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DUFOUR AND SORET EFFECT ON UNSTEADY MHD FREE CONVECTION FLOW PAST A SEMI-INFINITE MOVING VERTICAL PLATE IN A POROUS MEDIUM WITH VISCOUS DISSIPATION

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Abstract:

This paper investigates the Dufour and Soret effects on unsteady MHD free convection flow past a semi-infinite moving vertical plate in a porous medium considering the viscous dissipation effects. The resulting governing equations are transformed into non linear ordinary differential equations using similarity transformation. The set of non linear ordinary differential equations are first linearized by using Quasi-linearization technique and then solved numerically by using implicit finite difference scheme. Then the system of algebraic equations is solved by using Gauss-Seidal iterative method. The solution is found to be mainly dependent on six governing parameters including the magnetic field parameter M, the suction parameter v_0, Soret number Sr, Dufour number Du, Eckert number Ec, and Darcian parameter Da. Numerical results are tabulated for the local Nusselt number and Sherwood number. Velocity, Temperature and Concentration profiles drawn for different controlling parameters reveal the tendency of the solution.

Keywords**:** Magnetic field effects, Soret and Dufour effects, suction and viscous dissipation.

1. INTRODUCTION

Combined heat and mass transfer (or double-diffusion) in fluid-saturated porous media finds applications in a variety of engineering processes such as heat exchanger devices, petroleum reservoirs, chemical catalytic reactors and processes, geothermal and geophysical engineering moisture migration in a fibrous insulation and nuclear waste disposal and others. Double diffusive flow is driven by buoyancy due to temperature and concentration gradients. Bejan and Khair [1] (Bejan and Khair 1985) investigated the free convection boundary layer flow in a porous medium owing to combined heat and mass transfer.

Magneto hydrodynamic flows have applications in meteorology, solar physics, cosmic fluid dynamics, astrophysics, geophysics and in the motion of earth's core. In addition from the technological point of view, MHD free convection flows have significant applications in the field of stellar and planetary magnetospheres, aeronautical plasma flows, chemical engineering and electronics.

Raptis [2] (Raptis 1986) studied mathematically the case of time varying two dimensional natural convective flow of an incompressible, electrically conducting fluid along an infinite vertical porous plate embedded in a porous medium. Elabashbeshy [3] (Elabashbeshy 1997) studied heat and mass transfer along a vertical plate in the presence of magnetic field. Chamkha and Khaled [4] (Chamkha and Khaled 2001) investigated the problem of coupled heat and mass transfer by magneto hydrodynamic free convection from an inclined plate in the presence of internal heat generation or absorption.

In the combined heat and mass transfer processes, it is known that the thermal energy flux resulting from concentration gradients is referred to as the dofour or diffusion–thermal effect. Similarly, the soret or thermodiffusion effect is the contribution to the mass fluxes due to temperature gradients. The dofour and soret effects may be significant in the areas of geosciences and chemical engineering. kafoussias and Williams [5] employed the finite difference method to examine the dofour and soret effects on mixed free-forced convective heat and mass transfer along a vertical surface, various other influences that have been considered include magnetic field [6], variable suction [7], and chemical reaction [8].

Various other aspects dealing with the soret effect on the combined heat and mass transfer problems have been also studied. For example, joly et al [9] used the brinkman-extended Darcy model to examine the effect of the soret effect on the onset of convective instability. Mojtabi et al [10] presented, making use of as pseudo-spectral chebyshev collocation method, a stability analysis of the influence of vibration on soret-driven convection in porous media. Bourich et al [11] carried out an analytical and numerical study of the onset of soret convection in a horizontal porous layer subjected to a uniform vertical magnetic field.

However in the existing convective heat transfer literature on the non-Newtonian fluids, the effect of the viscous dissipation has been generally disregarded. Gnaneswara and Bhasker Reddy [12] have studied the effects of soret and dufour on steady MHD free convection flow in a porous medium with Viscous dissipation. Kishan and Shashidar Reddy [13] have studied the MHD effects on non-Newtonian power-law fluid past a continuously moving porous flat plate with heat flux and Viscous Dissipation. Recently Alam et al [14] has studied Dufour and Soret effects on unsteady MHD free convection and Mass transfer flow past a vertical porous plate in a porous medium. The present work aims to study the effects of Dufour and Soret on unsteady free convection and mass transfer flow past an infinite vertical porous flat plate in porous medium by taking into account the effect of viscous dissipation.

2. MATHEMATICAL ANALYSIS

An unsteady two-dimensional flow of an incompressible and electrically conducting viscous fluid, along an infinite vertical porous flat plate embedded in a porous medium is considered. The *x*-axis is taken on the infinite plate, and parallel to the free-stream velocity which is vertical and the *y*-axis is taken normal to the plate. A magnetic field B_0 of uniform strength is applied transversely to the direction of the flow.

Initially the plate and the fluid are at same temperature T_∞ in a stationary condition with concentration level C_∞ at all points. For t >0, the plate starts moving impulsively in its own plane with a velocity U_0, its temperature is raised to T_ω and the concentration level at the plate is raised to C_ω. The flow configuration and coordinate system are shown in the following fig-1.
The fluid is assumed to have constant properties except that the influence of the density variations with temperature and concentration, which are considered only in the force term.

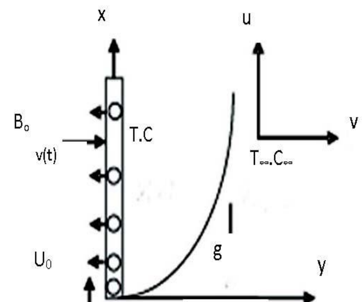

Fig-1

Under the above assumptions, the physical variables are functions of *y* and *t* only. Assuming that the Boussinesq and boundary-layer approximation hold and using the Darcy-Forchheimer model, the basic equations, which govern the problem, are given by (see Alam and Rahman[8]):

$$\frac{\partial v}{\partial x} = 0 \qquad \ldots(1)$$

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\sigma\beta_0^2 u}{\rho} - \frac{\nu}{k}u - \frac{b}{k}u^2 \qquad \ldots(2)$$

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y} = \propto \frac{\partial^2 T}{\partial y^2} + \frac{D_m K_T}{c_s c_p}\frac{\partial^2 C}{\partial y^2} + \frac{\nu}{C_p}\left(\frac{\partial u}{\partial y}\right)^2 \qquad \ldots(3)$$

$$\frac{\partial C}{\partial t} + v\frac{\partial C}{\partial y} = D_M\frac{\partial^2 C}{\partial y^2} + \frac{D_m K_T}{T_m}\frac{\partial^2 T}{\partial y^2} \qquad \ldots(4)$$

Initially (t=0) the fluid and the plate are at rest. Thus the no slip boundary conditions at the surface of the plate for the above problem for t >0 are:

$$u = U_0, v = v(t), T = T_w, C = C_w \ at \ y = 0 \qquad \ldots(5a)$$

$$u = 0, T = T_\infty, C = C_\infty \ as \ y \to \infty \qquad \ldots(5b)$$

The last but one term on the right-hand side of the energy equation (3) and concentration equation (4) signifies the Dufour or diffusion-thermo effect and the Soret or thermal-diffusion effect, respectively.

Here u, v are the Darcian velocity components in the x-and y-directions respectively, t is the time, **v** is the kinematic viscosity, g is the acceleration due to gravity, ρ is the density, β is the coefficient of volume expansion, β^* is the volumetric coefficient of expansion with concentration, k is Darcy permeability, b is the empirical constant, B_0 is magnetic induction, T and T_∞ are the temperature of the fluid inside the thermal boundary layer and the fluid temperature in the free stream, respectively, while C and C_∞ are the corresponding concentrations. Also σ is the electric conductivity, α is the thermal diffusivity, D_m is the coefficient of mass diffusivity, c_p is the specific heat at constant pressure, T_m is the mean fluid temperature, K_T is the thermal diffusion ratio and c_s is the concentration susceptibility.

3. METHOD OF SOLUTION:

Now in order to obtain a local similarity solution (in time) of the problem under consideration, we introduce a time dependent length scale δ as

$$\delta=\delta(t) \qquad \dots (6)$$

In terms of this length scale, a convenient solution of the equation (1) is considered to be in the following form:

$$V = v(t) = -v_0 \frac{\nu}{\delta} \qquad \dots(7)$$

Where $v_0 > 0$ is the suction parameter.

We now introduce the following dimensionless variables:

$$\eta = \frac{y}{\delta}, = u_0 f(\eta),\ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},\ \emptyset(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \qquad \dots(8)$$

Then introducing the relations (6)-(8) into the equations (2)-(3) respectively, we obtain (by using the analysis of Sattar and Hossain [9], see also Hasimoto[10]), the following dimensionless ordinary differential equations:

$$f'' + (2\eta + v_0)f' + G_r\theta + G_m\varphi - Mf - \frac{1}{D_a}f - \frac{R_eF_s}{D_a}f^2 = 0 \qquad \dots(9)$$

$$\theta'' + p_r(2\eta + v_0)\theta' + p_rD_u\varphi'' + p_rE_c(f'')^2 = 0 \qquad \dots(10)$$

$$\varphi'' + S_c(2\eta + v_0)\varphi' + S_cS_r\theta'' = 0$$
…(11)

Where primes denotes differentiation with respect to η and the dimensionless quantities are given by

$D_a = \frac{k}{\delta^2}$ is the local Darcy number,

$F_s = \frac{b}{\delta}$ is the local Forchheimer number,

$R_e = \frac{U_0\delta}{\nu}$ is the local Reynolds number,

$p_r = \frac{\nu}{\propto}$ is the Prandtl number,

$S_c = \frac{\nu}{D_m}$ is the Schmidt number,

$M = \frac{\sigma\beta_0^2\delta^2}{\rho\nu}$ is the Magnetic field parameter,

$S_r = \frac{D_m K_T (T_w - T_\infty)}{T_m \nu (C_w - C_\infty)}$ is the Soret number,

$D = \frac{D_m K_T (C_w - C_\infty)}{c_s c_p \nu (T_w - T_\infty)}$ is the Dofour number,

$G_r = \frac{g\beta (T_w - T_\infty)\delta^2}{\nu U_0}$ is the local Grashof number

$G_m = \frac{g\beta^* (C_w - C_\infty)\delta^2}{\nu U_0}$ is the modified local Grashof number

The corresponding boundary conditions for $t > 0$ are obtained as

$f = 1,\ \theta = 1,\ \varphi = 1$ at $\eta = 0$ …

(12a)

$f = 0,\ \theta = 0,\ \varphi = 0$ as $\eta \to \infty$ …(12b)

The equations (9)-(11) are locally similar in time but not explicitly time dependent. To solve the system of transformed governing equations (9) & (11) with the boundary conditions (12), we first linearized equation (9) by using Quasi linearization technique then equation (9) is transformed to

$$f'' + (2\eta + v_0)f' + G_r\theta + G_m\varphi - Mf - \frac{1}{D_a}f - \frac{R_eF_s}{D_a}(2Ff - F^2) = 0$$

…(13)

Where F is assumed to be a known function and the above equation can be rewritten as

$$A_0 f'' + A_1 f' + A_2 f + A_3 = 0 \quad \text{…(14)}$$

Where $A_0[i] = 1,\ A_1[i] = 2\eta + v_0\ A_2[i] = -M - \frac{1}{D_a} - \frac{2R_eF_s}{D_a}F$

$A_3[i] = -\frac{R_eF_s}{D_a}F^2 + G_r\theta + G_m\varphi$

Equation (10) can be expressed as

$$C_0\theta'' + p_r(2\eta + v_0)C_1\theta' + C_2 = 0 \quad \text{…(15)}$$

Where

$$C_0[i] = 1,\ C_1[i] = p_r(2\eta + v_0),\ C_2[i] = p_r D\varphi'' + p_r E_c (F'')^2$$

And equation (11) can be expressed as

$$E_0\varphi'' + E_1\varphi' + E_2\theta'' = 0 \qquad \ldots(16)$$

Where

$$E_0[i] = 1,\ E_1[i] = S_C(2\eta + v_0),\ E_2[i] = S_c S_r \theta''$$

Using implicit finite difference formulae, the equations (14),(15) &(16) are transformed to

$$B_0[i]f[i+1] + B_1[i]f[i] + B_2[i]f[i-1] + B_3[i] = 0 \qquad \ldots(17)$$

Where

$$B_0[i] = 2A_0[i] + hA_1[i], B_1[i] = -4A_0[i] - hA_1[i] + 2h^2A_2[i],$$
$$B_2[i] = 2A_0[i], \quad B_3[i] = 2h^2A_3[i]$$

$$D_0[i]\theta[i+1] + D_1[i]\theta[i] + D_2[i]\theta[i-1] + D_3[i] = 0 \qquad \ldots(18)$$

Where

$$D_0[i] = 2C_0[i] + hC_1[i], D_1[i] = -4C_0[i],\ D_2[i] = 2C_0[i] - hC_1[i], D_3[i] = 2h^2C_2[i]$$

And

$$H_0[i]\varphi[i+1] + H_1[i]\varphi[i] + H_2[i]\varphi[i-1] + H_3[i] = 0 \qquad \ldots(19)$$

Where

$$H_0[i] = 2E_0[i] + hE_1[i], H_1[i] = -4E_0[i],\ H_2[i] = 2E_0[i] - hE_1[i], H_3[i] = 2h^2E_2[i]$$

Here 'h' represents the mesh size in η direction. Equation (17), (18) & (19) are solved under the boundary conditions (12) by Thomas algorithm for various parameters entering into the problem and computations were carried out by using C programming.

Knowing the concentration field, the rate of mass transfer coefficient can be obtained which in the non-dimensional form in terms of Sherwood number is given by

$$S_h = -\left(\frac{\partial C}{\partial y}\right)_{y=0}$$

Knowing the temperature field, the rate of heat coefficient can be obtained which in the non-dimensional form in terms of the Nusselt number is given by

$$N_u = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0}$$

4. RESULTS AND DISCUSSIONS

A Parametric study is performed to explore the effect of suction parameter, on the velocity, Soret and Dufour numbers temperature and concentration profiles. The numerical computations have been done for different value of $V_0, S_r, D$and for fixed values of $P_r, S_c, G_r, G_m, R_e\, and\, F_s$. The values of S_r and D are taken in such a way that their product should be a constant provided the mean temperature is also constant. The value of prandtl number is chosen as 0.71 which corresponds to air and the value of Schmidt number is chosen to represent hydrogen at 25 ^{0}c and 1 atm. The values of Grashof numbers are taken to represent the free convection problem as G_r = 12 and G_m = 6. The value of local Reynolds number is taken as R_e = 100 and local forchheimer number is chosen as F_s = 0.09.

The values of local Nusselt number and the local Sherwood number which are directly proportional to $-\theta'(0)$ and $-\varphi'(0)$ are calculated and presented in a tabular form for various values of S_r, D in table 1. It is evident from the table that as S_r decreases there is an increase in Sherwood number but decrease in Nusselt number.

The effect of suction parameter on the velocity profiles in shown in figure 1. It is noticed from the figure that increases in the suction parameter decreases the velocity which indicates that suction stabilizes the growth in the boundary layer. Figure 2 shows the effect of suction parameter on the temperature profiles, from which it is evident that the temperature decreases as there is an increase in suction parameter.

The influence of suction parameter on concentration profiles is displayed in figure 3. It is observed from the figure that the concentration decreases with an increase in suction parameter away from the wall where as a reverse phenomenon is seen near the wall, i.e. the concentration increases with the increase in suction parameter very near to the wall. Hence suction reduces the growth of the thermal and concentration boundary layers.

The effects of Soret and Dufour numbers on velocity profiles and temperature profiles are shown in figure 4 and figure 5 respectively. As there is a decrease in the Soret number or an increase in the Dofour number, the velocity and temperature decreases. Here the variation in the profiles is very low. The variation in concentration profiles with the change in Soret and Dufour number is displayed in figure 6. The concentration increases as there is an increase in Dufour number or decrease in Soret number.

The effect of magnetic field on the velocity profiles is shown in figure 7. It is evident from the figure that the increase in the magnetic field parameter decreases the velocity profiles. The effect of magnetic field decreases the velocity. The variation in velocity profiles with the change in Darcy parameter is shown in figure 8. It is noticed from the figure that the velocity increases with an increase in Darcy parameter.

The effects of viscous dissipation on the temperature profiles is shown in figure 9. The viscous dissipation effect decelerate the temperature profiles.

5. CONCLUSIONS

1. Suction reduces the growth of the hydrodynamic, thermal and concentration boundary layers.
2. The increase in the Dufour number or decrease in the Soret number decreases the velocity and temperature profiles whereas increase the concentration
3. Viscous Dissipation decelerates the temperature.
4. Magnetic field retards the motion of the fluid whereas Darcy number accelerates the velocity.

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Table 1:

Sr	**Du**	**Nu**	**Sh**
2.0	0.03	1.121243	1.102232
1.0	0.06	1.009123	1.312547
0.5	0.12	0.921737	1.810015
0.4	0.15	0.885147	1.904895
0.2	0.30	0.863917	1.942347

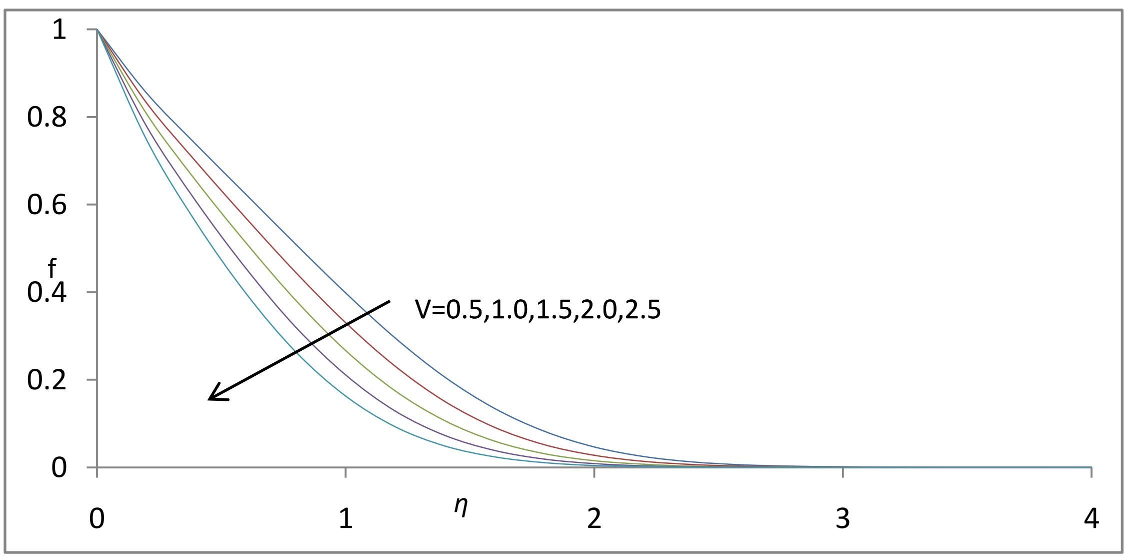

Fig.1 Velocity Profile for different values of v for Sr = 2.0, D = 0.03,Da =0.5 and M =0.3

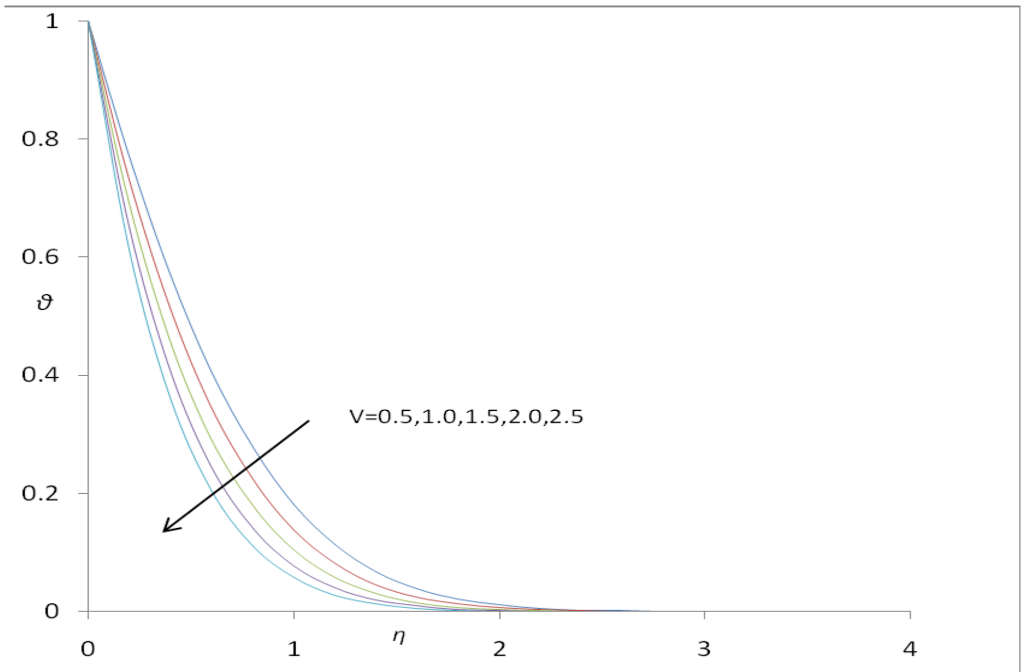

Fig2. Temperature Profile for different values of v for Sr = 2.0, D = 0.03,Da =0.5 and M =0.3

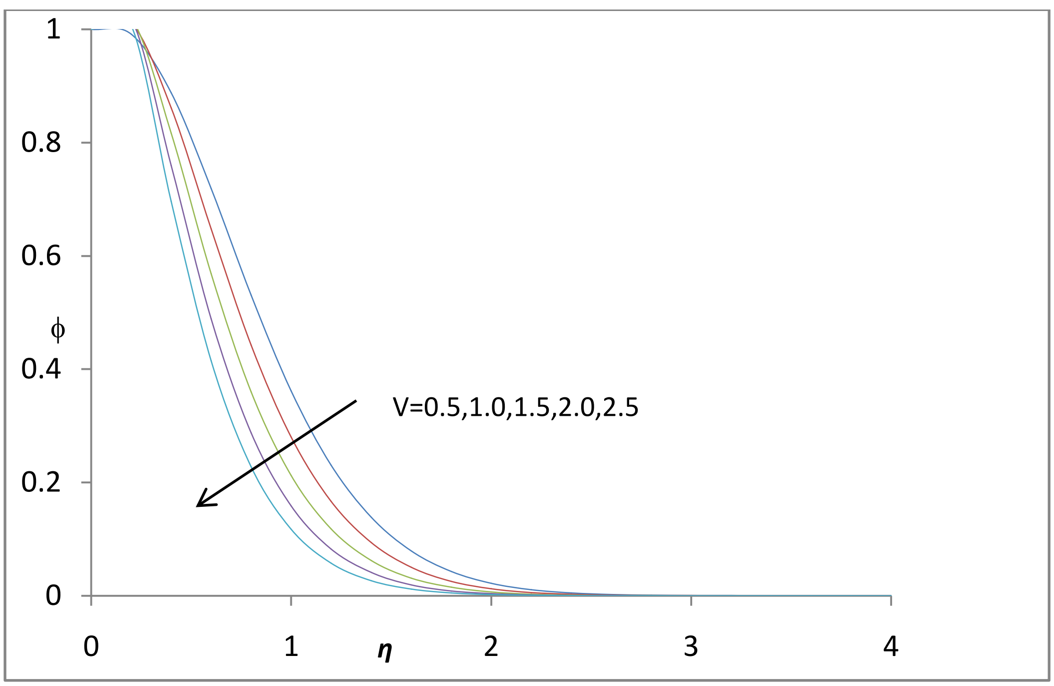

Fig. 3 Concentration Profile for different values of v for Sr = 2.0, D = 0.03, Da =0.5 and M =0.3

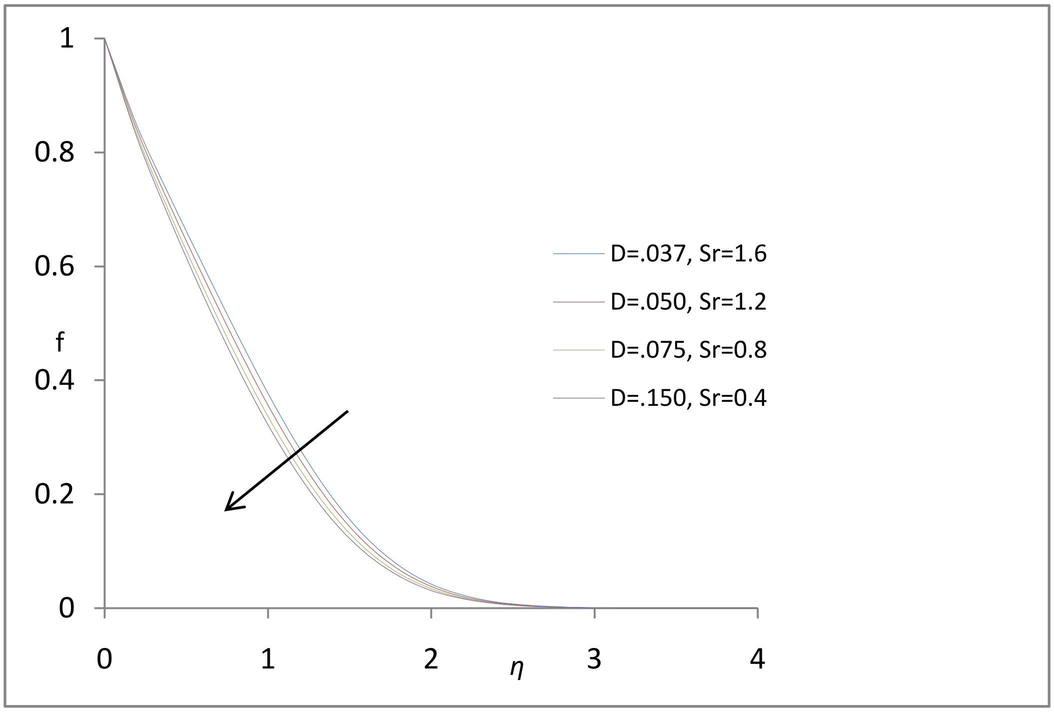

Fig. 4 Velocity Profile for different values of D, Sr for v = 0.5, Da =0.5 and M =0.3

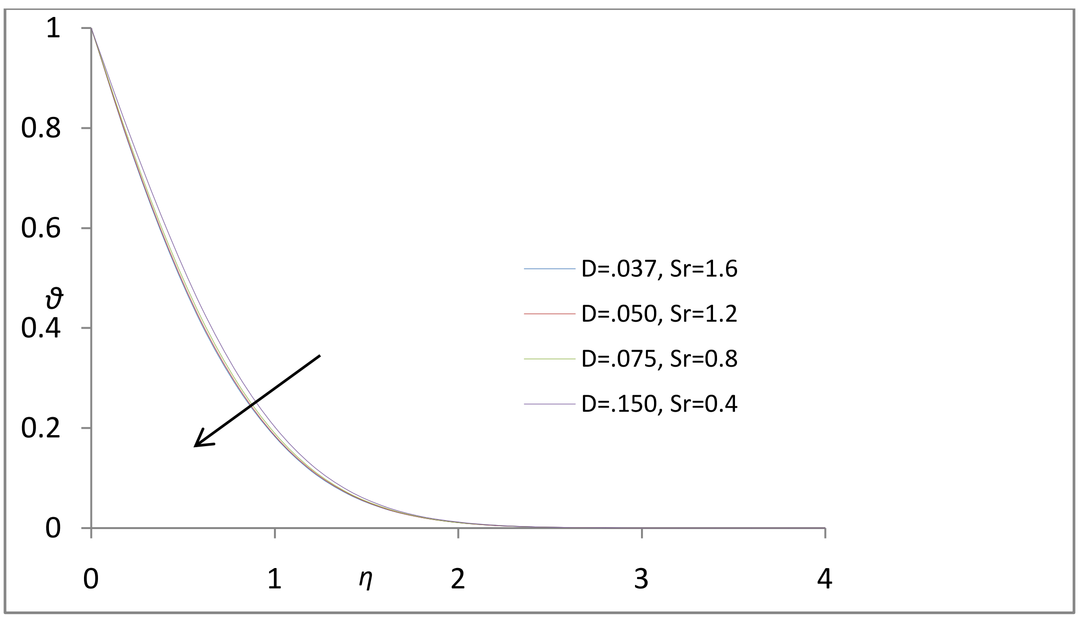

Fig 5 Temperature Profile for different values of D, Sr for v = 0.5, Da =0.5 and M =0.3

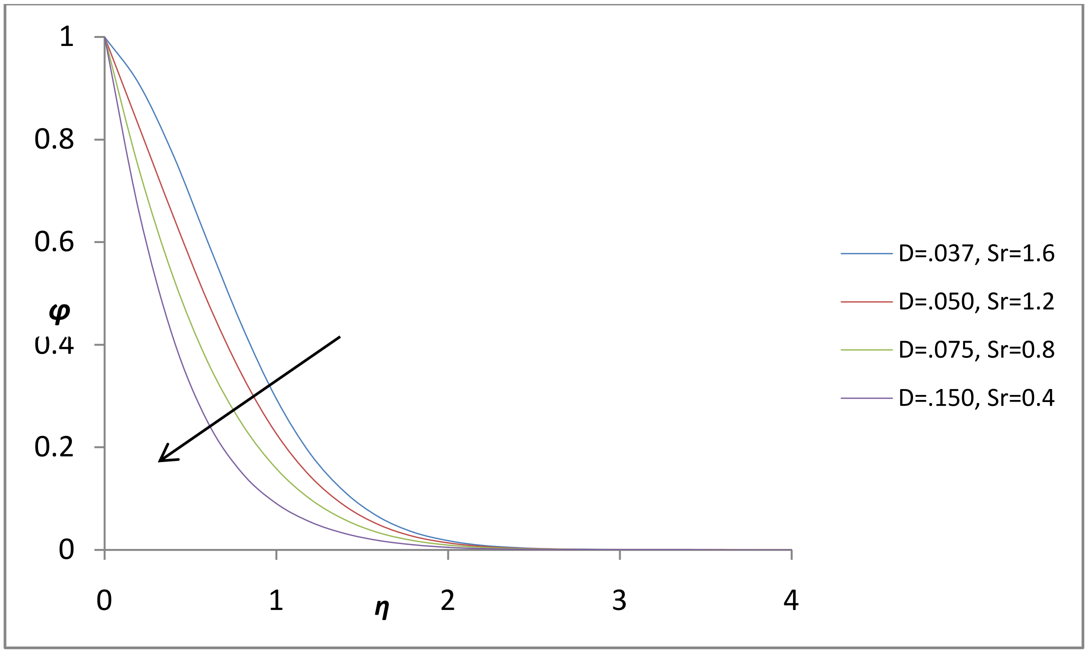

Fig.6 Concentration Profile for different values of D, Sr for v = 0.5, Da =0.5 and M =0.3

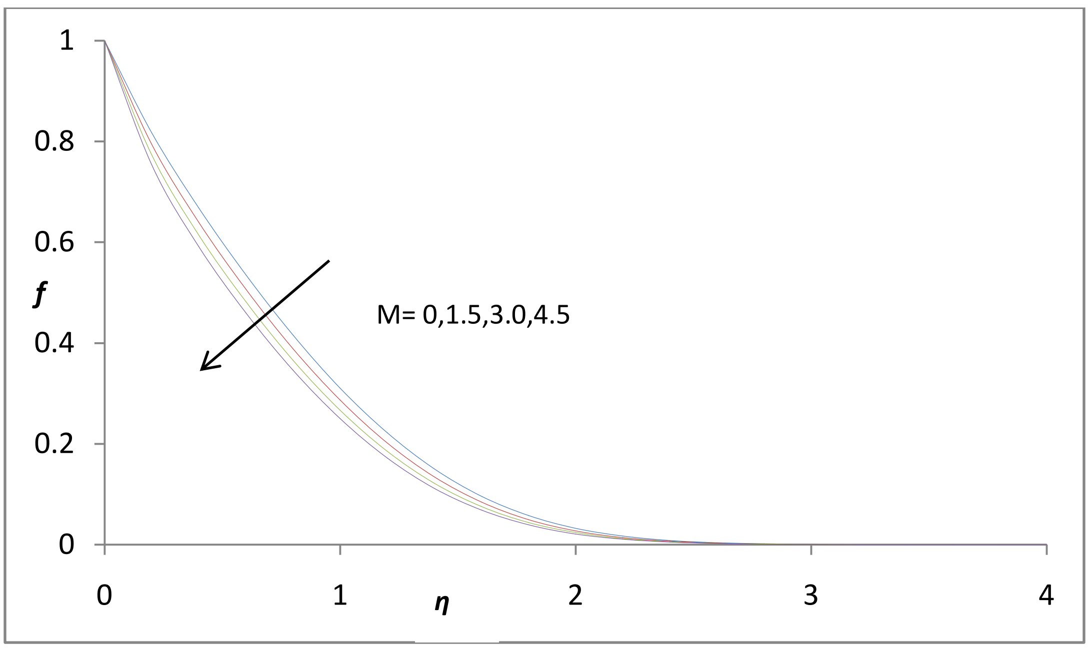

Fig. 7 Velocity Profile for different values of M for v = 0.5, D =0.03 , Da = 0.5 and Sr =2.0

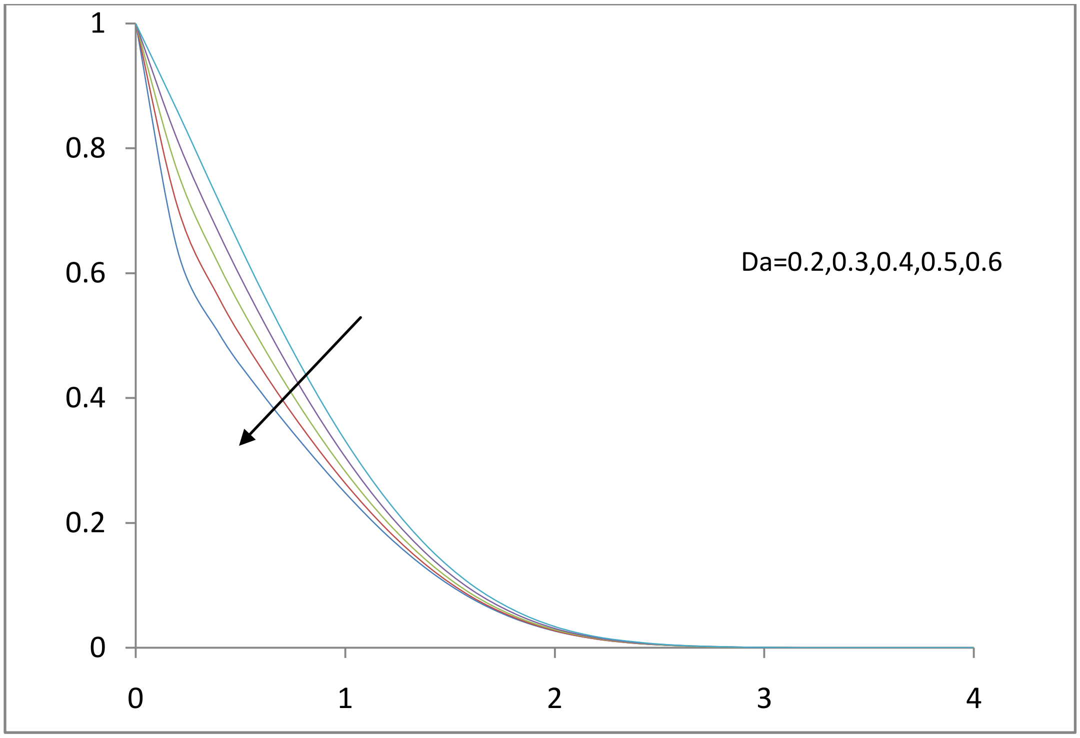

Fig. 8 Velocity Profile for different values of Da for v = 0.5, M=0.3,D =0.03 , Da = 0.5 and Sr =2.0

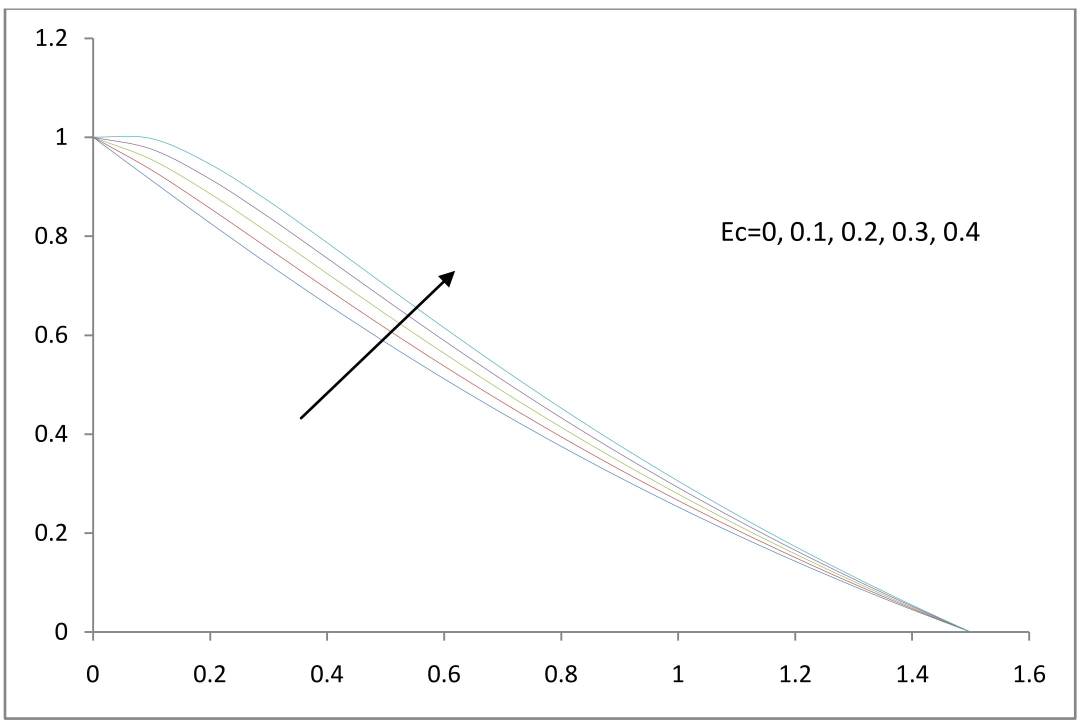

Fig 9 Temperature profile for different values of Ec for v = 0.5, M=0.3,D =0.03 , Da = 0.5 and Sr =2.0