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Dufour Effect on Unsteady MHD Free-Convective Flow of a Dissipative Fluid Past a Vertical Porous Plate

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Abstract

In the present paper a numerical attempt is made to analyze the effect of Dufour number variation in the presence of viscous dissipation, on unsteady free convection flow of a electrically conducting, viscous fluid past an infinite vertical porous plate embedded in porous medium. A transverse magnetic field is applied normal to the flow. The governing non-linear coupled partial differential equations with boundary conditions are solved using Crank-Nicholson method, for which a numerical code is executed by C-Program. Graphical results for velocity, temperature and concentration fields and tabular values of Skin-friction and Nusselt numbers are presented and discussed. It is found that the velocity and Temperature ,Skin–friction and Nusselt number are observed to be increasing in the presence of viscous dissipation and for the increasing values of Dufour number.

Key words: MHD Flow; Vertical Plate; Dufour, viscous dissipation, , Crank-Nicholson method.

Heat transfer is the energy interaction due to a temperature differences in a medium or between media. Heat is not a storable quantity and is defined as energy in transit due to a temperature difference. In nature, many flows exist which are caused not only by the temperature differences but by concentration differences also. These mass transfer differences show the effect in the rate of heat transfer. In industries, many transport processes exist in which, heat and mass transfer takes place, simultaneously, as a result of combined buoyancy effect of thermal diffusion of chemical species. The phenomenon of heat and mass transfer has been the object of extensive research due to its applications in science and technology. Such phenomenon is observed in buoyancy-induced motions in the atmosphere, in bodies of water, quasi- solid bodies such as earth so on. Unsteady oscillatory free connective flows play an important role in chemical engineering, turbo machinery and aerospace technology. Such flows arise due to unsteady motion of a boundary or boundary temperature. Besides, unsteadiness may also be due to oscillatory free stream velocity and temperature. In the past decades an intensive research effort has been devoted to problems on heat and mass transfer in view of their application to astrophysics, geophysics and engineering. In addition, the phenomenon of heat and mass transfer frequently exists in chemical processed industries such as food processing and polymer production.

Eckert *et al* [1] have done pioneer work on heat and mass transfer. The equations governing the mass transfer phenomenon are complicated. However, Gebhart [2] simplified these equations by assuming the presence of species concentration at very low levels and made extensive studies on combined heat and mass transfer flow, to highlight the insight of the phenomenon. Due to importance of these flows, several authors [3-11] have studied the problems on free convection and mass transfer flow of a viscous fluid through porous medium. In these studies, the permeability of the porous medium is assumed to be constant. However, a porous material containing the fluid is a non-homogeneous medium and the porosity of the medium may not necessarily be constant. Shreekanth *et al* [13] studied the effect of permeability variation on free convective flow past a vertical porous wall in a porous medium when the permeability varies with time. Singh *et al* [14] studied hydro magnetic free convection and mass transfer flow of a viscous stratified fluid considering variation in permeability with direction. Acharya *et al* [12] discussed magnetic field effects on the free convection flow through porous medium with constant suction and constant heat flux.

Recently, Singh *et al* [15] studied the effects of permeability variation and oscillatory suction velocity on free convection and mass transfer flow of a viscous fluid past an infinite vertical porous plate to a porous medium when the plate is subjected to a time dependent suction velocity normal the plate in the presence of a uniform magnetic field.The permeability of the porous medium is considered to be $K(t') = K_0(1+\varepsilon e^{in't'})$ and the suction velocity is assumed to be $V(t') = V_0(1+\varepsilon e^{in't'})$ where $V_o>0$ and $\varepsilon<<1$ is a positive constant.

In all the above stated problems, the effect of Soret and Dufour on the flow field has not been studied. Such effect is significant when density differences exist in the flow regime. For example when species are introduced at a surface in fluid domain, with different (lower) density than the surrounding fluid, both Soret and Dufour effects can be significant. Also, when heat and mass transfer occur simultaneously in a moving fluid, the relations between the fluxes and the driving potentials are of more intricate nature. It has been found that an energy flux can be generated not only by temperature gradients but also by composition gradients. The mass fluxes can be created by temperature gradients is called the Soret effect. The thermal-diffusion (Soret) effect, for instance, has been utilized for isotope separation and in mixture between gases with very light molecular weight (H2, He) and of medium molecular weight (N2, air). The thermal-diffusion effect was found to be of a considerable magnitude such that it cannot be ignored (Eckert and Drake [1]). In view of the importance of this effect, Jha and Singh [18] studied the free-convection and mass transfer flow about an infinite vertical flat plate moving impulsively in its own plane, taking into account the Soret effects. Kafoussias [19] studied the same problem in the case of MHD flow. Srihari et al [20] analyzed the Soret number variation on free convection hydromagnetic flow flow of a viscous, electrically conducting fluid, past an infinite vertical porous plate with oscillatory suction velocity with heat sink. Anand Rao et al. [21] analysed the effect of Soret number on an unsteady two-dimensional laminar mixed convective boundary layer flow of a chemically reacting fluid, along a semi-infinite vertical permeable moving plate with viscous dissipation. Srihari and Kesavareddy [22] have made the investigation to study the effects of Soret and Magnetic field on unsteady laminar boundary layer flow of a radiating and chemically reacting incompressible viscous fluid along a semi-infinite vertical plate.

Fluid supports an exothermic chemical or nuclear reaction is very common today and the correct processes design requires accurate correlation for the heat transfer coefficients at the boundary surfaces. Despite of its increasing importance in technological and physical problems, the magneto-hydrodynamic flow of a dissipative fluid past an infinite plate have received much attention because of the non-linearity of the governing equations.

In most of the earlier stated studies analytical or perturbation methods were applied to obtain the solution of the non linear problem. However, in the present paper a numerical attempt is made to study the effect of Dufour in the presence of viscous dissipation on free convection flow of an incompressible, viscous electrically conducting fluid past an infinite vertical porous plate. A magnetic field of uniform strength is applied normal to the fluid flow. In order to obtain the approximate solution and to describe the physics of the problem, the present non-linear boundary value problem is solved numerically using Crank-Nicholson method. The present study is used as a bridge to fill the knowledge gap among the researchers.

Mathematical Formulation of the Problem

Unsteady free-convection flow of an incompressible, electrically conducting viscous fluid, past an infinite vertical plate embedded with porous medium is considered. In Cartesian coordinate system, let x'-axis be along the plate in the direction of the flow and y'-axis normal to it. A uniform magnetic field is introduced normal to the direction of the flow. In addition, the analysis is based on the assumptions. (i) the plate temperature and species concentration are instantly raised to $T' = T'_w(1+\varepsilon e^{in't'})$ and $C' = C'_w(1+\varepsilon e^{in't'})$ are maintained as such; (ii) the fluid properties are not affected by the temperature differences except that of the density in the body force term; (iii) the influence of density variations in other terms of the momentum, energy and concentration equations and the variation of the expansion coefficient with temperature is negligible;(iv) the magnetic Reynolds number is much less than unity so that, the induced magnetic field is neglected; (v) the plate is electrically non conducting so that, the equation of conservation of electric charge $\vec{\nabla}.\vec{J} = 0$ given by $J_{y'}$ =constant = 0, everywhere in the flow; (vi) the joule heating effect and viscous dissipation terms have been neglected.

Within the above mentioned framework, under usual Boussinesq's approximation, the equations relevant to the problem are:

$$\frac{1}{4}\frac{\partial u}{\partial t} - (1+\varepsilon\, e^{int)}\,\frac{\partial u}{\partial y} = Gr\,T + Gm\,C + \frac{\partial^2 u}{\partial y^2} - \frac{u}{K_0(1+\varepsilon e^{int})} - M^2 u \qquad (1)$$

$$\frac{1}{4}\frac{\partial \theta}{\partial t} - (1+\varepsilon\, e^{int)}\,\frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} + Du\frac{\partial^2 \phi}{\partial y^2} + Ec\left(\frac{\partial u}{\partial y}\right)^2 \qquad (2)$$

$$\frac{1}{4}\frac{\partial \phi}{\partial t} - (1+\varepsilon\, e^{int)}\,\frac{\partial \varphi}{\partial y} = \frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2} \qquad (3)$$

The relevant boundary conditions in dimensionless form are:

$$u = 0,\ \theta = 1 + \varepsilon e^{int},\ \Phi = 1 + \varepsilon e^{int} \quad \text{at } y = 0. \tag{4}$$

$$u \to 0,\ \theta \to 0,\ \Phi \to 0 \quad \text{as } y \to \infty.$$

The non-dimensional quantities introduced in the above equations are defined as:

$$y = \frac{v_0 y'}{4v},\quad t = \frac{v_0^2 t'}{4v},\quad n = \frac{4\upsilon n'}{v_0^2},\quad u = \frac{u'}{v_0},$$

$$\theta = \frac{T' - T_\infty}{T_w - T_\infty},\quad \phi = \frac{C' - C_\infty}{C_w - C_\infty}.$$

$S = \frac{S'\upsilon}{v_o^2}$ (heat source parameter), $Gr = \frac{\upsilon g \beta^* (T_w - T_\infty)}{v_0^3}$ (Grashof number),

$Sc = \frac{\vartheta}{D}$ (Schmidt number), $\Pr = \frac{\mu C_p}{K_T}$ (Prandtl number),

$M = \frac{B_0}{v_0}\sqrt{\frac{\sigma\upsilon}{\rho}}$ (Magnetic parameter), $Du = \frac{D_m k_T (C_w - C_\infty)}{\upsilon Cs\, Cp (T_w - T_\infty)}$ (Dufour number)

$Gm = \frac{\upsilon g \beta (C_w - C_\infty)}{v_0^3}$ (Modified Grashof number),

$K_0 = \frac{K_0' v_0^2}{\upsilon^2}$ (Constant Permeability of the medium)

$Ec = \frac{U_0^2}{C_p (T_w - T_\infty)}$ (Eckert number)

Method of Solution

The equations (1)-(3) are coupled, non-linear partial differential equations. Obtaining exact solution of these equations under the given conditions is very difficult. So the problem is solved numerically as follows [15].

Using the following finite difference formulae

$$\frac{\partial f}{\partial t} = \frac{f_i^{j+1} - f_i^j}{\Delta t},\qquad \frac{\partial f}{\partial y} = \frac{f_{i+1}^j - f_i^j}{\Delta y}$$

$$\frac{\partial^2 f}{\partial \eta^2} = \frac{1}{2}\left(\frac{f_{i-1}^j - 2 f_i^j + f_{i+1}^j}{(\Delta y)^2} + \frac{f_{i-1}^{j+1} - 2 f_i^{j+1} + f_{i+1}^{j+1}}{(\Delta y)^2} \right),$$ where f stands u, θ and ϕ

into the equations (1),(2) and (3) and simplifying according to the **Crank and Nicholson method** , we get the following system of equations

$$A_1 u_{i-1}^{j+1} + B_1 u_i^{j+1} + A_1 u_{i+1}^{j+1} = C_{1i}^{j} \quad (5)$$

$$A_2 \theta_{i-1}^{j+1} + B_2 \theta_i^{j+1} + A_2 \theta_{i+1}^{j+1} = C_{2i}^{j} \quad (6)$$

$$A_3 \phi_{i-1}^{j+1} + B_3 \phi_i^{j+1} + A_3 \phi_{i+1}^{j+1} = C_{3i}^{j}, \quad (7)$$

Where

$$C_{1i}^{j} = (r/2)u_{i-1}^{j} + (-r - r\Delta yp - \Delta t/k_0 p - \Delta t M^2 + 1/4)u_i^j + (r\Delta yp + r/2)u_{i+1}^{j} + \Delta t G_r T_i^j + \Delta t G_m C_i^j$$

$$C_{2i}^{j} = (r/2)\theta_{i-1}^{j} + (P_r/4 - P_r r\Delta yp - r - S\,\mathrm{Pr}\,\Delta t)\theta_i^j + (P_r r\Delta yp + r/2)\theta_{i+1}^{j}$$

$$C_{3i}^{j} = (r/2)\phi_{i-1}^{j} + (S_c/4 - S_c r\Delta yp - r - Ch\;Sc\;\Delta t)\phi_i^j + (S_c r\Delta yp + r/2)\phi_{i+1}^{j}$$

A_1=-r/2, B_1=r+1/4, A_2=-(r)/2, B_2=(r)+ P_r /4 , A_3=-r/2, B_3=r+ S_c /4

And $r = \Delta t/(\Delta y)^2$, $p = 1 + \varepsilon e^{int}$, Δy and Δt are mesh sizes along space and time direction respectively.

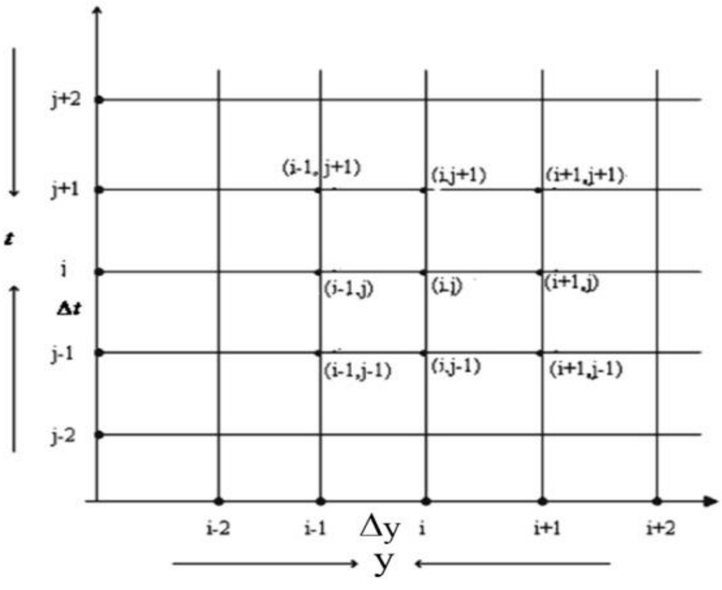

Diagram 1: Grid meshing for finite-difference scheme

To obtain the difference equations, the region of the flow is divided into a grid or mesh of lines parallel to η and t axes. Solutions of difference equations are obtained at the intersection of these mesh lines called nodes. The finite-difference equations at every internal nodal point on a particular *n*-level constitute a tri-diagonal system of equations. These equations are solved by using the Thomas algorithm [17]. In order to prove the convergence of finite difference scheme, the computation is carried out for slightly changed values of $\Delta\eta$ and Δt, running same program. Negligible change is observed in the values of u, θ and ϕ and also after each cycle of iteration the convergence checking is performed, i.e. $\left|u^{n+1}-u^{n}\right|<10^{-8}$ is satisfied at all points. Thus, it is concluded that, the finite difference scheme is convergent and stable.

Skin-friction,

Skin-friction Coefficient (τ) at the plate is

$$\tau=\left(\frac{\partial u}{\partial y}\right)_{y=0} \quad (8)$$

Nusselt number

Nusselt number (N_u) at the plate is

$$N_u=\left(\frac{\partial T}{\partial y}\right)_{y=0} \quad (9)$$

Results and Discussion

In order to obtain the approximate solution and to describe the physics of the problem, in the present work, numerical solution is obtained. The numerical calculations for the distribution of the velocity, temperature, Skin-friction coefficient and Nusselt number across the boundary layer for various values of the parameter have been carried out. In the current section Dufour and viscous dissipation effects on the flow field are discussed and analysed.

The effect of Dufour number variation on velocity and temperature field is shown in figures (1) and (5) respectively. Dufour number signifies the contribution of the concentration gradients to the thermal energy flux in the flow. It is observed that an increase in the value of Dufour number, leads to increase in the temperature and velocity of the fluid. It is also observed from figure (5) that the thermal boundary layer thickness increases as the value of Dufour number increases. This due to the fact that as Dufour number increases, thermal acceleration comes into play causing the enhancement of fluid velocity. Further, it is noted that the velocity and concentrations profiles are found to be more sensible to the changes with the variation of Dufour number.

Figures (2) and (6) show the effect of Eckert number on velocity and temperature profiles respectively. The analysis of these figures reveals that the an increase in Eckert number leads increase in the temperature. Consequently the velocity of its particles also increases with the increasing values of Eckert number as heat energy is stored in the fluid due to the frictional heating. Fig (3) shows the effect of magnetic parameter M on velocity field u. It is observed from figure that the velocity of the fluid flow decreases with the increasing values of M. This due to the fact that the introduction of transverse magnetic field in an electrically conducting fluid has a tendency to give rise to a resistive-type force called the Lorentz force, which acts against the fluid flow and hence results in retarding the velocity profile. Figure (4) show that an increase in Gr and Gm leads to increase in the velocity of the flow. This is due to the fact that with the increasing values of thermal Grashof number and mass Grashof number has the tendency to increase the thermal and mass buoyancy effect. This gives rise to an increase in the induced flow.

Skin-friction coefficient (τ) values are shown in table (1) for various values of Du, Ec, Gr, Gm, M, Sc, Pr, and K_0, in the case of cooling of the plate. From this table it is oserved that an increase in Du, Ec, Gr, Gm and K_0 leads to increase in the Skin-friction but an increase in M, Pr, and Sc leads to decrease in the Skin-friction. Table (2) shows the Nusselt number (N_u). From this table it is noted that an increase in source parameter Ec and Du, leads to increase in the the Nusselt number. But an increase in Pr decreases the Nusselt number (N_u).

Conclusions

Temperature and Nusselt number increase as the value of Eckert number increases. It is explained by the fact that heat energy is stored in the fluid due to the frictional heating between fluid and plate. Therefore Velocity, Skin–friction and Nusselt number increase in the presence viscous dissipation.

(1) The Temperature profile is severely affected by the Dufour effect i.e for increasing values of Dufour parameter, there is a considerable enhancement in the velocity of the fluid is observed.

Nomenclature

u	velocity along the x-axis
K_T	Thermal conductivity
υ	kinematic coefficient of viscosity
β	coefficient of volume expansion for the heat transfer
β^*	volumetric coefficient of expansion with species concentration
T_∞	fluid temperature at infinity
C_∞	species concentration at infinity,
D	chemical molecular diffusivity
K_0	constant permeability of the medium
μ	coefficient of viscosity
C_p	specific heat at constant pressure
n	frequency of oscillation
ρ	density of the fluid

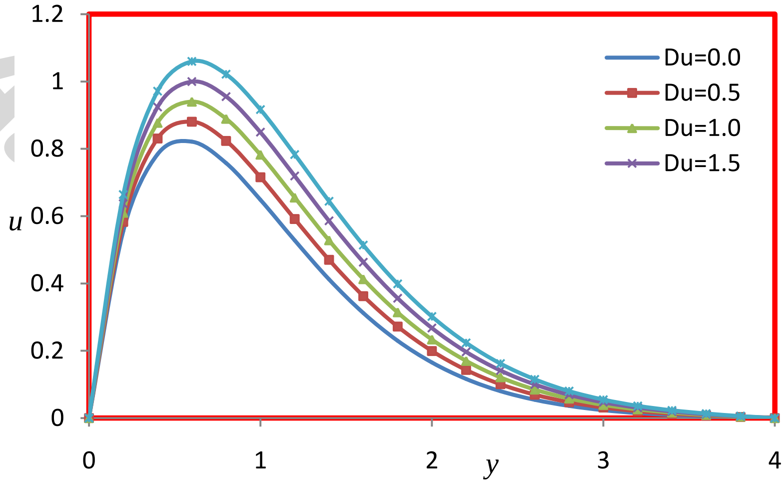

Fig1: Effect of Dufour Du on velocity field u
(G_r=5.0,Gm=5.0, M=0.5,Sc=0.22,Pr=0.71,K_o=1.0,Ec=0.5, ε =0.005 and nt=π/2)

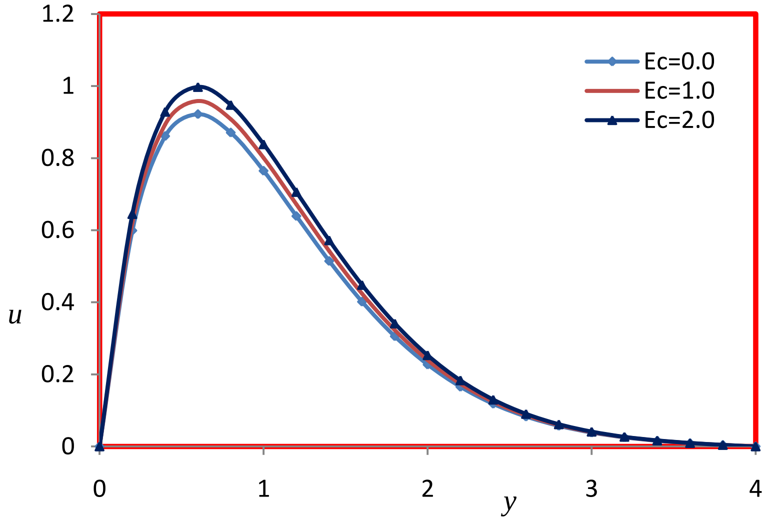

Fig 2: Effect of viscous dissipation on velocity field u
(G_r=5.0,Gm=5.0, M=0.5,Sc=0.22,Pr=0.71,K_o=1.0,Du=1.0, ε =0.005 and nt=π/2)

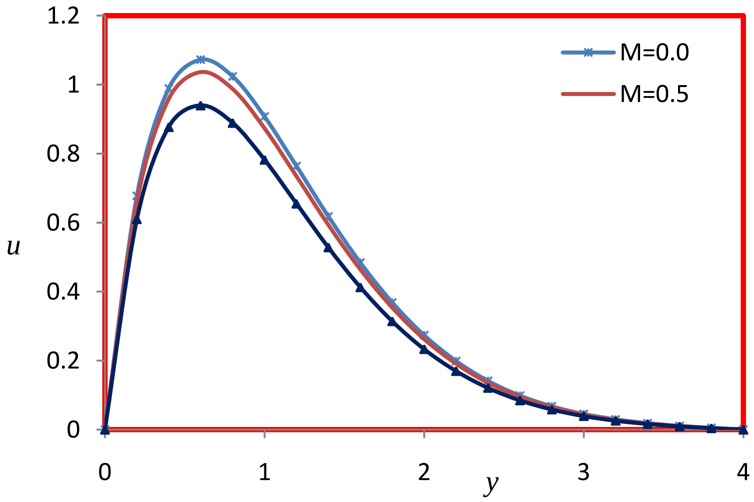

Fig 3: Effect of Magnetic parameter M on velocity field u
(G_r=5.0,Gm=5.0, Sc=0.22, Pr=0.71, Du=1.0, K_o=1.0, Ec=0.5, ε =0.005 and nt=π/2)

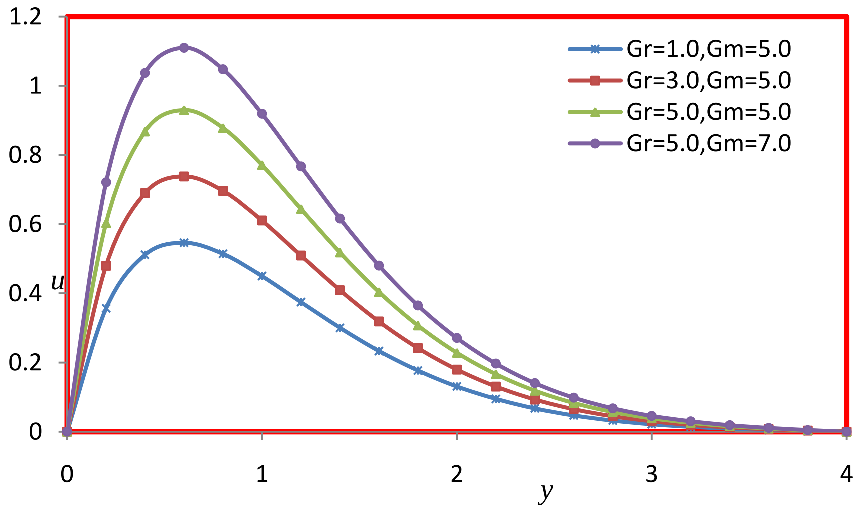

Fig 4: Effect of Gr and Gm on velocity field u
(M=1.0, Du=1.0,Sc=0.22,Pr=0.71,K_o=1.0,Ec=0.5, ε =0.005 and nt=π/2)

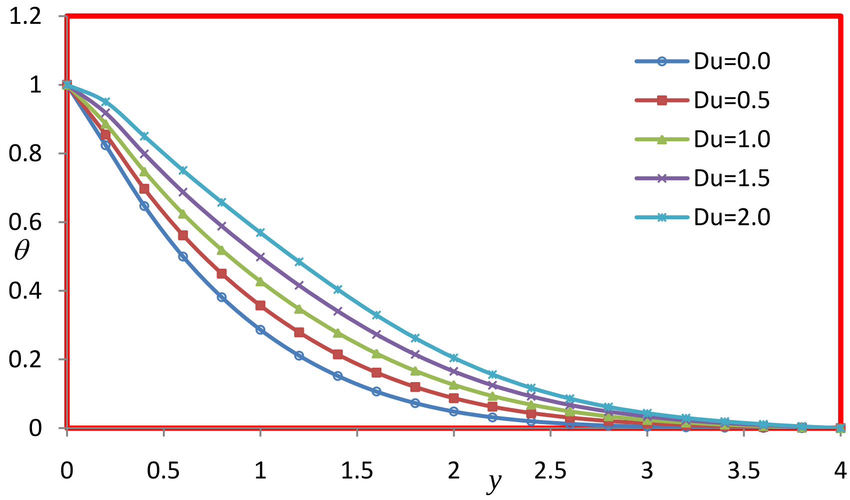

Fig 5: Effect Dufour Du on temperature field θ
(Gr=5.0, Gm=5.0, Pr=0.71, M=1.0, Ec=0.5, K_o=1.0, ε=0.005 and nt =π/2)

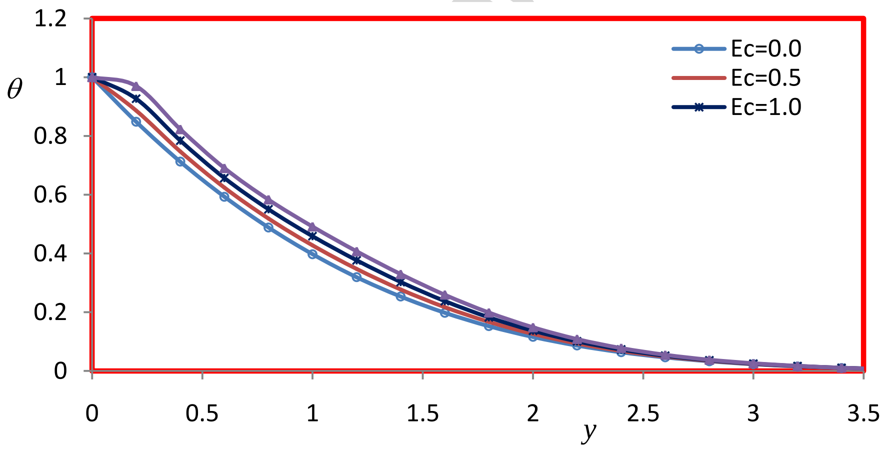

Fig 6: Effect of viscous dissipation Ec on temperature field θ
(Gr=5.0, Gm=5.0, Pr=0.71, M=1.0, Du=1.0, K_o=1.0, ε=0.005 and nt =π/2)

Table1: Skin-friction coefficient (τ)

Gr	Gm	M	Du	Pr	Sc	K_0	Ec	τ
5.0	5.0	1.0	0.0	0.71	0.22	0.5	0.5	3.301038
5.0	5.0	1.0	0.5	0.71	0.22	0.5	0.5	3.442620
5.0	5.0	1.0	1.0	0.71	0.22	0.5	0.5	3.584781
10.0	5.0	1.0	0.5	0.71	0.22	0.5	0.5	5.257878
5.0	10.0	1.0	0.5	0.71	0.22	0.5	0.5	5.282882
5.0	5.0	2.0	0.5	0.71	0.22	0.5	0.5	2.726658
5.0	5.0	1.0	0.5	0.71	0.22	2.0	0.5	3.605184
5.0	5.0	1.0	0.5	0.71	0.22	0.5	1.0	3.499813
5.0	5.0	1.0	0.5	7.0	0.22	0.5	0.5	2.797158
5.0	5.0	1.0	0.5	0.71	0.66	0.5	0.5	3.222829

Table2: Nusselt number Nu for Gr=5.0, Gm=5.0,M=1.0, K_0=1.0

Pr	Du	Ec	Nu
0.71	0.0	0.5	-0.831495
0.71	1.0	0.5	-0.491660
0.71	1.0	1..0	-0.173085
7.0	1.0	0.5	-0.637034
0.71	1.0	0.5	-0.627205

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