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HEAT TRANSFER IN MHD FLOW AT THE THERMAL ENTRANCE REGION OF FLAT DUCT

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ABSTRACT:

In this paper heat transfer of MHD, fully developed incompressible cowling flow at thermal entrance region of a flat duct under constant heat flux at walls is investigated by finite difference technique. The flow is in X-direction and applied magnetic field in the y-direction. The flow pattern is determined for different values of Hartmann number (m), heat generation (η) parameter and Electric field factor (e). Comparisons have been made with ordinary fluid flow and MHD flows. It is observed at MHD has effect of increasing temperature in cowlings flow. The variation of the wall temperatures for different values of M, e, η are shown graphically. Pseudo local Nusselt number variations shown graphically

INTRODUCTION:

Heat transfer in electrically conducting fluid under the influenceof magnetic field has attracted the attention of many researchers. It has wide applications in magneto- hydrodynamics occillarater, generator, space research and wave propagation. Romig[1], Siegel[2] have investigated heat transfer in a duct in the region where temperature distributions is fully developed and heat flux at the wall is uniform. Alpher [3], Yen [4] and Snyder [5] investigated the same problem but they assumed that the duct walls are electrically conducting. Regiver [6] and Grshuni et. al. studied the problem, neglecting joul heating in the fluid. Nigam and Singh [7] have included Jouls heating with constant wall temperature, viscous and electrical dissipation in the thermal entrance. Erickson [8] applied “Finite Difference Technique” for the same problem. Jain and Srinivasn[9] extended this problem by including effect of electrically conducting walls.

N. Rudraiah et. al [11] have studied effect of magnetic field in free convention in rectangular enclosure.

Zniber et.al [12] have obtained the analytical solution to the problem of heat transfer in a MHD flow inside channel with prescribed sinusoidal heat flux. J.N.Lin.et.al [13] have investigated convective instability of heat and mass transfer for laminar forced convection in the thermal entrance region of horizontal rectangular channels. M.Eissa Sayyed-Ahmed[14] have studied laminar heat transfer for thermally developing flow of a Herschel-bulkley fluid in a square duct. J.Lahjomri.et.al [15] have studied heat transfer by laminar Hartmann flow in thermal entrance region with a step change in a wall temperatures.

In view of interest shown by researchers in this field heat transfer in MHD flow at the thermal entrance region of flat duct is invested. The fluid is assumed to be incompressible and the flow is fully developed Hartmann flow which is independent of temperature. Heat flux at walls is considered to be constant.

Mathematical formulation:

The fluid is assumed to be incompressible and the flow is developed Hartmann flow which is independent of temperature . Heat flux at walls is consider to be constant.

Semi infinite parallel plates are extending in x-direction. The magnetic field is imposed in Y-direction. The distance between two parallel plates is taken as origin at entrance region. The equation of plates is now y = $\pm$ d, d hence the flow is symmetrical about y = 0 and every flow parameter is function of y and t. The flow is in x-direction between walls y = $\pm$ d. The applied magnetic field H_o is in y-direction under these assumptions the fully developed Hartmann flow is given by cowling [10].

$$u = \frac{PM}{\sigma_e \, \mu_e^2 \, H_0^2}\left[\frac{\cosh M - \cos h M(y/d)}{\sin \, hM}\right] \qquad (3.1.1)$$

Which is independent of electric field.

The general form of the energy equation for unidirectional steady flow of an incompressible fluid with constant properties and negligible heat conduction in the fluid flow direction can be simplified to

$$u \, \frac{\partial T}{\partial x} = \frac{K}{\rho C_p} \, \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho \, C_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{J^2}{\rho C_p \sigma_e} \qquad (3.1.2)$$

The electric current intensity J can be expressed by

$$J = u_0 \, \sigma_e \, B_0 \left[-e + \frac{u}{u_0}\right] \qquad (3.1.3)$$

With this value for j the energy equation becomes

$$u\frac{\partial T}{d x} = \frac{k}{\rho \, c_p}\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{u_0^2 \, \sigma_e \, B_0^2}{\rho \, C_p}\left(-e + \frac{u}{u_0}\right)^2 \qquad (3.1.4)$$

Where 'd' is one half of duct height 'A' is the surface area of channel wall through which heat is being transferred, B_O is magnetic field induction, C_p is specific heat, E is electric field strength , e is electric field magnitude factor and e $= \frac{E}{u_0 B_0}$, H is magnetic field intensity, H_o is magnetic field imposed perpendicular to bounding walls, h is heat transfer coefficient, J is electric current K- is thermal conductivity, M $= \mu_e H_0 d \sqrt{\left(\frac{\sigma_e}{\mu}\right)}$ is Hartmann number

$Nu_x = \frac{h_x D_e}{k}$ is local Nusselt number, P is fluid pressure gradient in Equation(3.1.5),

$Pr = \frac{\mu c_p}{k}$ is prandtl number, q_x is rate of heat transfer, $q^{11} = \frac{-q}{A}$ negative rate of heat transfer per unit area, $Re_d = \frac{\rho U_0 d}{\mu}$ Reynolds number, T is temperature, T_0 is temperature of fluid at entrance of channel, U is dimensionless velocity, u is velocity in x –direction, u_o is average fluid velocity x $= \frac{k_x}{\rho d^2 u_0 c_p}$ is dimensionless variable distance along length of duct, $Y = \frac{y}{d}$ dimensionless variable distance across height of duct

$\eta = \frac{u_0^2 \mu}{d\ q^{11}}$ heat generation parameter ρ = density, μ is viscosity, μ_e is magnetic permeability, σ_e is electric conductivity, τ is time, $\theta = \frac{T - T_0}{q^{11}\ (d/k)}$ is dimension less temperature, $Nu_{xi} = \frac{-4}{\Delta\theta}$, pseudo – local Nusselt number. $(\Delta\theta)_{xi} = \theta_{wxi} - \theta_{bxi\ xi}$

The average velocity u_0 is between y $= \pm d$ is

$$u_0 = \frac{\int_{-a}^{a} u\ dy}{\int_{-a}^{a} dy} = \frac{P}{\sigma_e\ \mu_e^2\ H_0^2}\left[\frac{MCos\,hM - \sinh M}{\sinh M}\right] \qquad (3.1.5)$$

Hence the dimension less velocity profile is

$$U = \frac{u}{u_0} = M\left[\frac{Cosh\,M - Cosh\,MY}{MCos\,hM - Sin\,hM}\right] \qquad (3.1.6)$$

Using dimensionless quantities defined above equation (3.1.4) in the dimensionless form is the equation.

$$U\ \frac{\partial\theta}{\partial X} = \frac{\partial^2\theta}{\partial Y^2} + y\left(\frac{\partial U}{\partial Y}\right)^2 + M^2\eta\ (e - U)^2 \qquad (3.1.7)$$

This equation has to be solved sub boundary conditions

$$\theta = 0 \ \ at \ \ X = 0 \ and \ 0 \le Y \le 1$$
$$\frac{\partial \theta}{\partial X} = 0 \ at \ Y = 0 \ and \ \ X > 0 \qquad (3.1.8)$$
$$\frac{\partial \theta}{\partial Y} = 1 \ \ at \ Y = 0 \ and \ \ X > 0$$

Since the heat flux is constant at the boundary

Discretizing y-direction in which $\Delta Y = \frac{1}{10}$ *and* $\Delta x = \frac{1}{2}$ *in* x-direction. U (x y) is denoted as U (i j) we have the following difference formulae.

$$\frac{\partial \theta}{\partial y} = \frac{\theta (i, \ j+1) - \theta \ (i, j)}{\Delta y}$$

$$\frac{\partial \theta}{\partial x} = \frac{\theta (i+1, \ j) - \theta \ (i, j)}{\Delta x}$$

$$\frac{\partial^2 \theta}{\partial y^2} = \frac{\theta (i, \ j+1) - 2\theta \ (i, j) + \theta \ (i, \ j-1)}{\Delta y^2}$$
$$\frac{\partial U}{\partial y} = \frac{U (i \ \ j+1) - U(i, j)}{\Delta y}$$

Applying above difference formulae in equation (3.1.7) and using equation (6) we have the finite difference equation as

$$U \ (i, j).\left[\theta \ (i+1, \ j)\right] - \left[U \ (i \ \ j) - 2 \frac{\Delta X}{(\Delta Y)^2}\right] \theta (i \ \ j)$$
$$- \left(\frac{\Delta X}{(\Delta Y)^2}\right) \theta (i, j-1) - \left(\frac{\Delta Y}{(\Delta Y)^2}\right) \theta (i, \ j+1) = Q \ (i, j) \qquad (3.1.9)$$

Where

$$Q(i,j) = (\Delta X)\,(\Delta Y)^2 \ \eta \ M^4 \left[\frac{\sin h \ MJ.\Delta Y}{M \ Cos \, hM - \sin hM}\right]^2 + M \, \eta \left[e - M \left(\frac{\cosh M - \cosh M \ J.\Delta Y}{M \cosh M \ - \sin hM}\right)\right]^2$$

$$U \ (i \ \ j) \ = \ M \left[\frac{CoshM - \cosh MJ\Delta y}{M \cosh M - SinhM}\right]$$

Equation (3.1.9) has to be solved subject to the boundary conditions

$$\theta(\theta \ \ j) = 0 \quad j = 1 \ (1) \ 10$$
$$\theta(i+1, 0) = 0 \ \ (i \quad 0) \ \ : \ i \ > \ 0 \qquad (3.1.1.)$$

$$\theta(i,\ 10)\ = \theta\ (i,\ 9)\ - \Delta\ x \quad i\ = 1\ (1)\ 6$$

$$\theta_{wxi} = \theta_{(xi\ ,10)} \quad \theta_{bxi}\ = \sum_{j=1}^{9} \theta_{i,\,j}\, U_{j}$$

Discussion:

In this paper heat transfer of MHD fully developed incompressible cowling flow at the thermal entrance region of a flat duct under constant heat flux at walls is investigated by finite difference technique. The flow is in x-direction and applied magnetic field in y-direction. The flow pattern is determined for x = i for different values of Hartmann number M, Heat generation parameter η, and electric field factor e.

The equation (9) is sloved subject to boundary condition (10) by using MATLAB software for following values of parameters
M = 0, 4, 10
E = 0.5, 0.8, 1
η = -1, -0.5, 0, 0.5, 1

Comparison have been made with ordinary fluid flow and MHD flows on line x = 0.1 the effect of magnetic field is to push temperature upwards (Fig. 1, 2). Same phenomena observed at M = 4 (Fig. 3, 4). On line x = 0.2 an increase in Hartmann number M for η = -1, the temperature found to increase (Fig 5, 6). On x = 0.5 while on non MHD flow, the flow is uniform from y = 0.1 to y = 0.6 slight variations in temperature are found in this region in MHD flow and temperature found to fall y = 0.7, to y = 0.8 and increase there onwards (Fig ,7 8) .

On x = 0.6 MHD and non MHD flows behaves differently. In non MHD temperature falls, y = 0.6 to y = 0.7 and increases for y = 0.7 to y = 0.8 and temperature falls, in case of MHD, while temperature increases from y = 0.6 to y = 0.7 then falls from y = 0.7 to y = 0.8 and increases in MHD flow (Fig.9, 10). Hence MHD has effect of increasing temperature in cowling flow. An increasing in Hartmann number M makes cause unsteady temperature distribution in the region y = 0.1 to y = 0.8 (fig. 10, 13).

In case of non MHD on the x = 4 the temperature is almost uniform up to y = 0.6 and then falls up to y = 0.7 getting maximum at y = 0.8 and falls than onwards. In case of MHD flow on the line the temperature decreases y = 0.1 to y = 0.2 then almost uniform y = 0.6 and then increases up to y= 0.7 and takes a large positive gradient up to
y = 0.8 and then falls (Fig. 11, 12).

It is observed that variation of wall temperature is almost same up to x = 0.2 for all values of M, e and η there afterwards temperature is found to increase for positive η and decreases for negative η for all values of M, e and η fig. (14, 16, 17) pseudo local Nusselt number is calculated for M=4, e=1, η=0.5,-0.5,-1 they found to take symmetric values about Nu=0 in the region x=1, x=2there after wards they have same value zero from x=2 on wards (fig-18)
It is observed with increase of Hartman number the temperature is found to increase. Hence applied magnetic field increases the temperature. The pseudo-local Nusselt number is found change fromx=1 to x=2 and uniform there onwards.

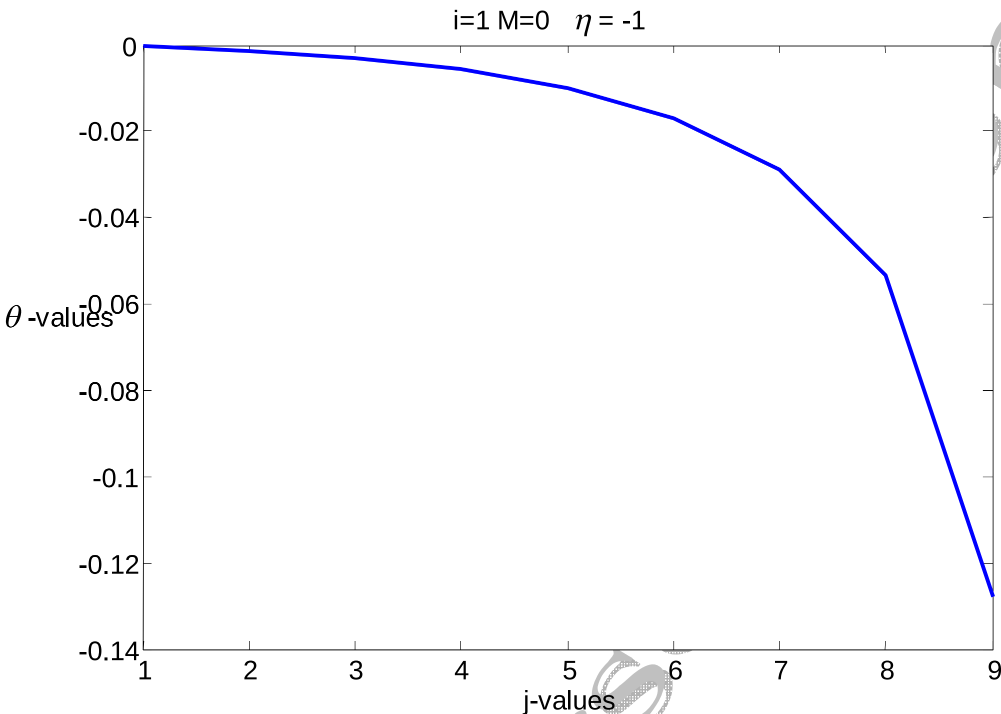

Fig-1: Development of temperature profile in the thermal entrance region M=0 η = -1

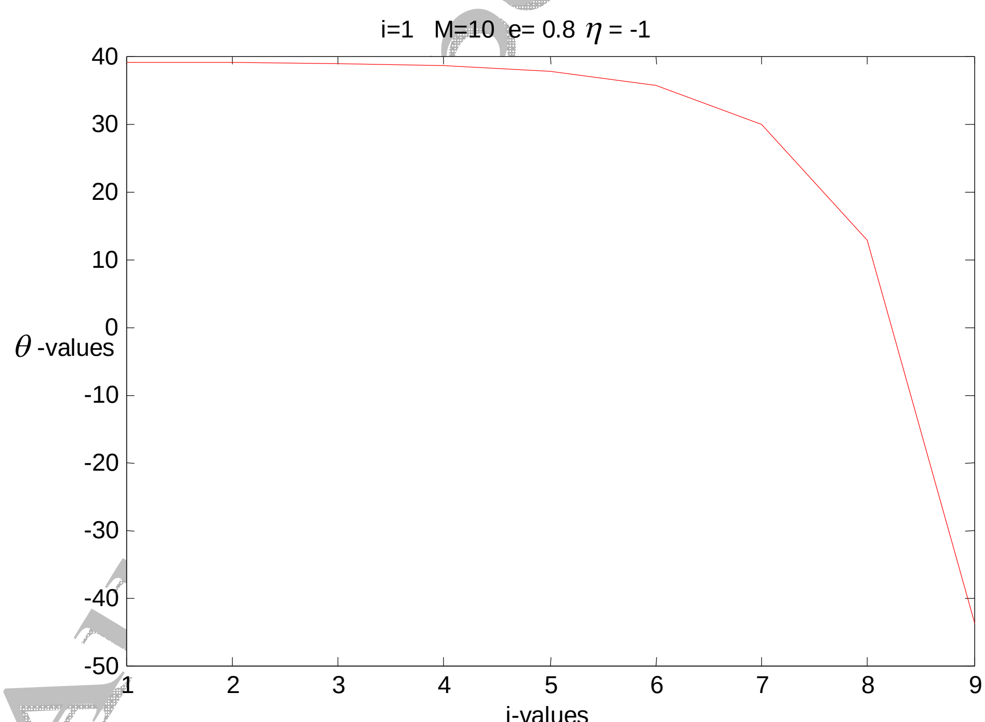

Fig-2: Development of temperature profile in the thermal entrance region M=10 η = -1

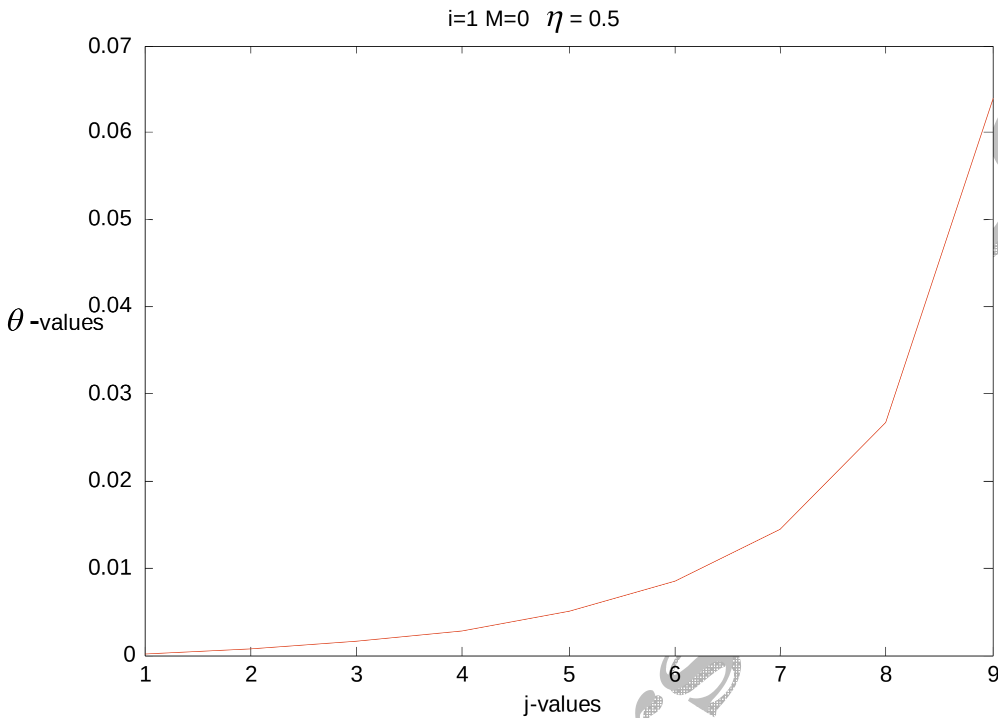

Fig-3: Development of temperature profile in the thermal entrance region M=0 η = -1

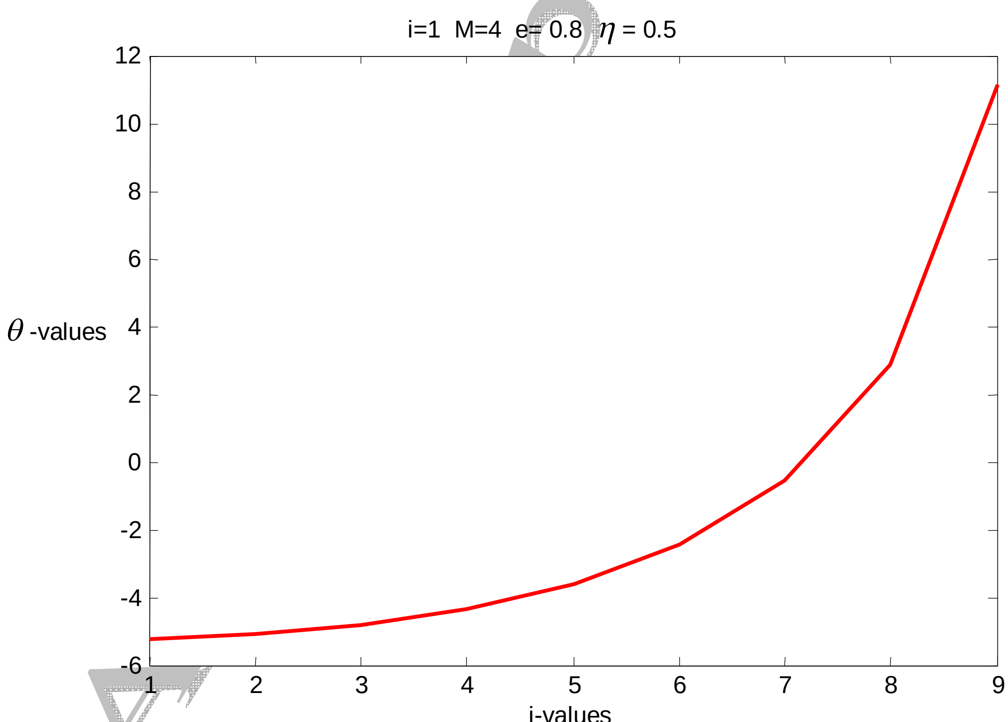

Fig-4: Development of temperature profile in the thermal entrance region M=4 η =0.5

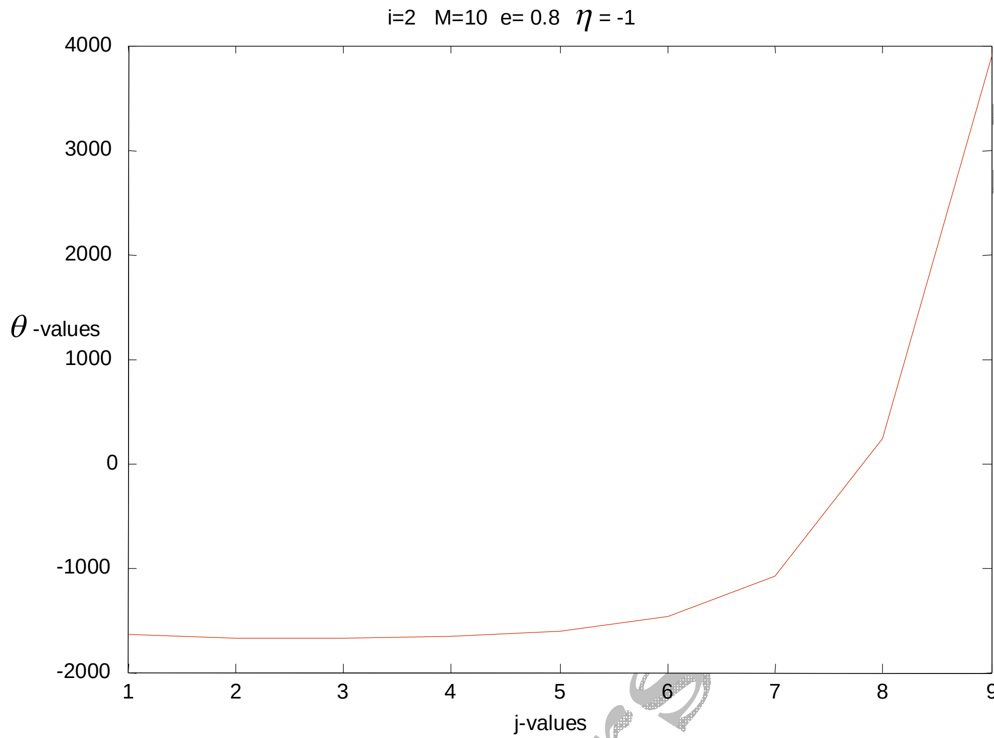

Fig-5: Development of temperature profile in the thermal entrance region M=10 η = -1

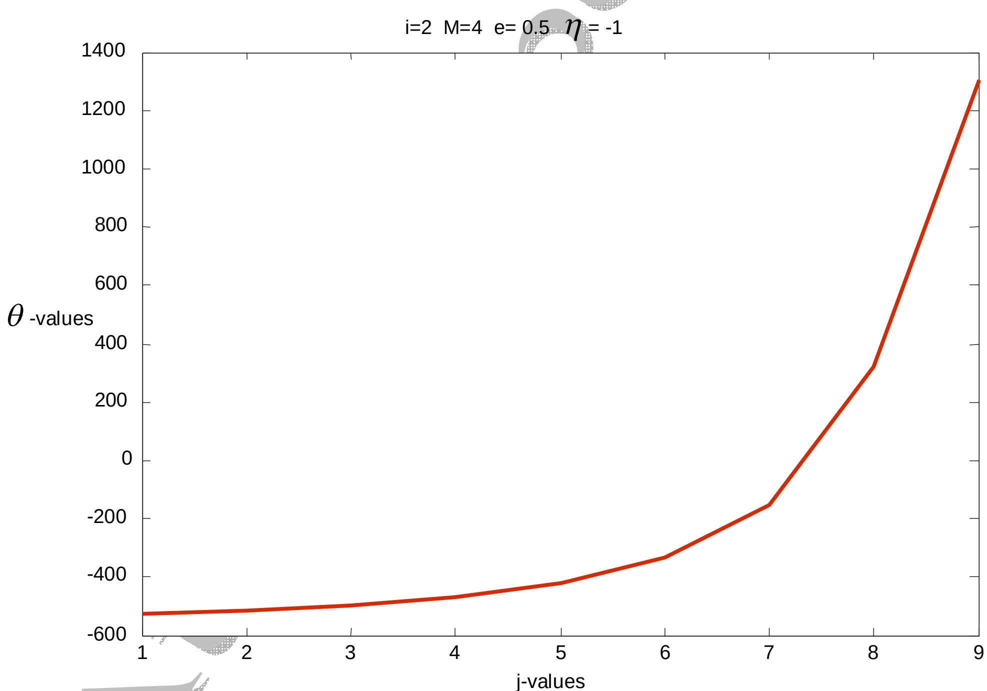

Fig-6: Development of temperature profile in the thermal entrance region M=4 η = -1

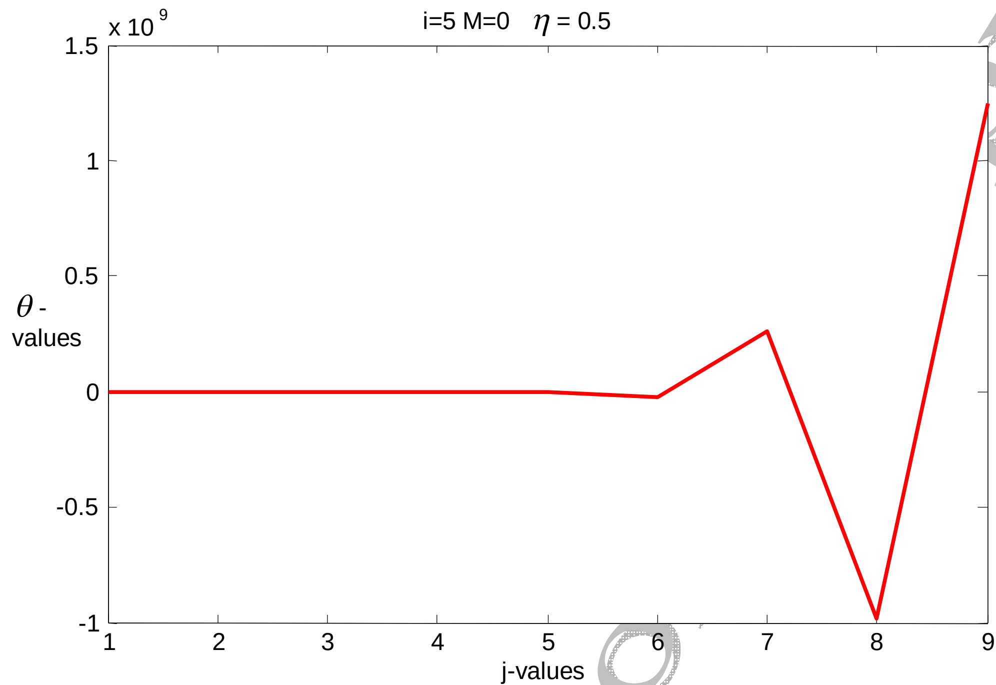

Fig-7: Development of temperature profile in the thermal entrance region M=0 η = 0.5

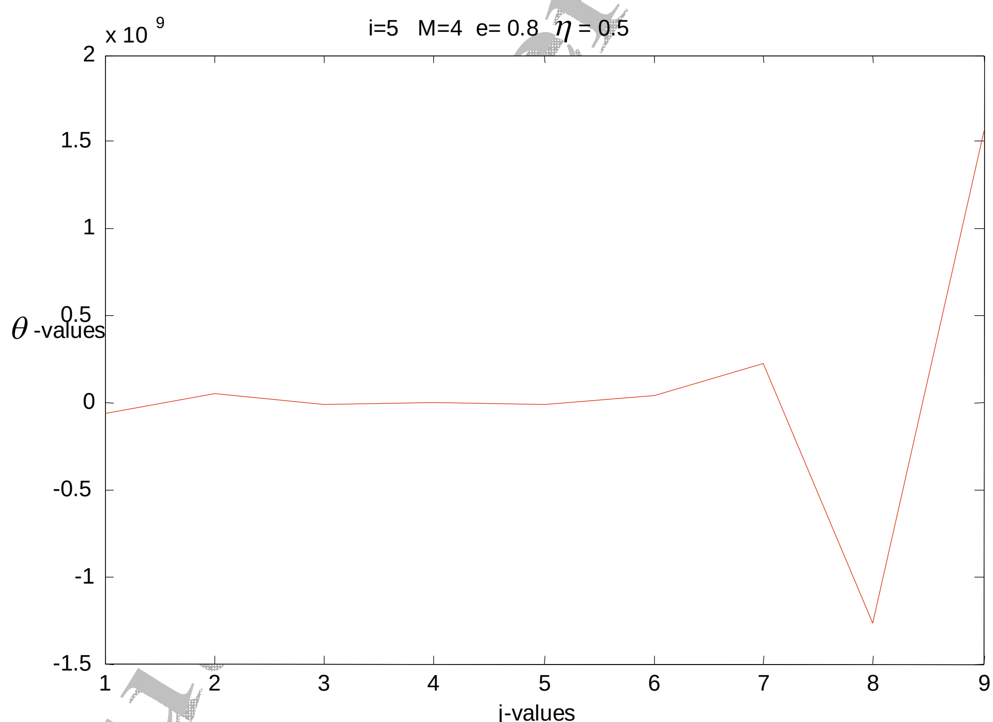

Fig-8: Development of temperature profile in the thermal entrance region M=4 η =0.5

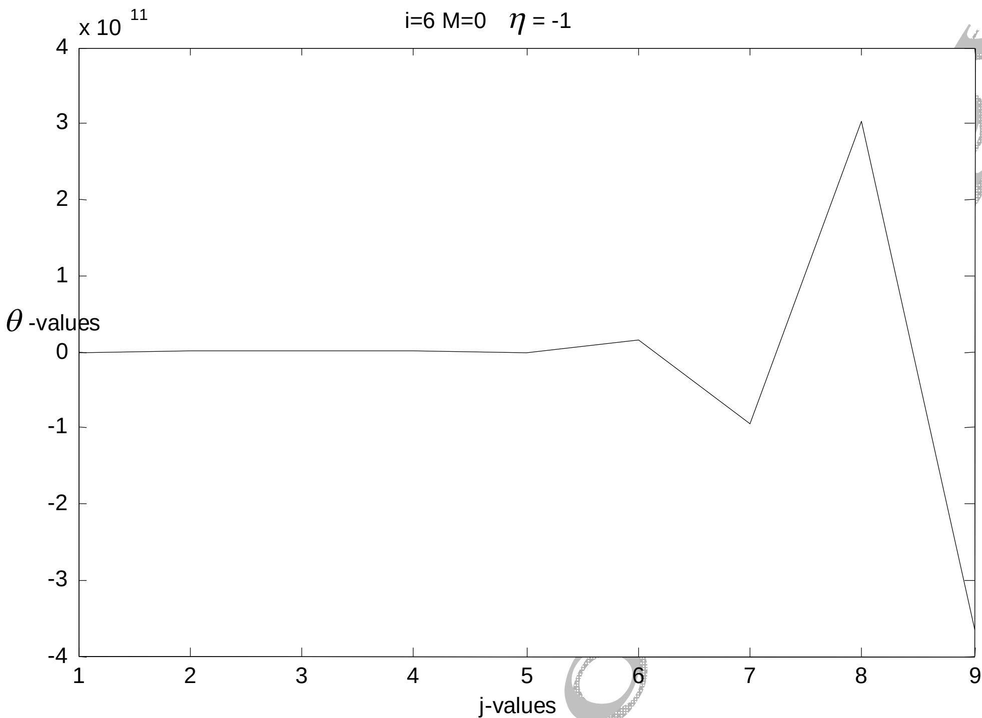

Fig-9: **Development of temperature profile in the thermal entrance region** M=0 η = -1

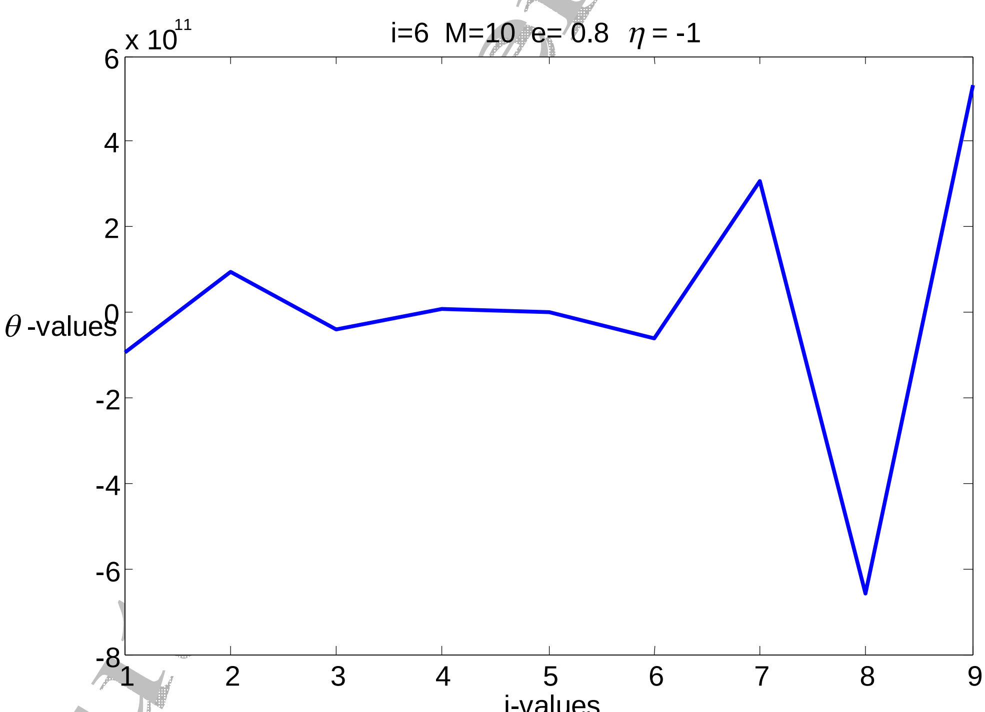

Fig-10: **Development of temperature profile in the thermal entrance region** M=10 η = -1

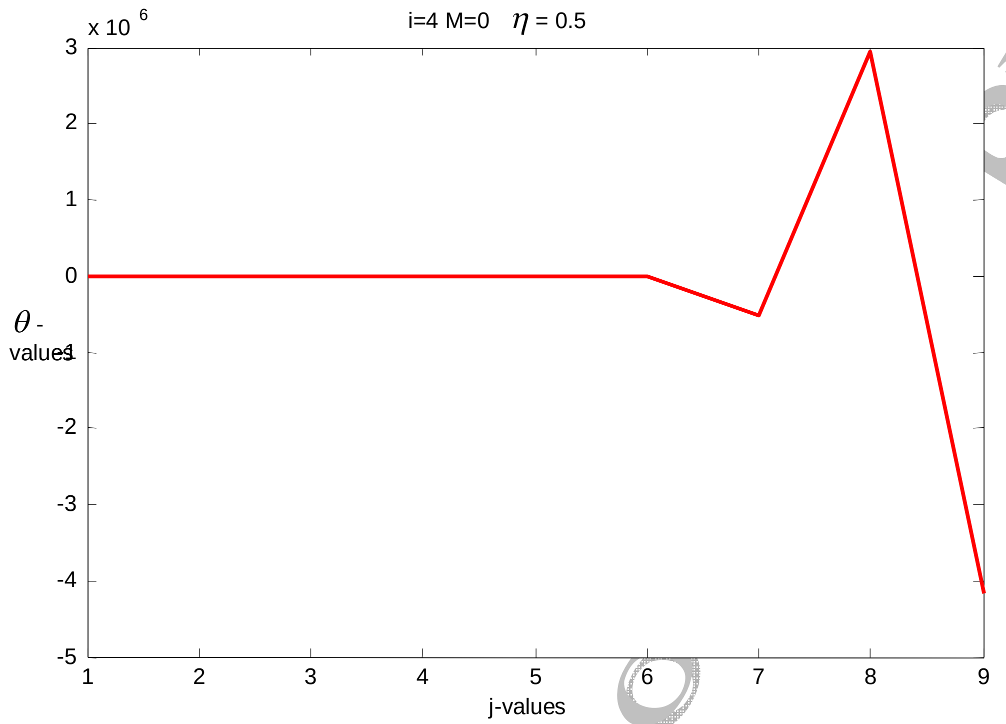

Fig-11:Development of temperature profile in the thermal entrance region M=0 η = 0.5

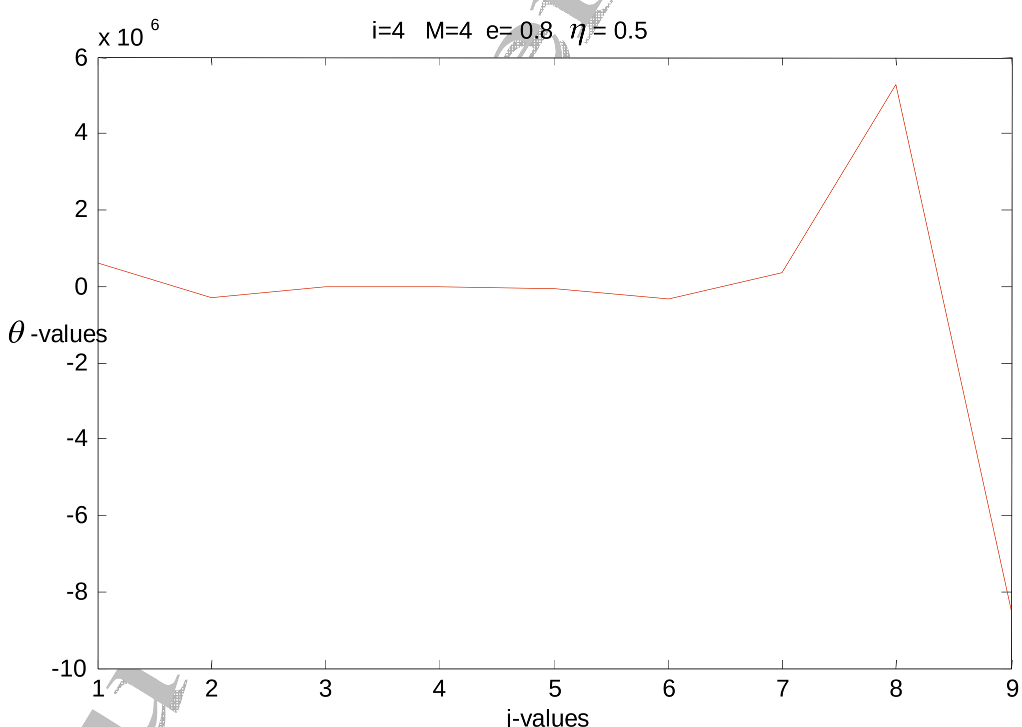

Fig-12: Development of temperature profile in the thermal entrance region M=4 η =0.5

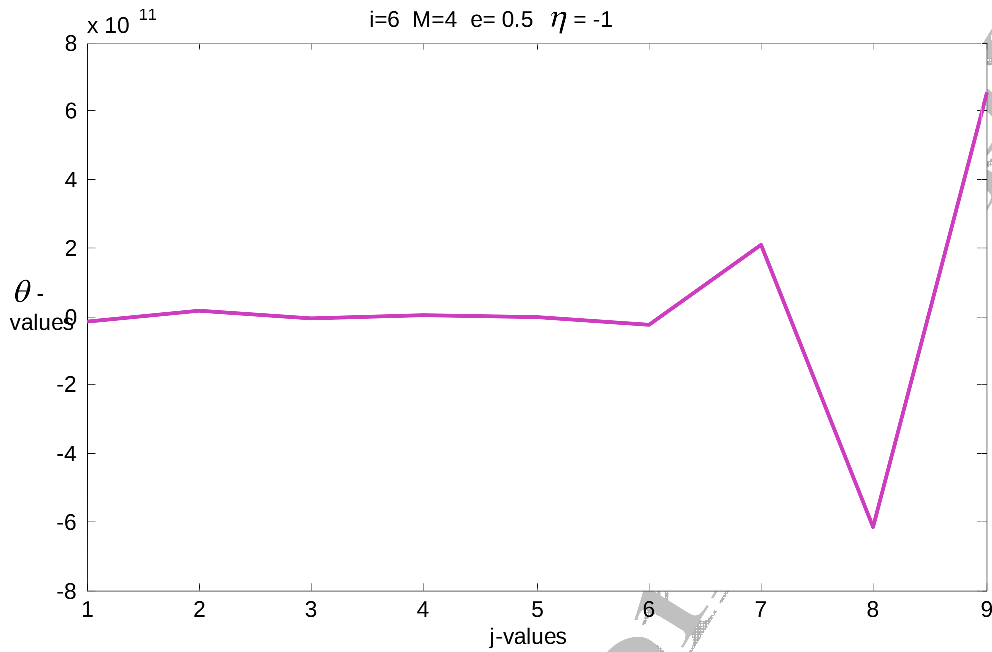

Fig-13: Development of temperature profile in the thermal entrance region M=4 η = -1

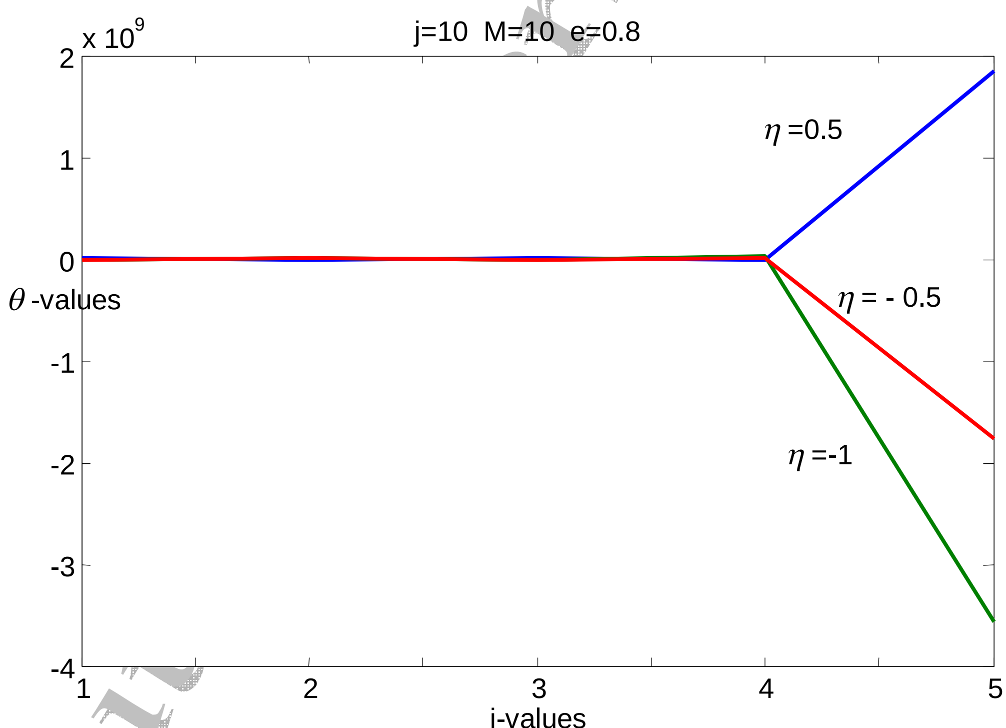

Fig-14: variation of wall temperature at M=10,e=0.8

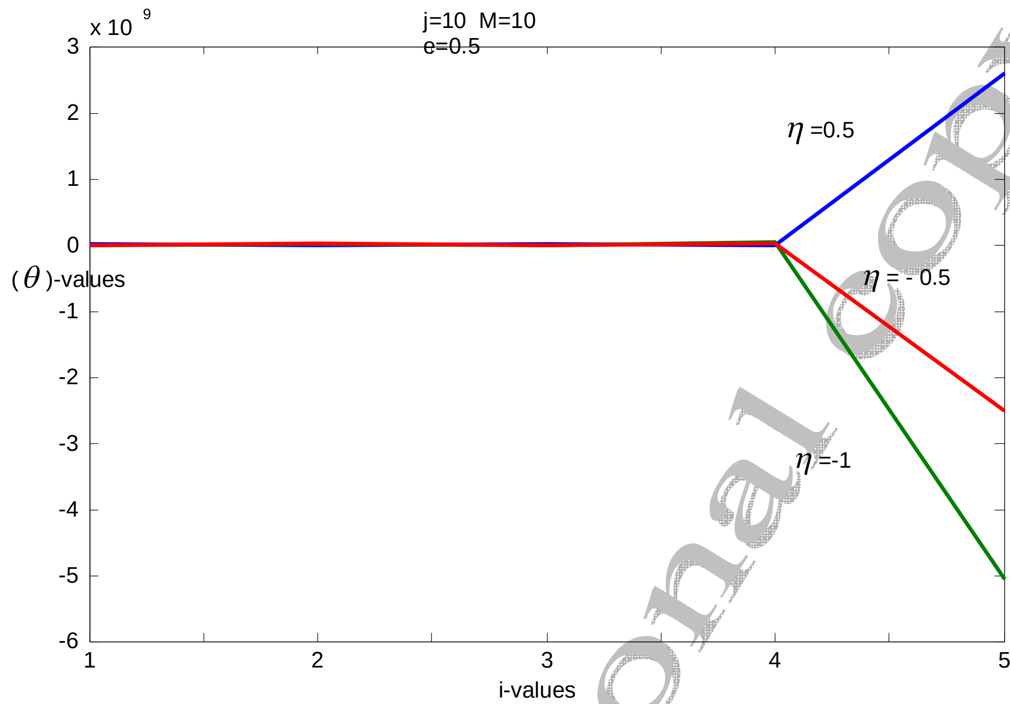

Fig-15:variation of wall temperature at M=10,e=0.5

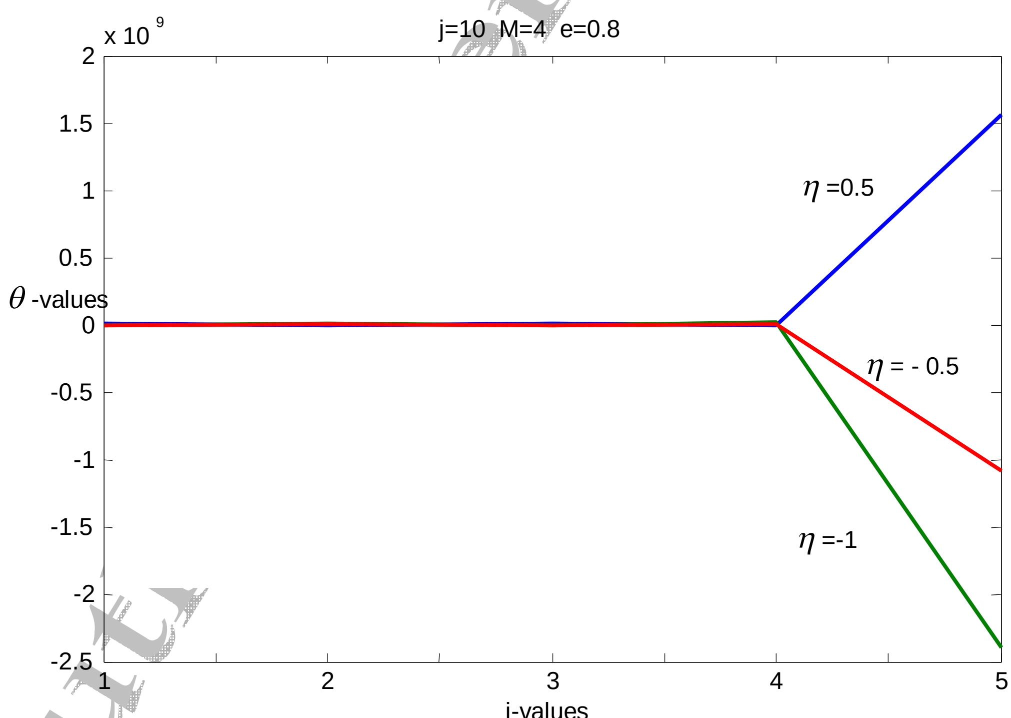

Fig-16: variation of wall temperature at M=4,e=0.

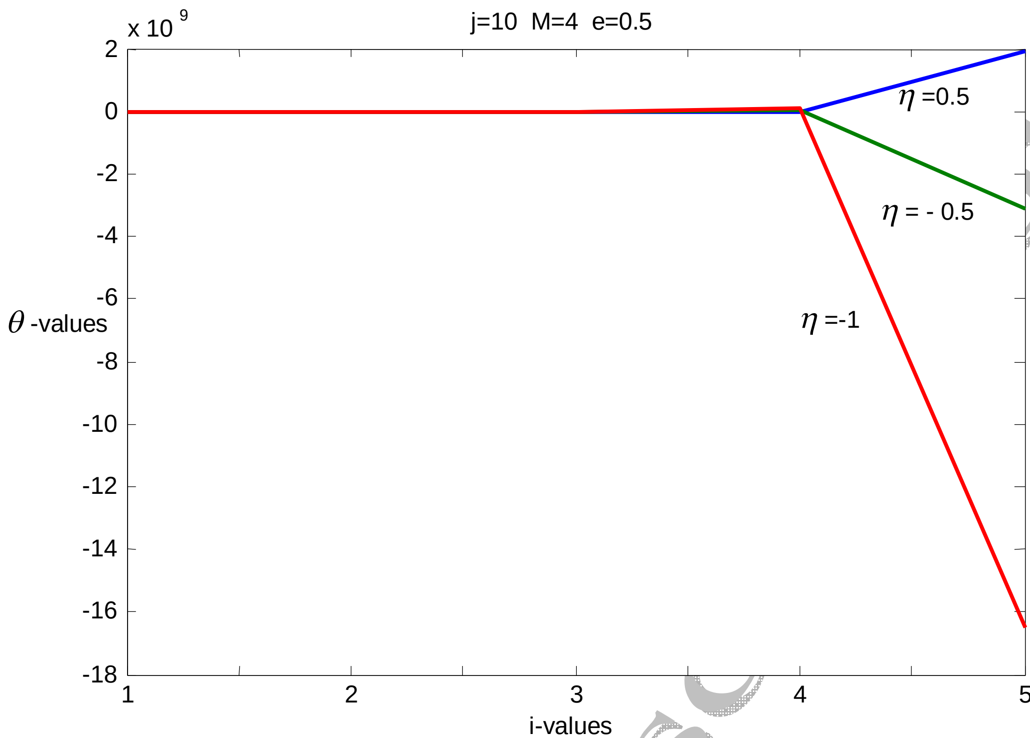

Fig-17: variation of wall temperature at M=4,e=0.5

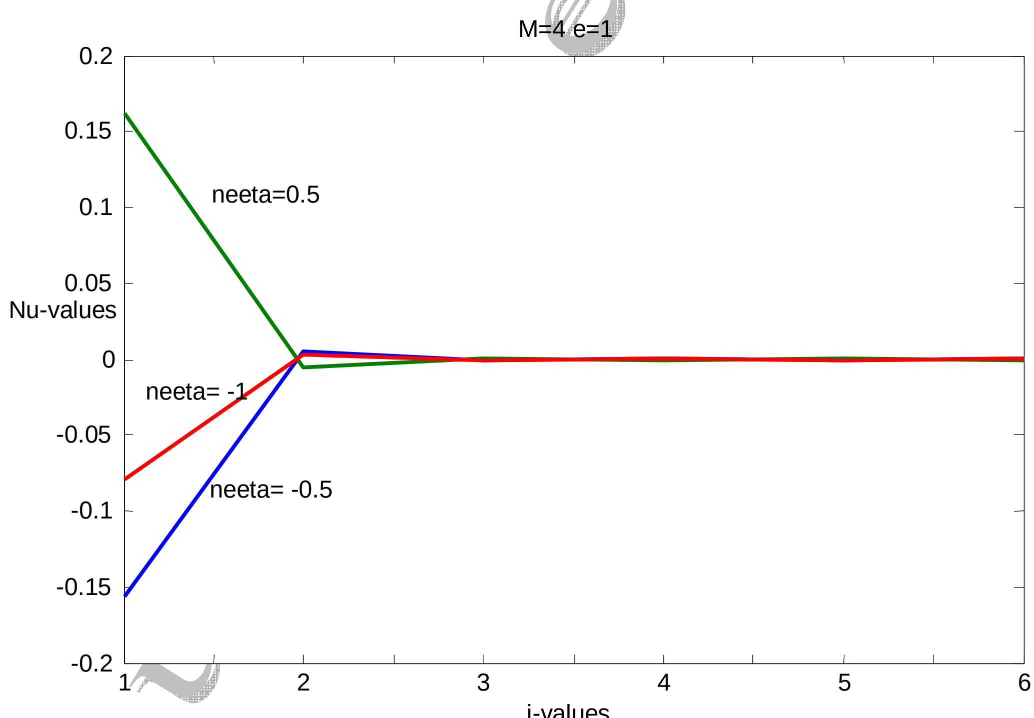

Fig-18:Pseudolocal nusselt numbers, M=4,e=1

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