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MHD FLOW OF A SECOND – ORDER FLUID THROUGH POROUS MEDIUM

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Abstract

The flow of a second order non-Newtonian fluid under the transverse magnetic field is considered in the presence of an oscillatory pressure gradient, through porous medium contained in an elliptic tube.

The effect of the permeability coefficient of the medium, the effect of the magnetic parameter and that of the oscillatory frequency are examined.

The flow of the classical Newtonian flow through elliptic tube and the flow of Newtonian flow through porous medium in elliptic tube and the flow through circular tube are deduced. The Darcian effect is seen in the core of the region near to the axis of the tube. The non-Darcian effect is seen near the boundary of the tube.

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1. Introduction

The flow of a non-Newtonian fluid has wide range of applications in the Industrial, Technological and scientific fields. Several investigations were made earlier, Vnguyen and Chandra [1] has studied applying hodograph transformations to obtain solutions of non-Newtonian MHD flow. The exact solutions of non-Newtonian MHD flow are obtained by Singh and Thakur [2]. Many investigators has derived general equations of motion for the flow of a viscous fluid through a porous medium on the principle suggested by Eringen. Some have discussed the influence of magnetic field on the velocity of a conducting fluid in porous media. Ashok Kumar etal [3]. Attia [4, 5] studied MHD flow of an incompressisble conducting fluid through elliptic, cylindrical ducts based on super posability of the fluids.

In this present paper the flow of a second order non-Newtonian fluid, through porous medium bounded by an elliptic tube is considered. A transverse magnetic field is applied normal to the flow. The effect of magnetic parameter and that of permeability coefficient of the porous medium is examined. The influence of oscillations on the flow is also discussed. Investigation of several situations of the flow problem is made. The general observation is that Darcian law holds near axis of the tube and Brinkman [6] generalized Darcian law explains the flow phenomenon near the boundary of the tube.

2. Formulation of the Problem

Let (x, y, z) be the rectangular coordinate system such that the z-axis lies along the length of the tube with impermeable boundary T. The velocity components in x, y and z directions are taken to be 0, 0 and w (x, y, t) respectively. A uniform magnetic field H_0 is applied perpendicular to the axis of flow. The induced magnetic effect is assumed to be negligible in comparison with the transverse magnetic field, due to low magnetic Reynolds number as a result of slightly conducting field (Sparrow and cess [7]). Further the electric force E given by ohms law $J = \left(E + V \text{ X } B\right)$ where B = (H_0, 0, 0) and electrical conductivity is assumed to be a null vector for the simplicity of the problem.

The equation of motion of the flow of the fluid is

$$\rho \frac{d\overline{V}}{dt} = \text{Div}S_{ij} - \sigma\, \mu_e^2 H_0{}^2 - \left(\frac{\mu}{K}\right)\overline{V} \qquad (2.1)$$

Together with the equation of continuity

$$\nabla \,.\, \overline{V} = 0 \qquad (2.2)$$

Where $\overline{V}$ velocity of the fluid is, ρ is density of the fluid, σ is electrical conductivity, μe is magnetic permeability, H_0 is intensity of magnetic field, S_{ij} is stress Tensor given by

$$\left.\begin{aligned} S_{ij} &= -P_i + \phi_1 E^{(1)} + \phi_2 E^{(2)} + \phi_3 E^{(3)} \\ \text{with } E_{ij}^{(1)} &= U_{ij} + U_{i.i} \\ E^{(2)} &= A_{i.j} + A_{j.i} + 2_{m.i} U_{m.j} \end{aligned}\right\} (2.3)$$

Where U_i, A_i are components of velocity and acceleration along the coordinate axis, P is isotropic field and ϕ_1, ϕ_2, ϕ_3 are material constants. The choice of velocity field $\overline{V}\left[0, 0, w(x, y, o)\right]$ satisfies the equation of continuity and the equation of motion becomes,

$$\frac{\partial w}{\partial t} = \frac{-1}{e}\frac{\partial p}{\partial z} + \left(V + \beta\frac{\partial}{\partial t}\right)\nabla^2 w - \sigma\mu_r^2 H_O^2 \frac{w}{e} - \left(\frac{\mu}{K}\right)w \tag{2.4}$$

$$O = \frac{-1}{e}\frac{\partial p}{\partial z} + 4\beta\frac{\partial w}{\partial y} \cdot \frac{\partial^2 w}{\partial y^2} \tag{2.5}$$

The modified pressure is

$$P_e = P - 2\phi_2\left(\frac{\partial w}{\partial y}\right)^2 \text{ or } \frac{\partial P_e}{\partial z} = 0 \tag{2.6}$$

Hence

$$\frac{\partial w}{\partial t} = \frac{-1}{e}\frac{\partial P_e}{\partial z} + \left(V + \beta\frac{\partial}{\partial t}\right)\nabla^2 w - \sigma\mu_e^2 H_O^2 \frac{w}{e} - \left(\frac{V}{K}\right)w \tag{2.7}$$

With $V = \frac{\phi_1}{e}, \beta = \frac{\phi_2}{e}$

Employing the non dimensional quantities

$$\left.\begin{aligned}&\left(w^*, w_1^*\right)=\left(w, w_1\right)\frac{a}{v},\ P_e^*=\frac{P_e a^2}{\rho v^2}\\&E^*=t\frac{v}{a^2},\ \left(x^*, y^*, z^*\right)=\frac{(x, y, z)}{a}\\&\beta=\frac{a}{\sqrt{k}},\ M=\mu_e^2 H_0^2\frac{\sigma_a^2}{v\rho},\ S=\frac{\beta}{a^2}\end{aligned}\right\}\quad(2.8)$$

The equation in non dimensional form with out astrix is given as

$$\frac{\partial w}{\partial t}=\frac{-\partial P_e}{\partial z}+\left(1+S\frac{\partial}{\partial t}\right)\nabla^2 w-\left(M+K^2\right)w \tag{2.9}$$

Where $$\beta^2=\frac{a^2}{K} \tag{2.10}$$

3. SOLUTION OF THE PROBLEM

The motion of the fluid is considered to be a sinusoidal function of time t and initially the fluid is assumed to be at rest.

Let

$$\left.\begin{aligned}&-\frac{\partial P_e}{\partial z}=\text{Real } A\exp(i\sigma t)\\&W=\text{Real } W_1\exp(i\sigma t)\\&Q=\text{Real } Q_0\exp(i\sigma t)\end{aligned}\right\}\quad(3.1)$$

Where A is a real number, W, Q_0 are complex and Q is the flux.

From (3.1) and (2.8), we get

$$\nabla^2 W-\frac{(\alpha-i\sigma)}{(1+is\sigma)}W_1=\frac{-A}{1+is\sigma} \tag{3.2}$$

With $\alpha=K^2+M$

Which reduces to

$$\nabla^2\phi-\alpha_1\phi=0 \tag{3.3}$$

With $\alpha_1=\dfrac{\alpha-i\sigma}{1+is\sigma},\ \phi=w-\dfrac{A}{\alpha-i\sigma}$

The boundary condition $v = 0$ on τ

The boundary of the tube gives

$$w = \frac{A}{\alpha - i\sigma} \tag{3.4}$$

The equation (3.3) transformed into elliptic coordinates (ξ, η) by the transformation

$x + iy = c \cosh(\xi + i\,\eta)$ as in McLachlan [8]

Therefore we get,

$$\frac{\partial^2 \phi}{\partial \xi^2} + \frac{\partial^2 \phi}{\partial \eta^2} - 2q(\cosh 2\xi - \cos 2\eta)\phi = 0 \tag{3.5}$$

Where $q = \frac{\alpha_1 c^2}{4}$ and $c^2 = a^2 - b^2$

With $a = c \cosh \xi$ and $b = c \sinh \xi$ as the half of the major and minor axes of the elliptic cross section on T.

The equation (3.5) is solved by taking

$\phi(\xi, \eta) = f(\xi)\, g(\xi)$ then we get,

$$\frac{d^2 f}{d\xi^2} - \left[\lambda^{(2n)} + 2q \cosh 2\xi\right] f = 0 \tag{3.6}$$

$$\frac{d^2 g}{d\eta^2} + \left[\lambda^{(2n)} + 2q \cosh 2\eta\right] g = 0 \tag{3.7}$$

Here $\lambda^{(2n)}$ is a separation constant. The flow is symmetric about the axis of the ellipse and is periodic with period □ in η. The solutions of equations (3.6) and (3.7) are given by Mc Lachlan [8].

As

$$\left.\begin{aligned} g &= C\, e_{2n}(\eta, -q) \\ f &= C\, e_{2n}(\xi, -q) \end{aligned}\right\} \tag{3.8}$$

Hence ϕ is given by

$$\phi = \sum_{n=0}^{\infty} P_{2n}\ Ce_{2n}\left(\xi e, -q\right) e_{2n}\left(\eta_e, -q\right) \tag{3.9}$$

Where

$$Ce_{2n}\left(\eta, -q\right) = \left(-1\right)^n \sum_{r=0}^{\infty} \left(-1\right)^r\ A_{2r}^{(2n)} \cosh 2r\eta \tag{3.10}$$

and

$$Ce_{2n}\left(\xi, -q\right) = \left(-1\right)^n \sum_{r=0}^{\infty} \left(-1\right)^r\ A_{2r}^{(2n)}\, c e_{2n}\left(\eta, -q\right) \tag{3.11}$$

and the coefficients $A_{2r}^{(2n)}$ are the functions of q. The constants P_{2n} are determined using the boundary conditions on $\xi = \xi_0$.

$$\frac{-A}{\left(\alpha - i\sigma\right)} = \sum P_{2n}\ C\, e_{2n}\left(\xi_0, -q\right) C\ e_{2n}\left(\eta, -q\right) \tag{3.12}$$

Multiplying both sides of equation (3.12) by $C\ e_{2n}\left(\eta, -q\right)$ and integrating wrt η from 0 to 2 □ and using the orthogonality of McLachlan[8], we tget

$$P_{2n} = \frac{\dfrac{A}{\left(\alpha - i\sigma\right)}\left(-1\right)^{n+1}\ .\ 2\pi\ A_0^{2n}}{C\ e_{2n}^2\left(\xi_0, -q\right) I_{2n}} \tag{3.13}$$

Where

$$I_{2n} = \int_0^{2\pi} C\ e_{2n}^2\left(\eta, -q\right) d\eta \tag{3.14}$$

Hence $W_1 = \phi + \dfrac{A}{\phi - i\ \sigma}$

and W = Real W_1 exp (i □ t)

$$W = \operatorname{Real} \frac{A}{\alpha - i\,\sigma}\left[1 + \sum_{n=0}^{\infty} \frac{(-1)^{n+1}.2\pi.A_0^{2n}}{C\,e_{2n}(\xi_0, -q) I_{2n}}.C\,e_{2n}(\xi, -q) C\,e_{2n}(\eta, -q)\right]\exp(i\sigma_1 t)$$

(3.15)

The total flux Q is given by

$$Q = \mu \int_T W_1 \, dA$$

$$= \frac{-C^2\mu}{\alpha_1} \iint \left(\frac{\partial^2 w_1}{\partial x^2} + \frac{\partial^2 w_1}{\partial y^2} \right) dx\, dy \qquad (3.16)$$

From Mc Lachlan, we get

$$w_1 = \frac{-4C^2\pi^2\mu w_0}{\alpha_1}\left[\sum_{n=0}^{\infty} \frac{\left(A_0^{2n}\right)^2 C\,e_{2n}(\xi, q)}{I_{2n}\; C\,e_{2n}(\xi_0, q)}\right] \qquad (3.17)$$

Where

$$w_0 = \sum_{n=0}^{\infty} P_{2n}\; C\,e_{2n}(\xi_0, -q)\; C\,e_{2n}(\eta, -q)$$

This represents the total flux with in an ellipse defined by ξ, ξ_0 and the total core flux is given by putting $\xi = \xi_0$ in Q.

4. SPECIAL CASES

CASE – 1: Flow of non-Newtonian fluid through porous medium bounded by elliptic tube when pressure gradient is constant i.e. □ = 0 in (3.15), we get,

$$w = \operatorname{Real} \frac{A}{\alpha}\left[1 + \sum_{n=0}^{\infty} \frac{(-1)^{n+1}.2\pi.A_0^{(2n)}}{C\,e_{2n}(\xi_0, -q) I_{2n}}.C\,e_{2n}(\xi, -q) C\,e_{2n}(\eta, -q)\right] \qquad (4.1)$$

CASE – 2: The oscillatory flow of non-Newtonian fluid through the elliptic tube with porous medium and with out magnetic field is given by putting M = 0 in (3.15), we get,

$$w = \text{Real}\frac{A}{\left(K^2 - i\sigma\right)}\left[1 + \sum_{n=0}^{\infty}\frac{(-1)^{n+1}.2\pi.A_0^{(2n)}}{C\,e_{2n}\left(\xi_0, -q\right)I_{2n}}.C\,e_{2n}\left(\xi, -q\right)C\,e_{2n}\left(\eta, -q\right)\right]\exp\left(i\sigma t\right) \tag{4.2}$$

CASE – 3: Flow of non-Newtonian fluid through highly porous medium i.e. $K \to \infty$, $K \to 0$ in the absence of magnetic field is given by

$$w = \text{Real}\left(\frac{-A}{i\sigma}\right)\left[1 + \sum_{n=0}^{\infty}\frac{(-1)^{n+1}.2\pi.A_0^{(2n)}}{C\,e_{2n}\left(\xi_0, -q\right)I_{2n}}.C\,e_{2n}\left(\xi, -q\right)C\,e_{2n}\left(\eta, -q\right)\right]\exp\left(i\sigma_1 t\right) \tag{4.3}$$

CASE – 4: The flow of Newtonian fluid under magnetic field through elliptic tube containing porous medium is obtained by putting S = 0.

$$w = \text{Real}\frac{A}{\alpha - i\sigma}\left[1 + \sum_{n=0}^{\infty}\frac{(-1)^{n+1}.2\Pi.A_0^{(2n)}}{C\,e_{2n}\left(\xi_0, -q\right)I_{2n}}.C\,e_{2n}\left(\xi, -q\right)C\,e_{2n}\left(\eta, -q\right)\right]\exp\left(i\sigma t\right)$$

Where $q = \dfrac{(\alpha - i\sigma)c^2}{4}$ (4.4)

CASE – 5: The flow of Newtonian fluid through elliptic tube, where there is no magnetic field and flow is non oscillatory the pressure gradient being a constant is given by,

$$w = \text{Real}\frac{A}{K^2}\left[1 + \sum_{n=0}^{\infty}\frac{(-1)^{n+1}.2\pi.A_0^{(2n)}}{C\,e_{2n}\left(\xi_0, -q\right)I_{2n}}.C\,e_{2n}\left(\xi, -q\right)\right] \tag{4.5}$$

Where $q = \dfrac{(\alpha - i\sigma)c^2}{4}$ and $I_{2n} = \displaystyle\int_0^{2\Pi} C\,e_{2n}^2\left(\eta, -q\right)d\eta$

$$C\,e_{2n}\left(\eta, -q\right) = (-1)^n\sum_{r=0}^{\infty}(-1)^r\,A_{2r}^{(2n)}\cosh 2r\eta$$

$$C\,e_{2n}\left(\xi, -q\right) = (-1)^n\sum_{r=0}^{\infty}(-1)^r\,A_{2r}^{(2n)}\cosh 2r\xi$$

And the coefficients $A_{2r}^{(2n)}$ are the functions of q.

CASE – 6: Flow through circular tube of non-Newtonian fluid can de derived by taking $C \to 0$, $\xi_0 \to \infty$ such that $a = \cosh \xi_0$. The ellipse turns to a radius a and also (Mc Lachlan, PP.367).

$$C\, e_0(\eta, -q) \to 2^{\frac{-1}{2}}$$

$$C\, e_{2n}(\eta, -q) \to \cos 2n\eta,\ n > 1$$

$$C\, e_{2n}(\xi, -q) \to P_{2\eta},\ I_{2n}(K_1, \eta),\ n > 0$$

$$C\, e_{2n}(\xi_0, -q) \to P_{2\eta},\ I_{2n}(K_1, a),\ n > 0$$

$$A_0^{(0)} \to 2^{\frac{-1}{2}}$$

$A_0^{(2n)} \to 0,\ n > 1$ and $I_{2n} \to \pi,\ n > 0$

Where $K_1 = K^{\frac{-1}{2}}$ and $r = \cosh \xi = \left(x^2 + y^2\right)^{\frac{1}{2}}$

with the help of these limiting values, we get the velocity of the fluid as,

$$w = \frac{A}{K^2}\left[1 - \frac{I_0\left(\frac{r}{K^{1/2}}\right)}{I_0\left(\frac{a}{K^{1/2}}\right)}\right] \tag{4.6}$$

which is the same as that obtained by Narsimha Charyulu et.al.[9].

CASE – 7: Flow through elliptic tube of a Newtonian fluid with no oscillations and no magnetic field and highly porous region is given by,

When $K \to \infty$, $q \to 0$ (Mc Lachlan PP.15)

$$C\, e_0(\eta, -q) + \left(\frac{1}{2}\right) q \cos 2\eta$$

$$C\, e_{2n}(\eta, -q) c\cos 2\eta + \left(\frac{-1}{4}\right)\left(\frac{1}{12}\right) q \cos 4\eta$$

$$C\,e_{2n}(\eta,-q)=C\,e_{2n}(i\xi,-q)$$

$$A_0^{(0)}=1,\ A_0^{(2)}=0 \quad \text{and} \quad A_0^{(2n)}=0 \quad \text{for all } n>2$$

$$P_0=\frac{A}{K^2}\left[1-\left(\frac{1}{2}\right)q\cosh 2\xi_0\right]$$

$$P_2=\frac{A}{K^2}\left(\frac{q}{2\cosh 2\xi_0}\right),\ P_{2n}=0,\ n>2$$

and the velocity of the fluid becomes

$$w=\frac{A}{2K^2}\frac{a^2b^2}{\left(a^2+b^2\right)}\left(1-\frac{x^2}{a^2}-\frac{y^2}{b^2}\right) \tag{4.7}$$

which is same as the velocity of viscous liquid in an elliptic tube when no resistance is offered by the medium.

CASE – 8: If the permeability of the medium is very small and there is no oscillations and magnetic effect in the case of Newtonian fluid the flow can be deduced by using,

When K is very small and q is very large.

$$A_0^{(0)}=1,\ A_0^{(2n)}\approx 0 \text{ for } n>1$$

and from the Mc Lachlan (PP. 230)

$$C\,e_{2n}(\xi,-q)=\frac{(-1)^n C_{2n}}{2^{2n-\frac{1}{2}(i\operatorname{Sinh}\xi)}}\cosh\left[2q^{\frac{1}{2}}\cosh\xi-(4n+1)\operatorname{Tanh}^{-1}\left\{\operatorname{Tan}\left(\frac{\Pi}{4}-\frac{i\xi}{2}\right)\right\}\right]$$

$$C_{2n}\cong\frac{(-1)^n 2^{2n-\frac{1}{2}}}{A_0^{(2n)}\left(\frac{\Pi}{\sqrt{q}}\right)^{\frac{1}{2}}}.C\,e_{2n}(0)\,C\,e_{2n}\left(\frac{\Pi}{2}\right)$$

$C_{2n}\cong 0$ for n>1 and for large |q|.

$$\phi\cong P_0\,C\,e_0(\xi,-q)\,C\,e_0(\eta,-q)$$

$$\cong \frac{-A}{K^2}\frac{\cosh\left(\frac{1}{2}\xi\right)}{\cosh\left(\frac{1}{2}\xi_0\right)}\exp\left(\frac{-2}{q^{\frac{1}{2}}}\right)\left(\cosh\xi_0-\cosh\xi\right).C\,e_0\left(\eta,-q\right)$$

Hence velocity of the fluid w is given by,

$$w=\frac{A}{K^2}\left[1-\frac{\cosh\frac{1}{2}\xi}{\cosh\frac{1}{2}\xi_0}C\,e_0\left(\eta,-q\right)\exp\left(\frac{-d}{K^{\frac{1}{2}}}\right)\right] \qquad (4.8)$$

where $d=\left(c\cosh\xi_0-\cosh\xi\right)$

5. CONCLUSION

The flow of non-Newonian fluid through porous region in an elliptic tube is considered under oscillating pressure gradient. In the case of a constant pressure gradient and no magnetic field the flow of the fluid depends on the permeability of the porous medium. When the permeability coefficient K increases the velocity is observed to be increasing. If the medium is highly porous the flow represents the flow of the non-Newtonian fluid in a clear medium (4.3).

When permeability of the medium is very small it is observed $\xi=\xi_0$ or $d\to\infty$, $w=\frac{a}{K^2}$ which is constant which implies that non Darcian effect is seen to exist predominantly near the boundary of the tube and classical Darcian effect is realized only in a core near the axis of the tube (4.8).

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