsteady flow is governed by the following

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u \tag{1}$$

$$\rho\, C_p\, \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\, \frac{\partial^2 C'}{\partial y^2} \tag{3}$$

With the following initial and boundary conditions:

$$\begin{array}{llll} t' \le 0: & u = 0, & T = T_\infty, & C' = C'_\infty \quad for\ all \quad y \\ t' > 0: & u = u_0 cos\omega' t', & T = T_w, & C' = C'_w \quad at \quad y = 0 \\ & u = 0, & T \to T_\infty, & C' \to C'_\infty \quad as \quad y \to \infty \end{array} \tag{4}$$

The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y} = -4a^*\, \sigma\left(T_\infty^4 - T^4\right) \tag{5}$$

It is assume that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T^4 \cong 4T_\infty^3\, T - 3T_\infty^4 \tag{6}$$

By using equations (5) and (6), equation (2) reduces to

$$\rho\, C_p\, \frac{\partial T}{\partial t'} = k\, \frac{\partial^2 T}{\partial y^2} + 16a^* \sigma T_\infty^3\, (T_\infty - T) \tag{7}$$

On introducing the following non-dimensional quantities:

$$\begin{gathered} U = \frac{u}{u_0}, t = \frac{t' u_0^2}{\nu}, Y = \frac{y u_0}{\nu}, \theta = \frac{T - T_\infty}{T_w - T_\infty}, \\ Gr = \frac{g\beta\nu(T_w - T_\infty)}{u_0^3}, C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, Gc = \frac{\nu g \beta^* (C'_w - C'_\infty)}{u_0^3}, \omega = \frac{\omega' \nu}{u_0^2}, \\ R = \frac{16a^* \nu^2 \sigma T_\infty^3}{k u_0^2}, Pr = \frac{\mu C_p}{k}, Sc = \frac{\nu}{D}, M = \frac{\sigma B_0^2 \nu}{\rho u_0^2} \end{gathered} \tag{8}$$

in equations (1) to (4), leads to

$$\frac{\partial U}{\partial t} = Gr\, \theta + Gc\, C + \frac{\partial^2 U}{\partial Y^2} - M\, U \tag{9}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial Y^2} - \frac{R}{Pr}\theta \tag{10}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} \tag{11}$$

The initial and boundary conditions in non-dimensional form are

$$\begin{array}{llllll} & U = 0, & \theta = 0, & C = 0, & for\ all & Y, t \leq 0 \\ t > 0: & U = cos\omega t, & \theta = 1, & C = 1, & at & Y = 0 \\ & U = 0, & \theta \to 0, & C \to 0 & as & Y \to \infty \end{array} \tag{12}$$

All the physical variables are defined in the nomenclature. The solutions are obtained for hydrodynamic flow field in the presence of first order chemical reaction. The equations (9) to (11), subject to the boundary conditions (12), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$\theta = \frac{1}{2}\left[\exp(2\eta\sqrt{Rt})\ erfc(\eta\sqrt{Pr} + \sqrt{at}) + \exp(-2\eta\sqrt{Rt})\ erfc(\eta\sqrt{Pr} - \sqrt{at})\right] \tag{13}$$

$$C = \frac{1}{2}\left[\exp(2\eta\sqrt{KtSc})\ erfc(\eta\sqrt{Sc} + \sqrt{Kt}) + \exp(-2\eta\sqrt{KtSc})\ erfc(\eta\sqrt{Sc} - \sqrt{Kt})\right] \tag{14}$$

$$U = \frac{\exp(i\omega t)}{4}\left[\exp(2\eta\sqrt{(M+i\omega)t})erfc(\eta + \sqrt{(M+i\omega)t}) + \exp(-2\eta\sqrt{(M+i\omega)t})erfc(\eta - \sqrt{(M+i\omega)t})\right]$$

$$+ \frac{\exp(-i\omega t)}{4}\left[\exp(2\eta\sqrt{(M-i\omega)t})\ erfc(\eta + \sqrt{(M-i\omega)t}) + \exp(-2\eta\sqrt{(M-i\omega)t})erfc(\eta - \sqrt{(M-i\omega)t})\right]$$

$$+ (d+e)\left[\exp\left(2\eta\sqrt{Mt}\right).erfc\left(\eta + \sqrt{Mt}\right) + \exp\left(-2\eta\sqrt{Mt}\right).erfc\left(\eta - \sqrt{Mt}\right)\right]$$

$$- d\exp(bt)\left[\exp\left(-2\eta\sqrt{(M+b)t}\right).erfc\left(\eta - \sqrt{(M+b)t}\right)\right.$$

$$\left. + \exp\left(2\eta\sqrt{(M+b)t}\right).erfc\left(\eta + \sqrt{(M+b)t}\right)\right]$$

$$- e\exp(ct)\left[\exp\left(-2\eta\sqrt{(M+c)t}\right).erfc\left(\eta - \sqrt{(M+c)t}\right)\right.$$

$$\left. + \exp\left(2\eta\sqrt{(M+c)t}\right).erfc\left(\eta + \sqrt{(M+c)t}\right)\right]$$

$$- d\left[\exp\left(2\eta\sqrt{Rt}\right).erfc\left(\eta\sqrt{pr} + \sqrt{a\,t}\right) + \exp\left(-2\eta\sqrt{Rt}\right).erfc\left(\eta\sqrt{pr} - \sqrt{a\,t}\right)\right]$$

$$-d\ \exp(bt)\left[\exp\left(-2\eta\sqrt{\Pr(a+b)t}\right)erfc\left(\eta\sqrt{\Pr}-\sqrt{(a+b)t}\right)\right.$$

$$\left.+\exp\left(2\eta\sqrt{\Pr(a+b)t}\right)erfc\left(\eta\sqrt{\Pr}+\sqrt{(a+b)t}\right)\right]$$

$$+\ e\ \exp(ct)\left[\exp\left(-2\eta\sqrt{ct\,Sc}\right)erfc\left(\eta\sqrt{Sc}-\sqrt{ct}\right)\right.$$

$$\left.+\exp\left(2\eta\sqrt{ctSc}\right)erfc\left(\eta\sqrt{Sc}+\sqrt{ct}\right)\right] \qquad (15)$$

where, $a=\dfrac{R}{Pr}, b=\dfrac{M-R}{Pr-1}, c=\dfrac{M}{Sc-1}, d=\dfrac{Gr}{2b(1-Pr)}, e=\dfrac{Gc}{2c(1-Sc)}, \eta=\dfrac{Y}{2\sqrt{t}}$.

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3 Discussion of Results

In order to get a physical view of the problem the numerical values of the velocity, tempaertaure and concentration fields are studied for different values of the phase angle, radiation parameter, magnetic field parameter, Schmidt number and time. The purpose of the calculations given here is to assess the effects of the parameters $\omega t, M, R, Gr, Gc$ and Sc upon the nature of the flow and transport. The solutions are in terms of exponential and complementary error function.

The temperature profiles are calculated for different values of thermal radiation parameter(R=2,5,7,10) from Equation (13) and these are shown in Figure 1. for air (Pr=0.71) at t=0.4. The effect of thermal radiation parameter is important in temperature profiles. It is observed that the temperature increases with decreasing radiation parameter.

Figure 2 represents the effect of concentration profiles at time t=1 for different Schmidt number(Sc=0.16,0.3,0.6,2.01). The effect of concentration is important in concentration field. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the wall concentration increases with decreasing values of the Schmidt number.

The velocity profiles for different phase angles ($\omega t = 0, \pi/4, \pi/3, \pi/2$), M=5, Gr=2, Gc=2, R=10, Sc=2.01, Pr=0.71 andt=1 are shown in figure 3. It is observed that the velocity increases with decreasing phase angle ωt. The effect of velocity for different values of the radiation parameter (R=5, 8, 15), $\omega t = \pi/4$, M=3, Gr=5, Gc=2, Pr=0.71, Sc=2.01 and t=0.3 are shown in figure 4. The trend shows that the velocity increases with decreasing radiation parameter. This shows that the velocity decreases in the presence of high thermal radiation.

The velocity profiles for different magnetic field parameter(M=2,5,10), $\omega t = \pi/6$, Gr=Gc=2, R=12, Sc=2.01, Pr=0.71 and t=0.2 are presented in Figure 5. It is clear that the velocity increases with decreasing magnetic field parameter. The effect of velocity profiles for different time (t=0.1, 0.2, 0.3), $\omega t = \pi/4$, M=2, Gr=5, Gc=2, R=3, Sc=2.01 and Pr=0.71

are are shown in Figure 6. In this case, the velocity increases gradually with respect to time t. The effect of velocity for different values of thermal Grashof number(Gr=2,5), Mass Grashof number(Gc=2,5), $\omega t = \pi/6$, R=5, M=2, Sc=2.01, Pr=0.71 and t=0.2 are presented in figure 7. It is observed that the velocity increases with increasing thermal Grashof number or mass Grashof number.

4. Conclusion

The study of MHD and thermal radiation effects on flow past an oscillating infinite isothermal vertical plate with uniform mass diffusion. The dimensionless equations are solved using Laplace transform technique. The effect of velocity, temperature and concentration for different parameters like $\omega t, M, R, Gr, Gc, Sc$ and t are studied. The study concludes the following results:

(i) The velocity increases with decreasing phase angle ωt, magnetic field parameter M and radiation parameter R. The trend is just reversed with respect to time t.

(ii) The temperature decreases due to high thermal radiation.

(iii) It is observed that the concentration increases with decreasing Schmidt number.

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Figure 1. Temperature profiles for different values of R

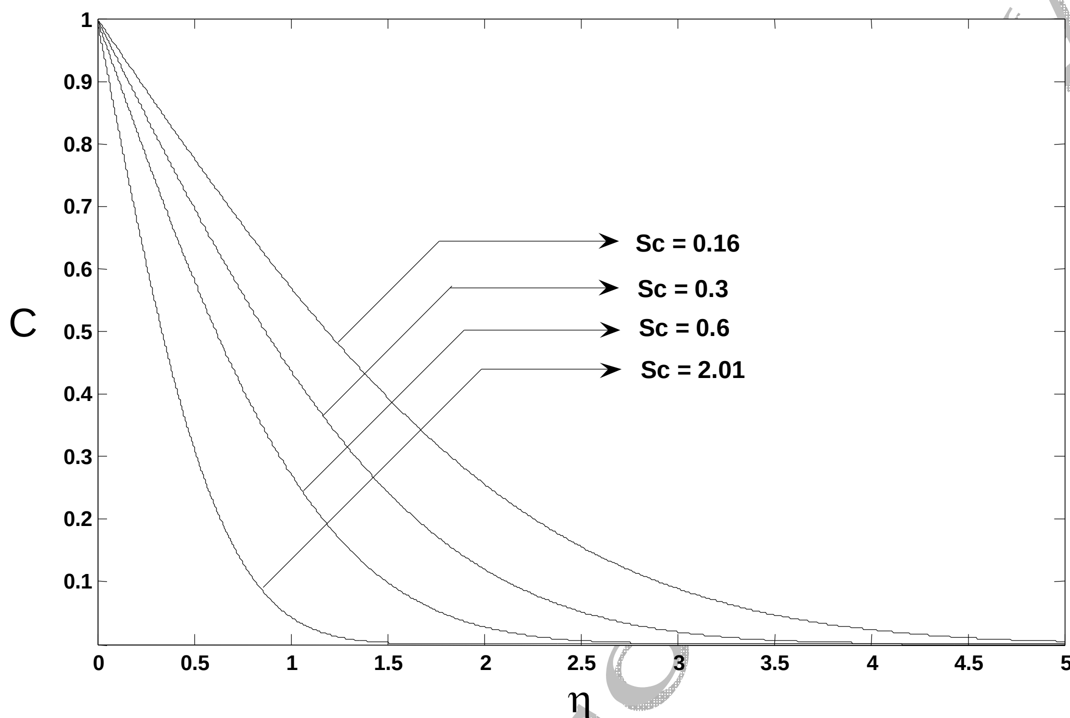

Figure 2. Concentration profiles for different values of Sc

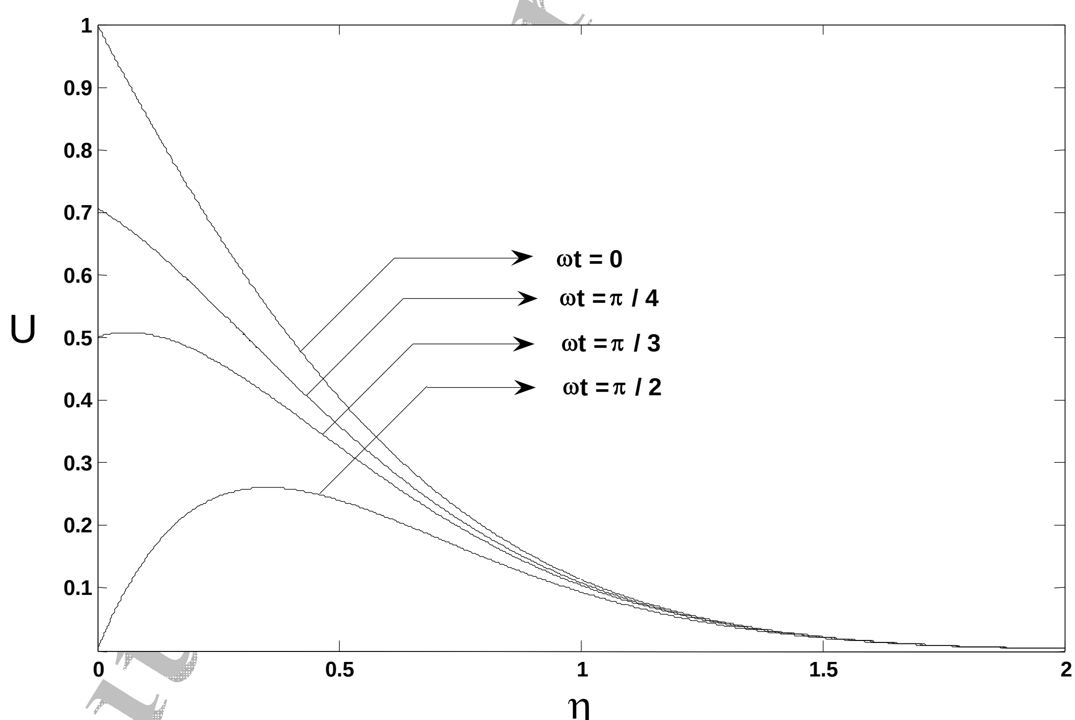

Figure 3.Velocity profiles for different values of ωt

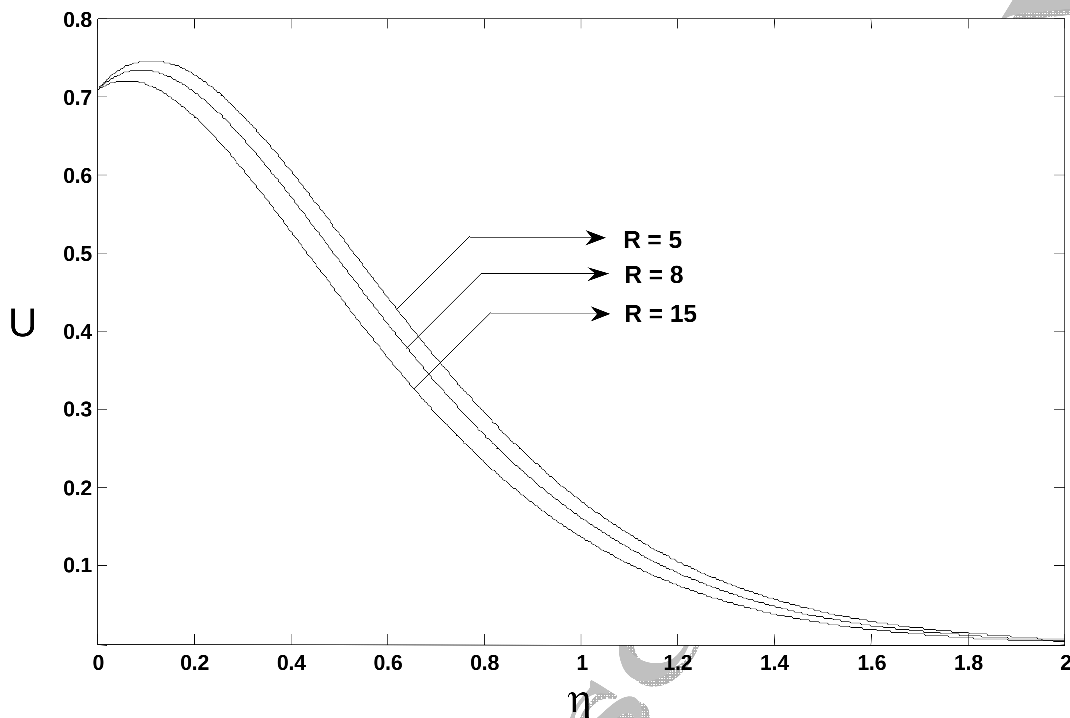

Figure 4. Velocity profiles for different values of R

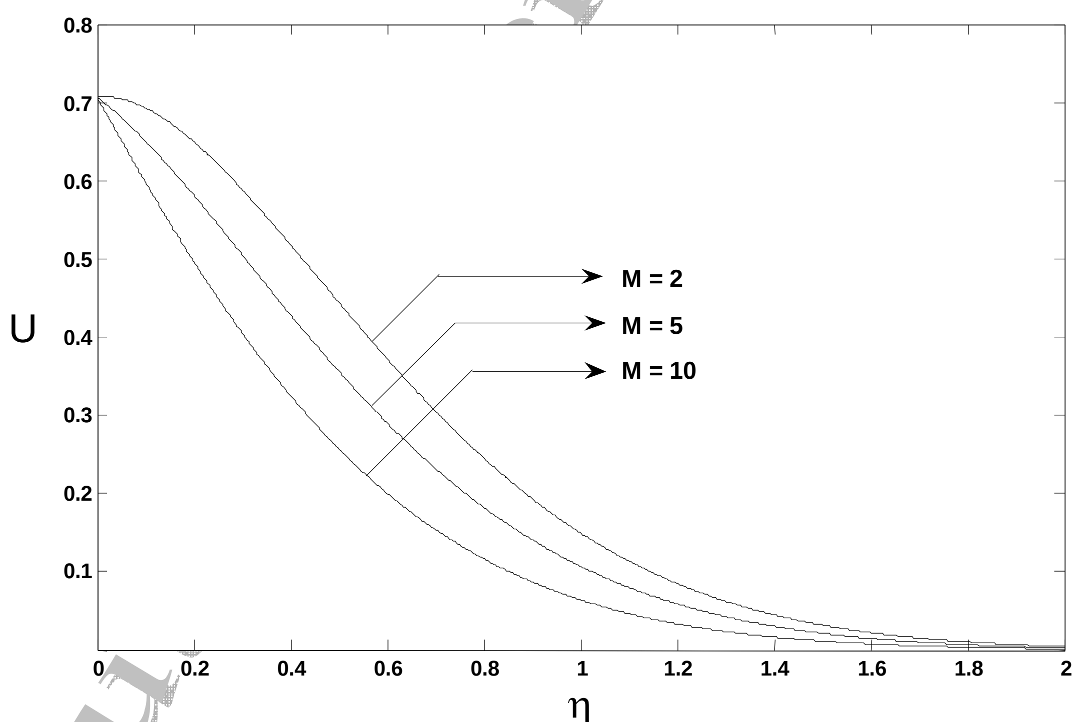

Figure 5. Velocity profiles for different values of M

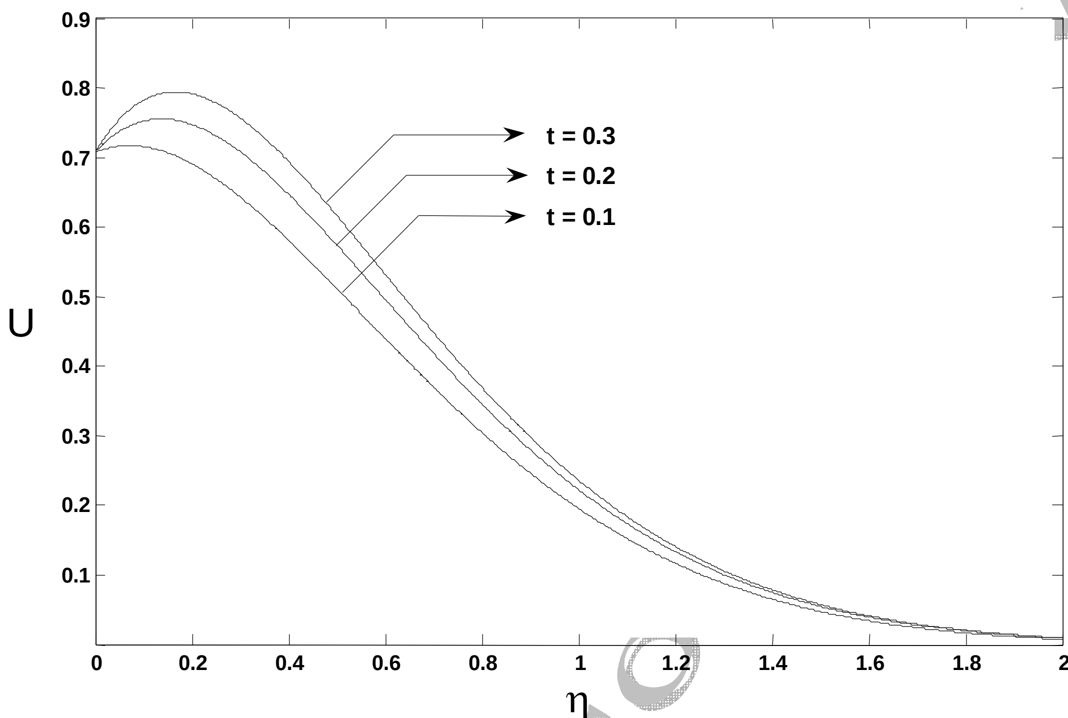

Figure 6. Velocity profiles for different values of t

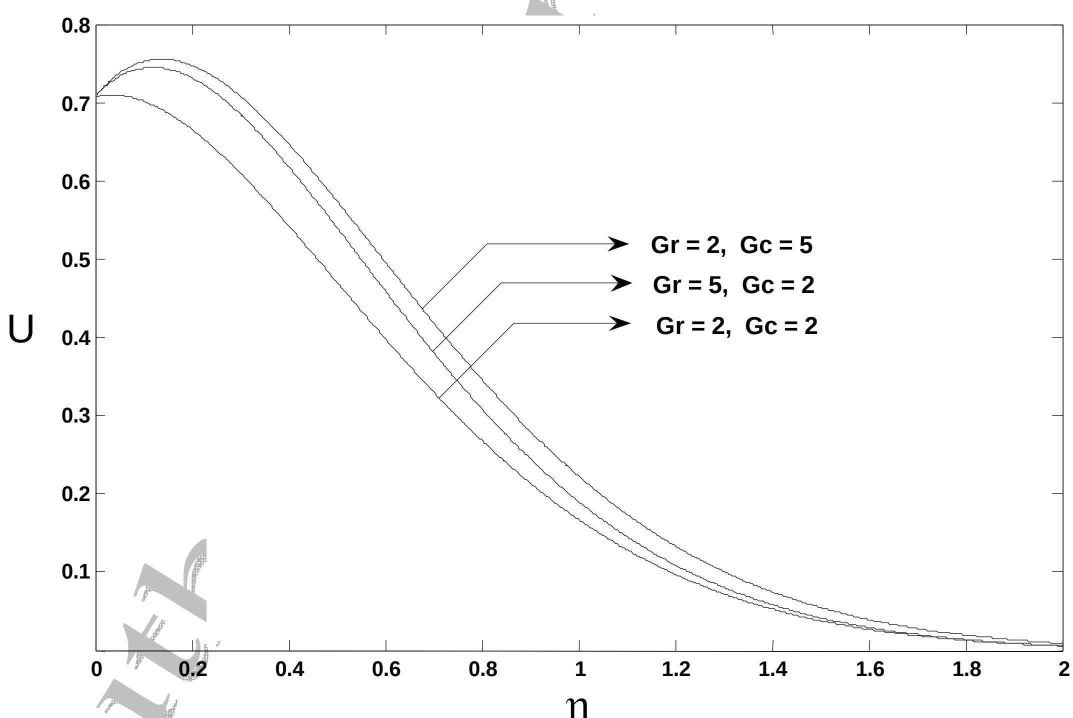

Figure 7. Velocity profiles for different values of Gr, Gc