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A RECURSIVE APPROACH FOR PREY-PREDATOR ECO – SYSTEM INCORPORATING DEATH RATE FOR PREDATOR

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ABSTRACT

In the present paper a four stage recursive procedure is designed to give an approximate solution to the mathematical model equations of a two-species Prey-Predator eco-system. Numerical examples are provided to explain trajectories of the solutions. The conclusions are recorded from the graphs.

Key words : Recursive procedure, Non-linear system, trajectories.
AMS Classification : 92 D 25, 92 D 40

INTRODUCTION

The exact solutions of first order non-linear differential equations obtained in the two-species competitive eco-systems can't be obtained directly because of intractability of non-linear terms. Techniques like self portrait analysis are used to give qualitative nature of solutions in treatises of Kapoor [ ], Meyer [ ], Kushing [ ] etal. In the present investigation we present a four stage recursive procedure to give an approximate solution for a first order non-linear differential equation and the same is applied to solve a two-species Prey-Predator eco-system .In the section 1, general four-stage procedure is presented for solving a first order non linear differential equation. In section II the recursive procedure is applied to a two-species Prey-Predator eco-system.Some numerical illustrations are given to explain the trajectories of the approximate solutions considering various situations. All the conclusions are recorded.

SECTION – I

General Recursive procedure with four-stage approximation for first order non-linear differential equations.

Consider the system

$$\dot{\vec{N}}(t)=A\vec{N}(t)+f\left(\vec{N},t\right)+\vec{h}(t) \qquad \ldots. \quad (1.1)$$

Where $\dot{\vec{N}}(t)=\frac{d\vec{N}(t)}{dt}$ with initial condition

$$\vec{N}(0)=\vec{N}_0 \qquad (1.2)$$

Where $A\vec{N}(t)$ denote the linear dependence of the system.

$f\left(\vec{N},t\right)$ corresponds to the non-linear dependence terms of the system

$\vec{h}(t)$ denote replenishment / renewal rates of the system.

The total procedure is represented in the following four stages.
Stage I : Consider the linear system of (1.1)

$$\dot{\vec{N}}(t)=A\vec{N}(t) \qquad (1.3)$$

with same initial conditions

$$\vec{N}(0)=\vec{N}_0$$

This system is obtained by suppressing the non-linear part in the right hand side in the absence of replenishments or renewals.

Let the solution of (1.3) together with (1.2) be given by $\vec{N}^{(1)}(t)$

Stage II : Compute the resdenishment / renewal rate that could maintain above solution $\vec{N}^{(1)}(t)$ in the non-linear system for the system (1.1) i.e.

$$\vec{h}(t)=\vec{N}^{(1)}(t)-A\vec{N}^{(1)}(t)-f\left(\vec{N}^{(1)},t\right)$$

$$\vec{h}(t)=\dot{\vec{N}}^{(1)}(t)-A\vec{N}^{(1)}(t)-f\left(\vec{N}^{(1)}(t),t\right)$$

$$\therefore\ \vec{h}(t)=-f\left(\vec{N}^{(1)}(t),t\right) \qquad (1.4)$$

Stage III : Consider the system

$$\vec{N}'(t)=A\vec{N}(t)-f\left(\vec{N}^{(1)},t\right) \qquad \ldots. \quad (1.5)$$

with the homogeneous initial condition

$$\vec{N}(0)=0 \qquad \ldots. \quad (1.6)$$

Let $\vec{N}^{(2)}(t)$ be the solution of system (1.5) together with (1.6).

Stage IV : An approximate solution of non-linear system (1.1) with (1.2) is given by

$$\vec{N}(t)=\vec{N}^{(1)}(t)-\vec{N}^{(2)}(t) \quad \text{….} \quad (1.7)$$

SECTION – II

2.1 In this section we obtain the approximate solution of a typical two-species Prey-Predator eco-system with death rate for Predator and replenishments for both prey and predator.

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The system under study is

$$\frac{dN_1}{dt}=a_1N_1-a_{12}N_1N_2-a_{11}N_1^{\,2}+h_1 \quad (2.1)$$

$$\frac{dN_2}{dt}=a_2N_2+a_{21}N_1N_2-a_{22}N_2^{\,2}+h_2$$

with initial conditions

$N_i(0) = N_{i0} = i = 1, 2$ …. (2.2)

The computations of recursive procedure explained in section –1 are carried out.

Stage I : Taking the system (2.1) with linear terms along with (2.2) we get

$$\frac{dN_1}{dt}=a_1N_1 \quad (2.3)$$

$$\frac{dN_2}{dt}=a_2N_2$$

along with $N_i(0) = N_{i0}$: i = 1, 2 …. (2.4)

The solution of (2.3) with (2.4) be given by

$$N_1^{(1)}=N_{10}\exp\{a_1t\} \quad (2.5)$$

$$N_2^{(1)}=N_{20}\exp\{a_2t\}$$

Stage II : Basing on (2.5) h_1, h_2 of (2.1) are calculated as

$$h_1(t)=a_{11}N_{10}^{\;2}\exp(2a_1t)+a_{12}N_{10}N_{20}\exp\{(a_1+a_2)t\} \quad (2.6)$$

$$h_2(t)=a_{22}N_{20}^{\;2}\exp(2a_2t)-a_{21}N_{10}N_{20}\exp\{(a_1+a_2)t\}$$

Stage III : Using (2.6) along with homogeneous initial conditions we set up the linear system

$$\frac{dN_1}{dt}=a_1N_1+h_1(t) \quad (2.7)$$

$$\frac{dN_2}{dt}=a_2N_2+h_2(t)$$

along with $N_1(0) = N_2(0) = 0$ …. (2.8)

The solution of (2.7) along with (2.8) is now given as,

$$N_1^{(2)} = N_{10}\exp(a_1 t)\left\{\frac{a_{11}N_{10}}{a_1}(\exp(a_1 t)-1)+\frac{a_{12}N_{20}}{a_2}(\exp(a_2 t)-1)\right\} \quad (2.9)$$

$$N_2^{(2)} = N_{20}\exp(a_2 t)\left\{\frac{a_{22}N_{20}}{a_2}(\exp(a_2 t)-1)-\frac{a_{21}N_{10}}{a_1}(\exp(a_1 t)-1)\right\}$$

Stage IV :

An approximate solution to the system (2.1) is now provided as

$$N_1 = N_1^{(1)} - N_1^{(2)}$$

$$N_1(t) = N_1^{(1)} - N_1^{(2)} \quad (2.10)$$

$$N_2(t) = N_2^{(1)} - N_2^{(2)}$$

i.e.

$$N_1(t) = N_{10}\exp(a_1 t)\left\{1-\frac{a_{11}N_{10}}{a_1}(\exp(a_1 t)-1)-\frac{a_{12}N_{20}}{a_2}(\exp(a_2 t)-1)\right\} \quad (2.10)$$

$$N_2(t) = N_{20}\exp(a_2 t)\left\{1-\frac{a_{22}N_{20}}{a_2}(\exp(a_2 t)-1)+\frac{a_{21}N_{10}}{a_1}(\exp(a_1 t)-1)\right\}$$

2.2 **Numerical computation** :

$N_1(t)$, $N_2(t)$ are obtained numerically for a sampled initial values $N_{10} = 1$, $N_{20} = 0.5$ and with species competing parameters.

$a_1 = a_{12} = 1$
$a_2 = 0.5, 1.0, 1.5, 2$
$a_{22} = 0.5, 1.0, 1.5, 2.0$
$a_{21} = 0.1, 1, 1.5$

All the graphs showing trajectories are illustrated in Fig. 1 – Fig. 12.

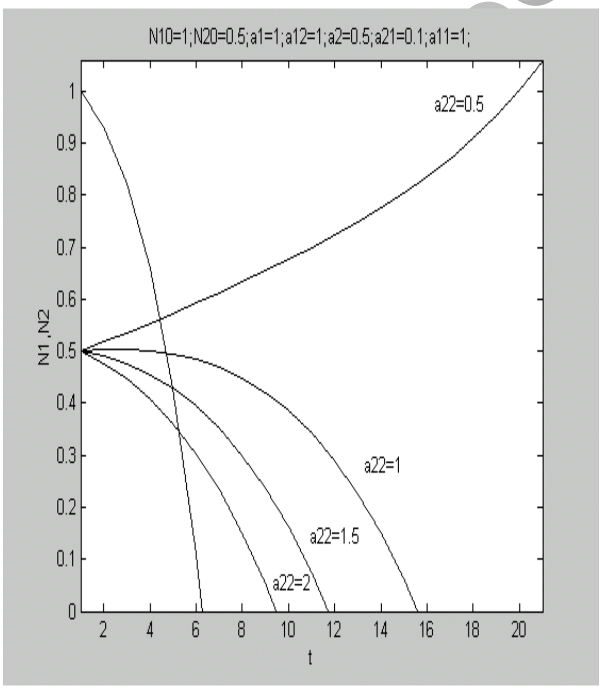


Fig. 1

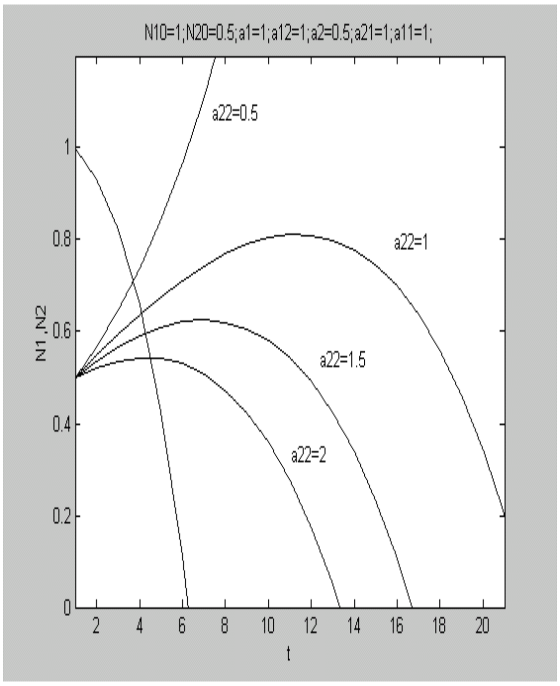
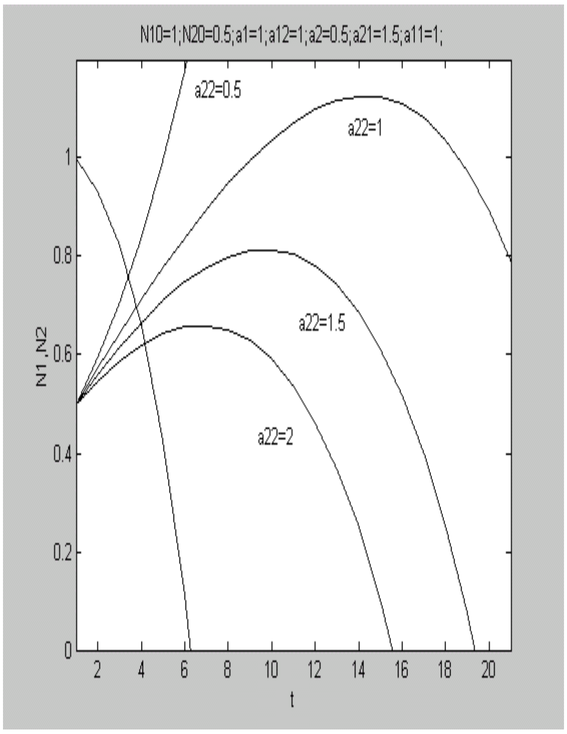


Fig. 2



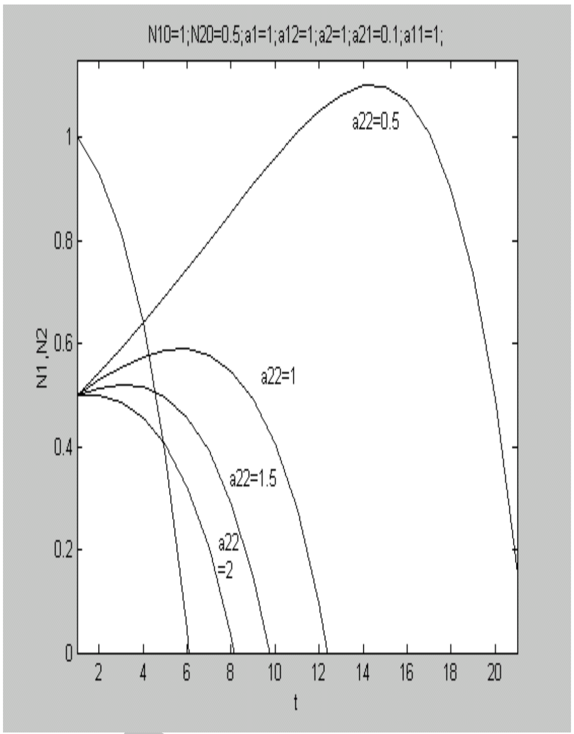


Fig. 3

Fig. 4

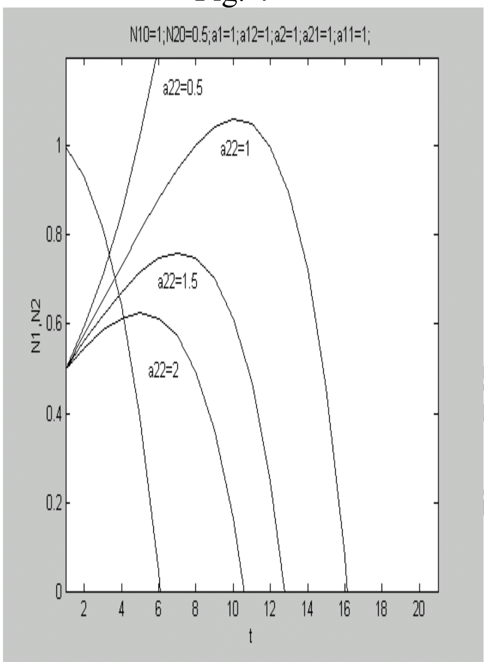


Fig. 5

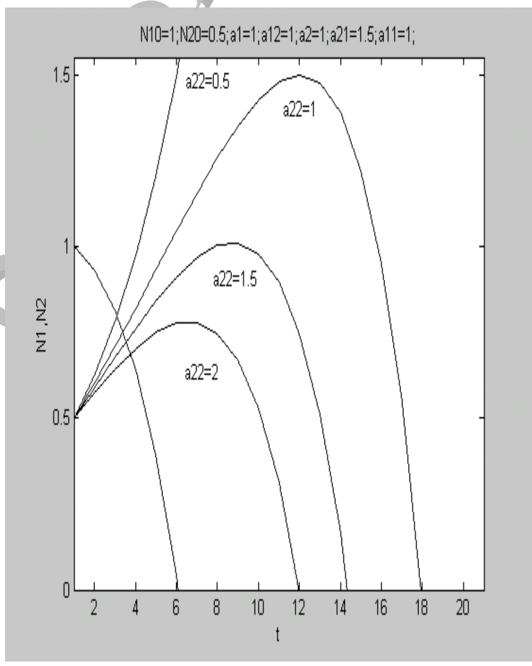


Fig. 6

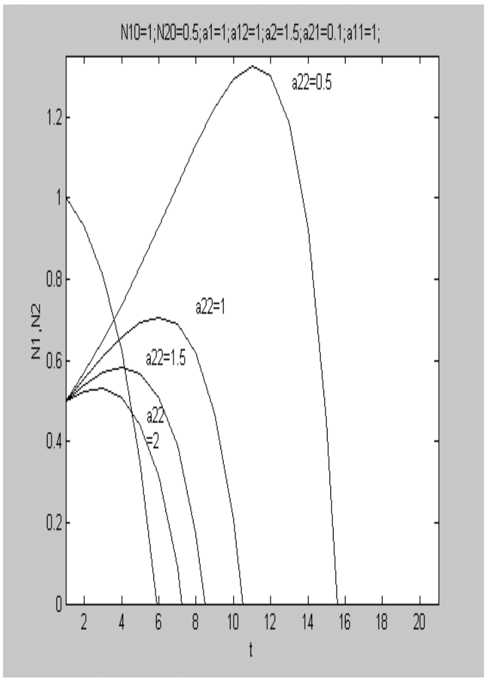


Fig. 7

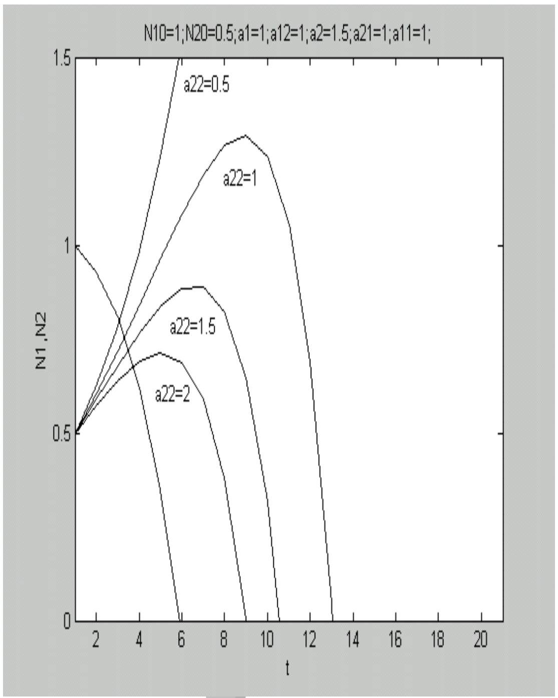


Fig. 8

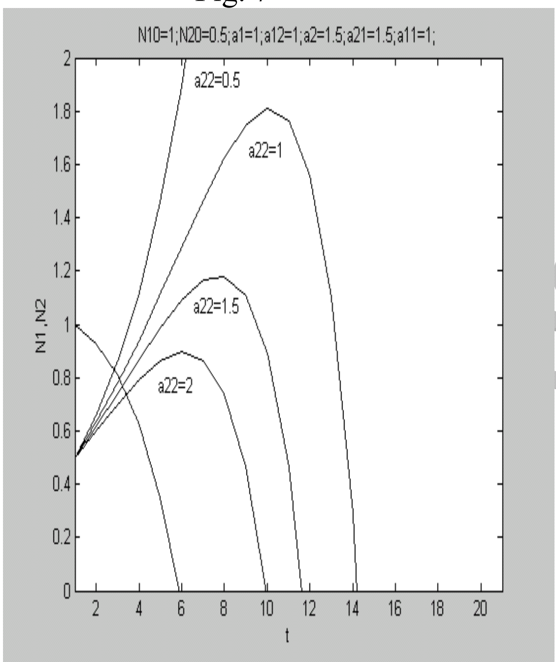


Fig. 9

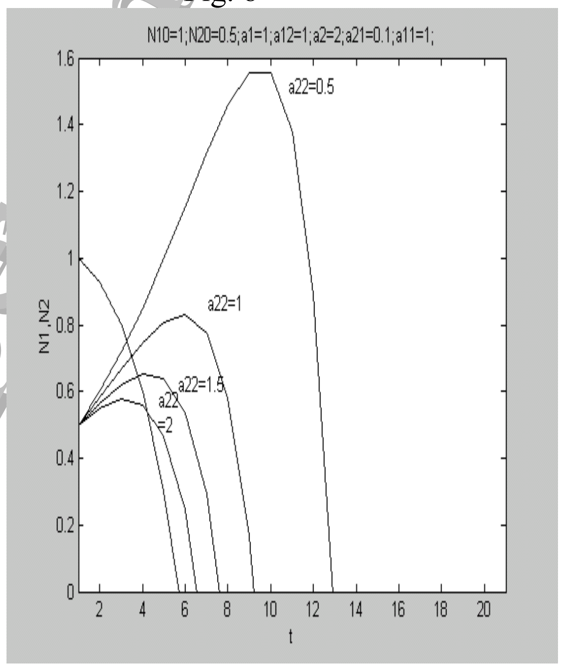


Fig. 10

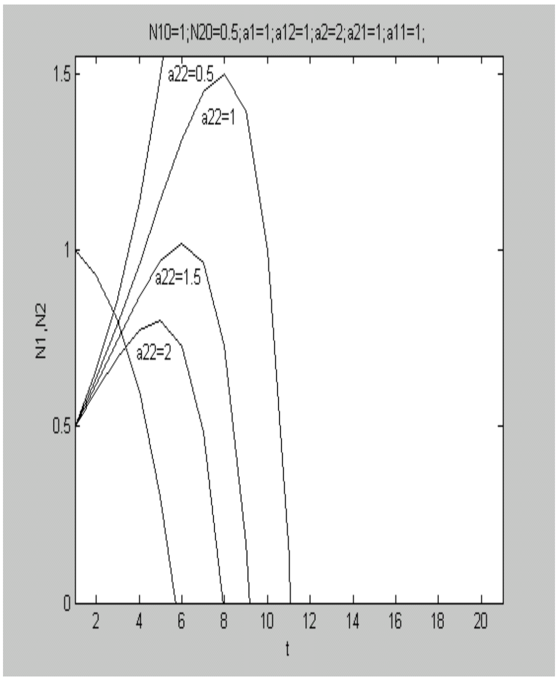


Fig. 11

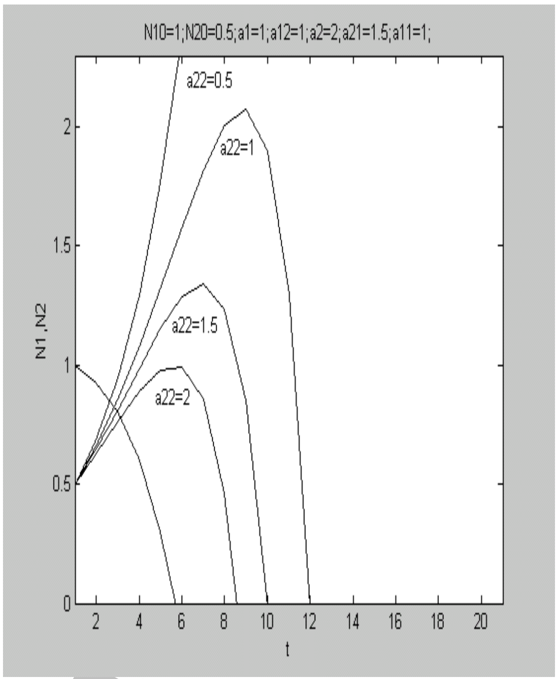


Fig. 12

3.3 **Conclusions from the graphs**:

Following conclusions are derived from the graphs (Fig.1 – Fig.12)

1. If Growth rate of second species S_2 decreases the N_2 curves are with decreasing Steepness.

2. Both N_1, N_2 decrease with t as it is expected in a competing process because of utilization of energy during the predation.

3. For $a_{21}<1$, weak competition of S_2 over S_1, N_2 falls with slower rate than N_1. However with increasing growth rate a_2 of S_2 steepness increases for the N_2 curves and falling rate is decreased with comparison with N_1.N_2 slowly falls down. With increasing a_{22} steepness decreases.

4. For $a_{21}= 1$, equal competition of S_1 & S_2 the steepness of N_2 increase with a_2. However falling of N_2 is faster along with increasing a_{22}.

5. For $a_{21}>1$, S_2 is stronger than S_1 it is observed that N_2 is increasing . This tendency is observed at slow rate in case of $a_{21}=1$. This behaviour is expected of S_2 because of resource limiting term and death rate are present.

6. Inspite of death rate of S_2, N_1 falls much faster than N_2. However this fall becomes slower in case of $a_{21}>1$.

7. As the resource limiting coefficients a_{22} of S_2 increases N_2 falls down rapidly.

Trend index:	N_1	N_2
a_2 ↑	decreases	steepness increases
a_{22} ↑	decreases	steepness falls
a_{21} ↑	decreases	steepness increases

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