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EFFECTS OF SUCTION AND INJECTION ON THE UNSTEADY FREE CONVECTION FLOW AND HEAT TRANSFER FROM A POROUS VERTICAL FLAT PLATE WITH CONSTANT HEAT FLUX

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ABSTRACT

The effect of suction and injection on unsteady natural convection boundary layer flow and heat transfer from a porous vertical flat plate with constant heat flux is investigated. The unsteadiness in the flow is caused due to constant heat flux between plate and ambient fluid. The governing partial differential equations are solved by an implicit finite difference method. It is found that, the unsteadiness and heat flux has significant effects on both skin friction parameter and surface temperature. In fact, these parameters are strongly affected by initial transient time – dependent flow. With the increase of stream wise coordinate, the velocity and temperature found to decrease in the case of suction while they increase in the case of injection.

INTRODUCTION

The problem of natural convection flows of viscous incompressible fluids past a vertical flat plate have received a great deal of attention in recent years because of its many practical applications, such as in electronic components, chemical processing equipment etc. Numerous works have been under taken by many researchers on this problem under different situations. Harries.et.al. Investigated the transient free convection from a vertical plate when the plate temperature is suddenly changed, obtaining an analytical solution and numerical solution until steady state reached. Kassem solved the problem for unsteady free – convection flow from a vertical moving plate subjected to constant heat flux. Pohlhausen did not consider viscous dissipation but obtained a solution employi the integral method. Polidori et.al. proposed a theoretical approach to the transient dynamic behavior of a natural convection boundary layer flow when a step variation of the uniform heat flux is applied using the Karman – Pohlhausen integral method. Other author's viz., Rees and Li et.al. studied the effect of the surface temperature oscillation. Recently Srinivasa et.al. studied an impulsive mixed convection MHD flow and heat transfer in the stagnation region of a vertical plate with constant heat flux.

The objective of the present paper is to study the effects of suction and injection on the unsteady free convection flow and heat transfer from a porous vertical flat plate with constant heat flux.

MATHEMATICAL MODEL

Consider a semi-infinite porous plate which is played vertical in a quiescentfluid of infinite extent. The plate is fixed in a vertical position with leading edge horizontal. The physical co-ordinates (x,y) are chosen such that x is measured from the leading edge in the stream wise direction and y is measured normal to the surface of the plate. Initially, the flow is assumed to be steady (i.e., at t=0) and at a certain instant of time (i.e., at t>0) the plate is subjected to constant heat flux in the direction normal to the surface. All fluid properties are considered to be constant, except for the density variation which induces the buoyancy force. Further, the fluid added (injection) or removed (suction) is the same as that involved in flow.

Under the aforesaid assumptions with Boussinesq's approximation, the equations governing the unsteady laminar two-dimensional boundary-layer flow are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\beta(T-T_\infty)+\nu\frac{\partial^2 u}{\partial y^2} \qquad (2)$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 u}{\partial y^2} \qquad (3)$$

The initial and boundary conditions are

$$t\le 0;\ u=0, v=0, T=T_\infty$$

$$x=0,\ y>0,\ t>0;\ u=0,\ T=T_\infty$$

$$y=0,\ x>0,\ t>0;\ u=0,\ v=-v_0(\,for\,suction),\ v=+v_0\,(\,for\,blowing\,)$$

$$t>0; u=0, -k\frac{\partial T}{\partial y}=q_w(x)=a,\ \ at\ y=0, a>0$$

$$y\to\infty,\ x>0,\ t>0;\ u=0,\ T=T_\infty \qquad (4)$$

Introducing the following transformations

$$\psi=\frac{\nu^4 g\beta\left(\frac{q_0}{k}\right)\xi^4}{V_0^4}\left[f(\eta,\xi,t^*)\pm\frac{\xi}{5}\right];\ \ t^*=\frac{V_o{}^2 t}{\nu\,\xi^2};\ T=T_\infty+\left(\frac{q_0}{k}\frac{\nu}{V_0}\xi\right)G(\eta,\xi,t^*)$$

$$\eta=\frac{V_o\, y}{\nu\xi};\qquad \xi=V_o\left(\frac{5x}{\nu^4 g\beta\left(\frac{q_0}{k}\right)}\right)^{1/5};\qquad u=\frac{\partial\psi}{\partial y};\ v=-\frac{\partial\psi}{\partial x} \qquad (5)$$

to Eqns.(1) – (3), we see that the continuity Eq.(1) is identically satisfied and Eqns.(2) – (3) reduces, respectively, to

$$F'' + G + 4fF' - 3F^2 \pm \xi F' - F_{t^*} = \xi\left(F F_\xi - F' f_\xi\right) \quad (6)$$

$$\Pr^{-1} G'' + 4fG' - FG \pm \xi G' - G_{t^*} = \xi\left(FG_\xi - G' f_\xi\right) \quad (7)$$

where

$$u = \frac{V_0^2 5x}{\nu \xi^2} F \;; \qquad v = -\frac{V_0}{\xi}\left(4f + \xi f_\xi - \eta F - 2t^* f_{t^*} \pm \xi\right)$$

$$f = \int_0^\eta F \, d\eta \;; \qquad \Pr = \frac{\nu}{\alpha} \quad (8)$$

It is remarked here that the mathematical model is designed with the help of unique transformation so that upper sign in Eqns. (6) and (7) is taken throughout for suction and the lower sign for blowing (injection).

The transformed boundary conditions are

F = 0; G = 1; G' = -1.0 at $\eta = 0$

F = 0; G = 0 as $\eta \to \infty$ for $\xi \ge 0$, $t^* \ge 0$ (9)

The surface temperature is given by

$$Q = \frac{x(q_o/k)}{T_w - T_\infty} = \frac{1}{G\xi} \quad (\xi \neq 0) \quad (10)$$

Here, u and v are velocity components in x and y direction; F is dimensionless velocity; T and G are dimensional and dimensionless temperatures, respectively; ξ, η, t^* are transformed co-ordinates; ψ and f are the dimension and dimensionless stream functions respectively; Pr is the Prandtl number; ν, α are respectively kinetic viscosity and thermal diffusivity; w_0 and ∞ denote conditions at the edge of the boundary layer on the wall at time t=0 and in the free stream respectively and prime $(')$ denotes derivatives with respect to η.

If we use the transformations

$$\psi = \frac{\nu^2 g\beta(T_{w0} - T_\infty)\xi^3}{V_0^3}\left[f(\eta,\xi,t^*) \pm \frac{\xi}{4}\right]; \quad T = T_\infty + \left(T_{wo} - T_\infty\right)G(\eta,\xi,t^*)$$

$$\xi = V_o\left[\frac{4x}{\nu^2 g\beta\left(T_{wo} - T_\infty\right)}\right]^{1/4}$$

in Eqns. (1) – (3) they reduces to

$$F'' + G + 3fF' - 2F^2 \pm \xi F' = \xi\left(F F_\xi - F' f_\xi\right) \quad (11)$$

$$\Pr^{-1} G'' + 3fG' \pm \xi G' = \xi\left(FG_\xi - G' f_\xi\right) \quad (12)$$

which are exactly same as those of Merkin for steady flow, while, they reduce to:

$$F'' + G + 3fF' - 2F^2 \pm \xi F' - F_{t^*} = \xi\left(F F_\xi - F' f_\xi\right) \quad (13)$$

$$\Pr^{-1} G'' + 3fG' \pm \xi G' - G_{t^*} = \xi\left(FG_\xi - G' f_\xi\right) \quad (14)$$

if we take the unsteady variable $t^* = \frac{V_o^2 t}{\nu \xi^2}$, which are exactly same as those of Jayakumar et.al. for constant wall temperature.

RESULTS AND DISCUSSION

The transformed coupled non – linear Eqns. (6) and (7) together with boundary conditions (9) are solved using a stable implicit finite difference method as described in Choi. To verify the problem the skin friction parameter and surface temperature ($F_w^{'}$, Q) for Pr = 1.0 are calculated by solving the Eqns. (11) and (12) for steady case and (13) and (14) for constant wall temperature. Our results are in excellent agreement with Merkin and Jayakumar et.al. For sake of brevity the comparison is not shown here.

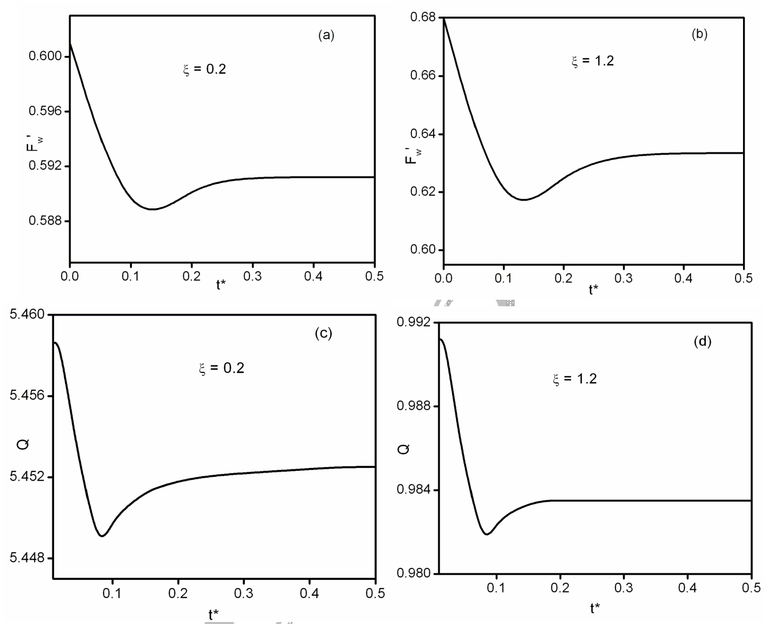

Fig.1. Skin friction parameter [1(a) – 1(b)] and Surface temperature [1(c) – 1(d)] results, for different values of ξ (Suction)

The variation of the skin friction parameter ($F_w^{'}$) and surface temperature (Q) with time t* for suction is displayed in Fig.2. As time increases, skin friction parameter $F_w^{'}$ increases with increase of ξ [Fig1(a) & 1(b)]. However, initially $F_w^{'}$ decreases for short time and then it increases with increase of time before attaining to a new steady state. The percentage of increase of $F_w^{'}$ is about 4.34% from ξ = 0.2 to ξ = 1.2 at t* = 0.5.

Fig. 1(c) and 1(d) shows that surface temperature Q decreases with the increase of ξ, which is opposite trend as compared to skin friction parameter $F_w^{'}$. This is because of the introduction of constant heat flux. The percentage of decrease of Q is about 447% at t* = 0.5 for 0.2 $\le$ ξ $\le$ 1.2.

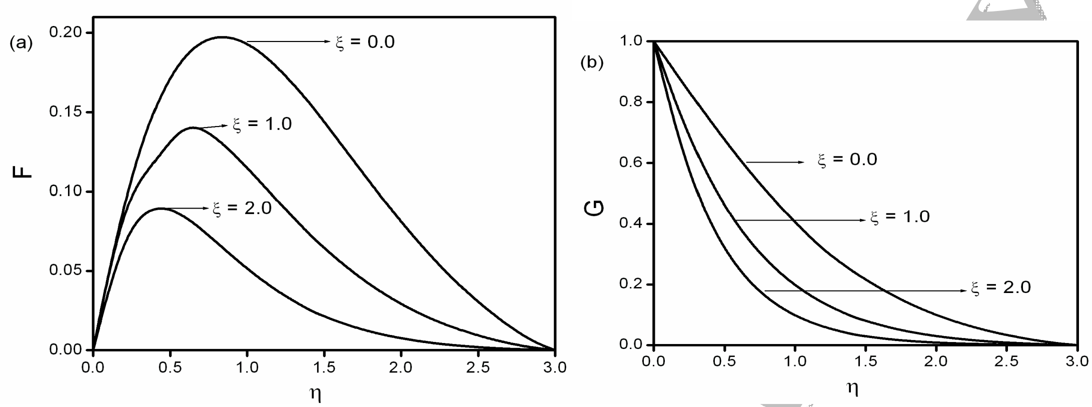

Fig.2. (a) Velocity and (b) Temperature profiles for different values of ξ at t* = 0.5 (Suction)

The velocity (F) and temperature (G) distributions obtained for suction are displayed in Fig.2. Relative to constant values of η, both the velocity and temperature decreases in magnitude as ξ increases. Hence, both the momentum and thermal boundary layer thickness decreases. In fact, the velocity boundary layer decreases about 10.39% from ξ = 0.0 to ξ = 2.0 at t* = 0.5 near η = 0.6. The corresponding decrease in the thickness of the thermal boundary layer is about 37.16% in the range 0.0 $\le \xi \le$ 2.0.

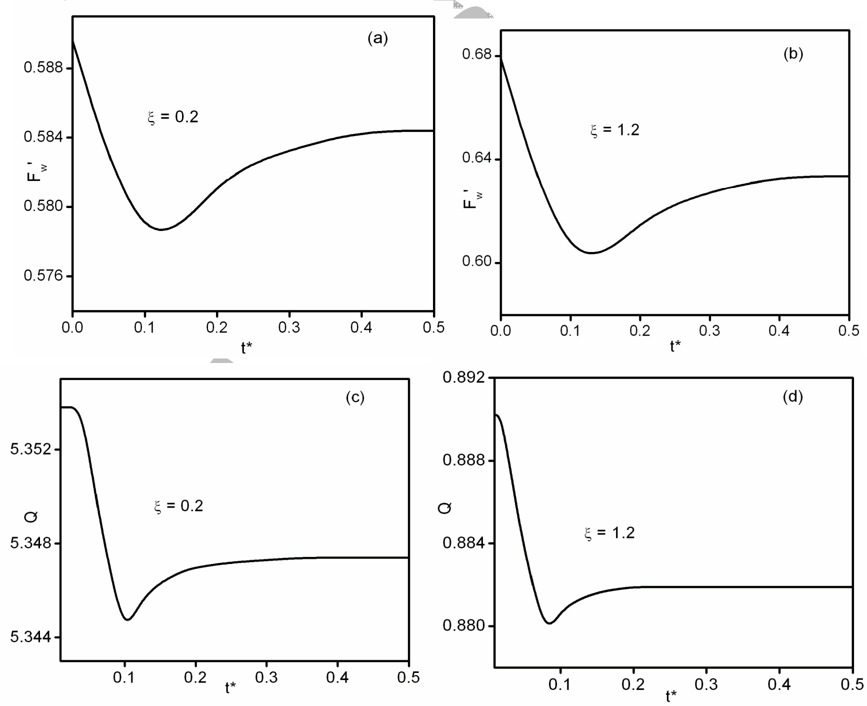

Fig.3. Skin friction parameter [3(a) – 3(b)] and Surface temperature [3(c) – 3(d)] results, for different values of ξ (Injection)

Fig.3. shows the corresponding results for skin friction parameter $F_w^{'}$ and surface temperature Q for different values of ξ in the case of injection. It is observed that the results are found to be qualitatively similar but quantitatively different as compared to suction. In fact, $F_w^{'}$ increases about 5.2% from ξ = 0.2 to ξ = 1.2 at t* = 0.5, while the percentage of decrease in Q is about 446% at t* = 0.5 in the range 0.2 $\le \xi \le$ 1.2.

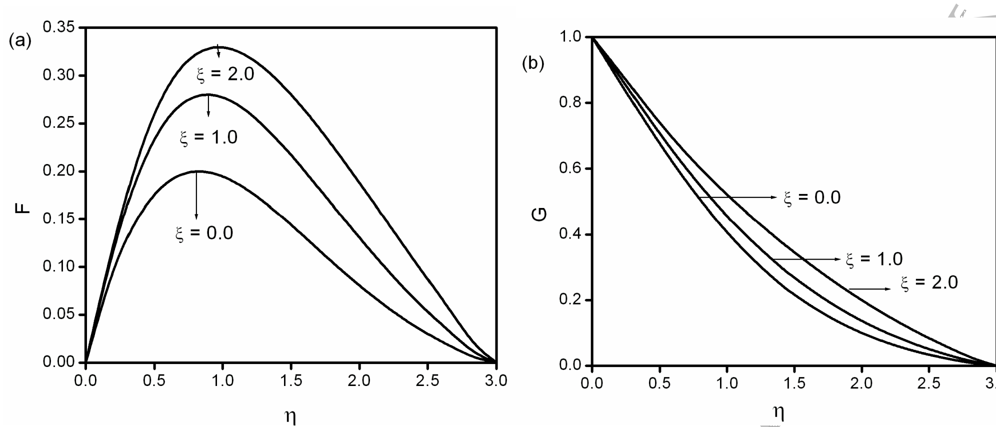

Fig.4. (a) Velocity and (b) Temperature profiles for different values of ξ at t* = 0.5 (Injection)

For the case of injection the velocity (F) and temperature (G) profiles for the values of ξ are displayed in Fig.4. It is observed that the velocity and temperature increases with increase of ξ. This results in the increase of both momentum and thermal boundary layer thickness. Indeed, the velocity boundary layer increases about 9.96% from $\xi = 0.0$ to $\xi = 2.0$ at $t^* = 0.5$ near $\eta = 0.6$, while the thermal boundary layer is about 7.41% in the range $0.0 \le \xi \le 2.0$.

CONCLUSIONS

The following conclusions are drawn from the present study of unsteady natural convection boundary layer flow and heat transfer from a porous vertical flat plate with constant heat flux:

(i) Both suction and injection increases skin friction parameter while, they reduce the surface temperature.

(ii) The momentum and thermal boundary layer thickness decreases along streamwise location in the case of suction, while the effect of injection is just opposite.

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