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MHD EFFECTS ON FLOW PAST AN EXPONENTIALLY ACCELERATED ISOTHERMAL VERTICAL PLATE WITH UNIFORM MASS DIFFUSION

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Abstract

MHD flow past an exponentially accelerated infinite isothermal vertical plate with uniform mass diffusion is studied here. The dimensionless governing equations are solved using Laplace-transform technique. The velocity, temperature and concentration are studied for different physical parameters like magnetic field parameter, thermal Grashof number, mass Grashof number, time and an accelerating parameter a. It is observed that for an increase in magnetic field parameter M, there is a fall in the velocity.

Key words: exponential, accelerated vertical plate, magnetic field.

Nomenclature

a' - accelerating parameter
a - dimensionless accelerating parameter
B_0 - magnetic field strength
C - dimensionless concentration
C_w - concentration of the plate
C_∞ - concentration of the fluid far away from the plate
C_p - specific heat at constant pressure
g - acceleration due to gravity
Gr - thermal Grashof number
Gc - mass Grashof number
k - thermal conductivity of the fluid
k^* - mean absorption coefficient
M - magnetic field parameter.
Pr - Prandtl number
p - pressure

Sc - Schmidt number
T - temperature of the fluid near the plate
T_w - temperature of the plate
T_∞ - temperature of the fluid far away from the plate
t' - time
t - dimensionless time
u' - velocity of the fluid in the x-direction
u_0 - velocity of the plate
u - dimensionless velocity
y - coordinate axis normal to the plate
y' - dimensionless coordinate axis normal to the plate

Greek symbols

α - thermal diffusivity
β - volumetric coefficient of thermal expansion
μ - coefficient of viscosity
ν - kinematic viscosity
ρ - density
σ - stefan-Boltzmann constant
τ - dimensionless skin-friction
θ - dimensionless temperature
η - similarity parameter
erfc - complementary error function

Introduction

Magneto convection plays an important in various industrial applications. Examples include magnetic control of molten iron flow in the steel industry, liquid metal cooling in nuclear reactors and magnetic suppression of molten semi-conducting materials. It is of importance in connection with many engineering problems, such as sustained plasma confinement for controlled thermonuclear fusion and electromagnetic casting of metals.

Sakiadis [2,3] studied the growth of the two dimensional velocity boundary layer over a continuously moving horizontal plate emerging from a wide slot at uniform velocity. Soundalgekar [5] was the first to present an exact solution for the flow of a viscous fluid past an impulsively started infinite isothermal vertical plate. The solution was derived by the usual Laplace transform technique and the effects of heating or cooling of the plate on the flow field were discussed through Gr. Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh and Naveen Kumar [4]. The Skin-friction for accelerated vertical plate has been studied analytically by Hossian and Shayo [1].

The object of the present paper is to study the MHD effects on flow past an exponentially accelerated infinite isothermal vertical plate with uniform mass diffusion. The dimensionless governing equations are solved using the Laplace-transform technique. The solutions are in terms of exponential and complementary error function.

Mathematical Formulation

Here the unsteady flow of a viscous incompressible fluid past an infinite isothermal vertical plate with uniform mass diffusion, in the presence of magnetic field is considered. The x-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to the plate. At time $t^{'} \leq 0$, the plate and fluid are at the same temperature T_{∞} and concentration C_{∞}. At time $t^{'}$ >0, the plate is exponentially accelerated with a velocity $u = u_0 \exp(a^{'} t^{'})$ in its own plane and the temperature of the plate is raised to T_w and the level of concentration near the plate is also raised to C_w. A transverse magnetic field of uniform strength B_o is assumed to be applied normal to the plate. The induced magnetic field and viscous dissipation is assumed to be negligible. Then under usual Boussinesq's approximation, the unsteady flow is governed by the following equations:

$$\frac{\partial u^{'}}{\partial t'} = g\beta\left(T - T_{\infty}\right) + g\beta^{*}\left(C - C_{\infty}\right) + \nu \frac{\partial^2 u^{'}}{\partial y'^2} - \frac{\sigma B_0^2}{\rho} u \qquad (1)$$

$$\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y'^2} \qquad (2)$$

$$\frac{\partial C}{\partial t'} = D \frac{\partial^2 C}{\partial y'^2} \qquad (3)$$

With the following initial and boundary conditions

	$u' = 0$,	$T = T_{\infty}$, $C = C_{\infty}$	for all y', $t^{'} \leq 0$	
$t' > 0$:	$u' = u_0 \exp(at)$,	$T = T_w$, $C = C_w$	at $y' = 0$	(4)
	$u' \to 0$,	$T \to T_{\infty}$ $C \to C_{\infty}$	as $y' \to \infty$	

On introducing the following non-dimensional quantities

$$u = \frac{u'}{u_0}, \quad t = \frac{t' u_0^2}{\nu}, \quad y = \frac{y' u_0}{\nu}, \quad \theta = \frac{T - T_{\infty}}{T_W - T_{\infty}}, \quad C = \frac{C - C_{\infty}}{C_W - C_{\infty}},$$

$$a = \frac{a^{'} \nu}{u_0^2}, \; Gr = \frac{g\beta\nu(T_W - T_{\infty})}{u_0^3}, \; Gc = \frac{g\beta^{*}\nu(C_W - C_{\infty})}{u_0^3}, \qquad (5)$$

$$\Pr = \frac{\mu C_p}{k}, \; Sc = \frac{\nu}{D}, \; M = \frac{\sigma B_0^2 \nu}{Q u_0^2}$$

in equations (1) to (4), leads to

$$\frac{\partial u}{\partial t} = Gr\,\theta + Gc\,C + \frac{\partial^2 u}{\partial y^2} - Mu \tag{6}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} \tag{7}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} \tag{8}$$

The initial and boundary conditions in a non-dimensional form are

$$\begin{array}{llll} & u = 0, & \theta = 0,\ C = 0 & \text{for all } y,\ t \le 0 \\ t > 0: & u = \exp(at), & \theta = 1,\ C = 1 & \text{at } y = 0 \\ & u \to 0, & \theta \to 0,\ C \to 0 & \text{for all } y \to \infty \end{array} \tag{9}$$

The dimensionless governing equations (6), (7) and (8), subject to the boundary conditions (9), are solved by the usual Laplace-transform technique and the solutions are derived as follows.

$$\begin{aligned} u = {} & \frac{e^{at}}{2}\left[\begin{array}{l} \bar{e}^{2\eta\sqrt{(a+M)t}}\, erfc(\eta - \sqrt{(a+M)t}) + \\ e^{2\eta\sqrt{(a+M)t}}\, erfc\ (\eta + \sqrt{(a+M)t}) \end{array}\right] \\ & + \frac{1}{2}\left(\frac{Gr}{(1-\Pr)b} + \frac{Gc}{(1-Sc)c}\right)\left[\begin{array}{l} \bar{e}^{2\eta\sqrt{Mt}}\, erfc(\eta - \sqrt{Mt}) + \\ e^{2\eta\sqrt{Mt}}\, erfc\ (\eta + \sqrt{Mt}) \end{array}\right] \\ & - \frac{Gr\ e^{bt}}{2(1-\Pr)b}\left[\begin{array}{l} \bar{e}^{2\eta\sqrt{(b+M)t}}\, erfc(\eta - \sqrt{(b+M)t}) - \\ e^{2\eta\sqrt{(b+M)t}}\, erfc\ (\eta + \sqrt{(b+M)t}) \end{array}\right] \\ & - \frac{Gc\ e^{ct}}{2(1-Sc)c}\left[\begin{array}{l} \bar{e}^{2\eta\sqrt{(c+M)t}}\, erfc(\eta - \sqrt{(c+M)t}) - \\ e^{2\eta\sqrt{(c+M)t}}\, erfc\ (\eta + \sqrt{(c+M)t}) \end{array}\right] \\ & - \frac{Gr}{(1-\Pr)b}\, erfc(\eta\sqrt{\Pr}) - \frac{Gc}{(1-Sc)c}\, erfc(\eta\sqrt{Sc}) \\ & + \frac{Gr\ e^{bt}}{2(1-\Pr)b}\left[\begin{array}{l} \bar{e}^{2\eta\sqrt{b\Pr t}}\, erfc(\eta\sqrt{\Pr} - \sqrt{bt}) + \\ e^{2\eta\sqrt{b\Pr t}}\, erfc\ (\eta\sqrt{\Pr} + \sqrt{bt}) \end{array}\right] \\ & + \frac{Gc\ e^{ct}}{2(1-Sc)c}\left[\begin{array}{l} \bar{e}^{2\eta\sqrt{cSct}}\, erfc(\eta\sqrt{Sc} - \sqrt{ct}) + \\ e^{2\eta\sqrt{cSct}}\, erfc\ (\eta\sqrt{Sc} + \sqrt{ct}) \end{array}\right] \end{aligned} \tag{10}$$

$$\theta = erfc(\eta\sqrt{\Pr}) \tag{11}$$

$$C = erfc(\eta\sqrt{Sc}) \tag{12}$$

where $\eta = \dfrac{y}{2\sqrt{t}},\ b = \dfrac{M}{\Pr - 1},\ and\ c = \dfrac{M}{Sc - 1}$.

Results and Discussion

For physical understanding of the problem numerical computations are carried out for different parameters M , a, Gr , Gc and t upon the nature of the flow and transport. The values of Prandtl number Pr are chosen such that they represent air (Pr=0.71) and water (Pr=7.0). The numerical values of the velocity and temperature are computed for different physical parameters like M, a, Prandtl number, thermal Grashof number, mass Grashof number and time.

The velocity profiles for different values of the magnetic field parameter are shown in Fig.1. It is observed that the velocity decreases in the presence of magnetic field than its absence. This shows that the increase in the magnetic field parameter leads to fall in the velocity. This agrees with expectations, since the magnetic field exerts a retarding force on the free convective flow.

The velocity profiles for different values of (a = 0.2, 0.5, 0.8) and time (t = 0.2, 0.4, 0.6) are shown in the Fig.2. It is observed that the velocity increases with increasing values of a or time.

The velocity profiles for different thermal Grashof number (Gr = 2, 5, 10) are shown in the Fig.3. It is observed that velocity increases with increasing values of Gr.

The temperature profiles are calculated for different values of the Prandtl number (Pr = 0.71 and Pr=7.0) are shown in Fig.4. It is observed that temperature increases in the presence of air than in water.

Figure 5 represents the effect of concentration profiles for different Schmidt number (Sc = 0.16, 0.3, 0.6, 2.01). The effect of concentration is important in concentration field. It is observed that the wall concentration increases with decreasing values of the Schmidt number. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity.

Conclusions

The theoretical solution of flow past an exponentially accelerated infinite isothermal vertical plate with uniform mass diffusion, in the presence of magnetic field is considered. The dimensionless governing equations are solved by the usual Laplace-transform technique. The effect of different parameters like magnetic field parameter, thermal Grashof number, mass Grashof number, a and t are studied graphically. It is observed that the velocity decreases with increasing values of magnetic field parameter but increases with increasing values of Gr, a and t.

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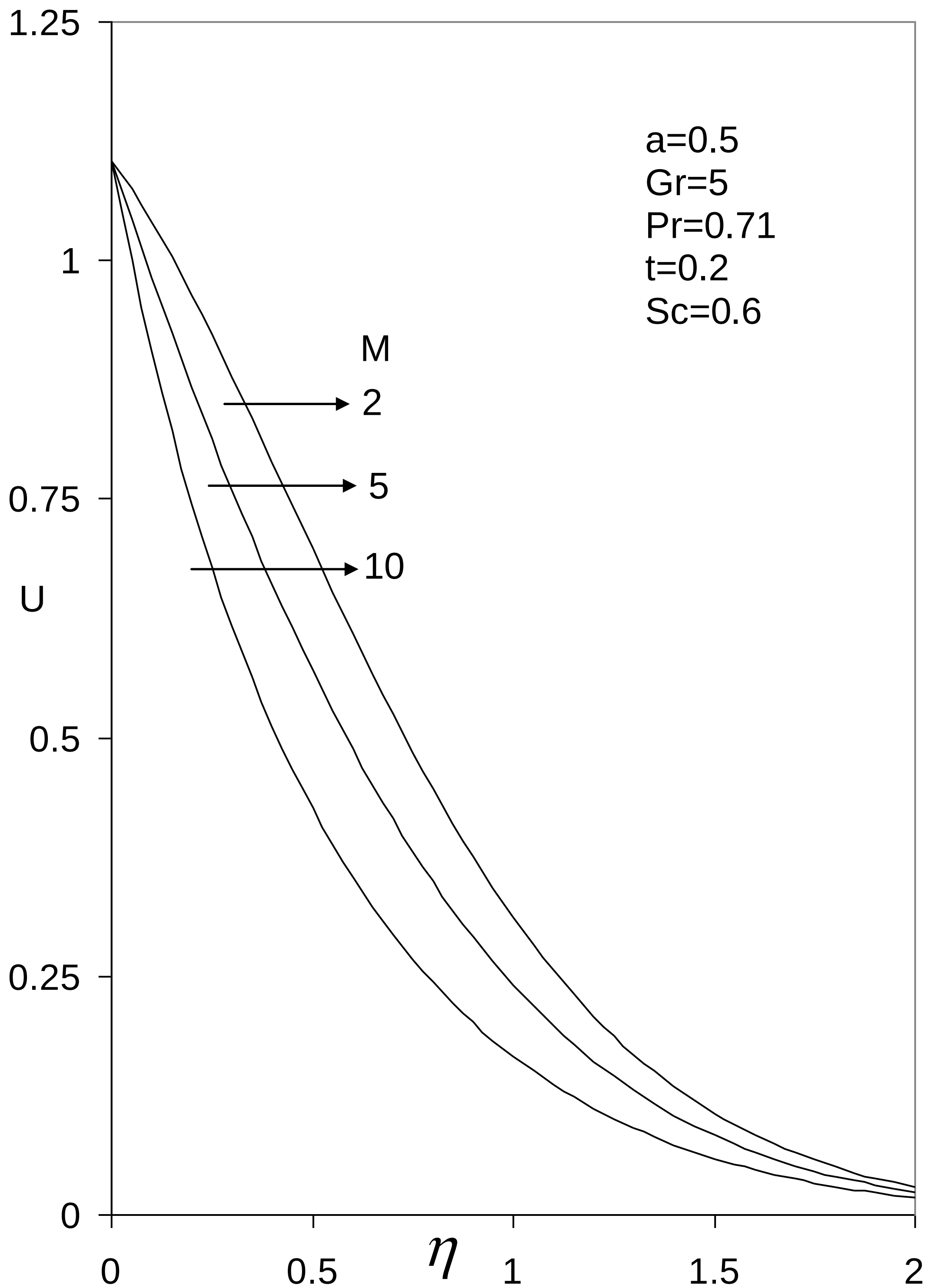

Figure 1: Velocity profiles for different M

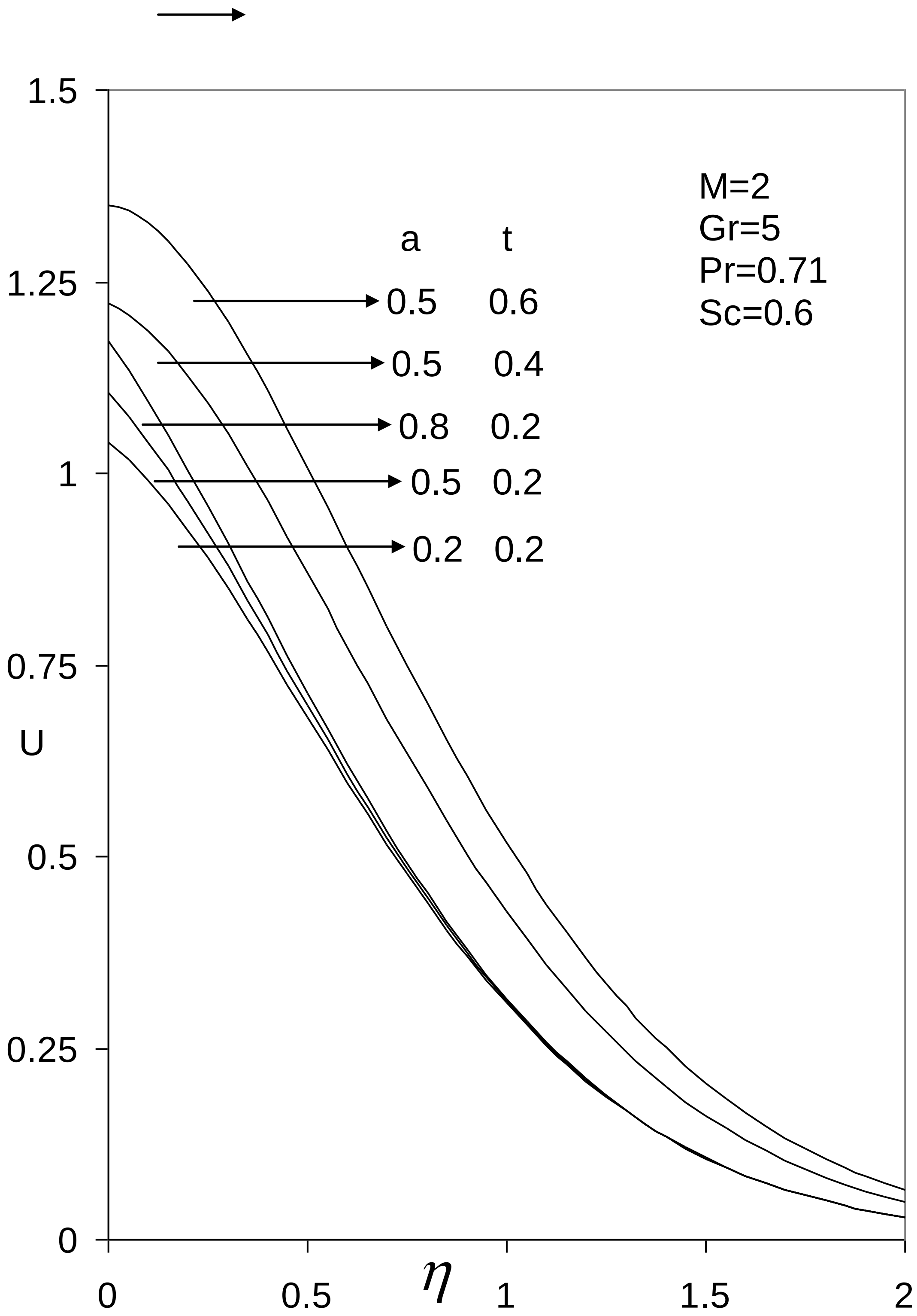

Figure 2: Velocity profiles for different a and t

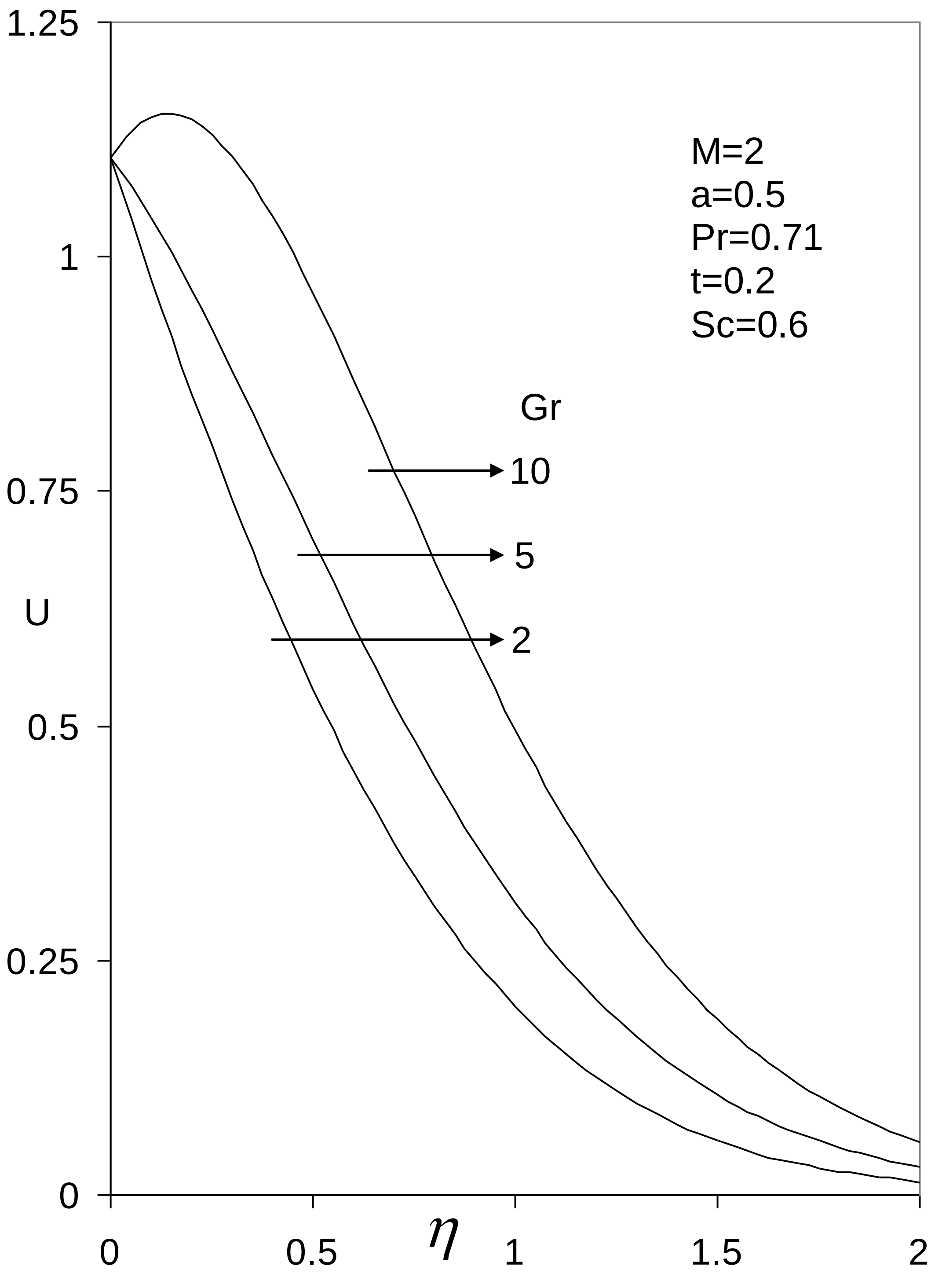

Figure 3: Velocity profiles for different Gr

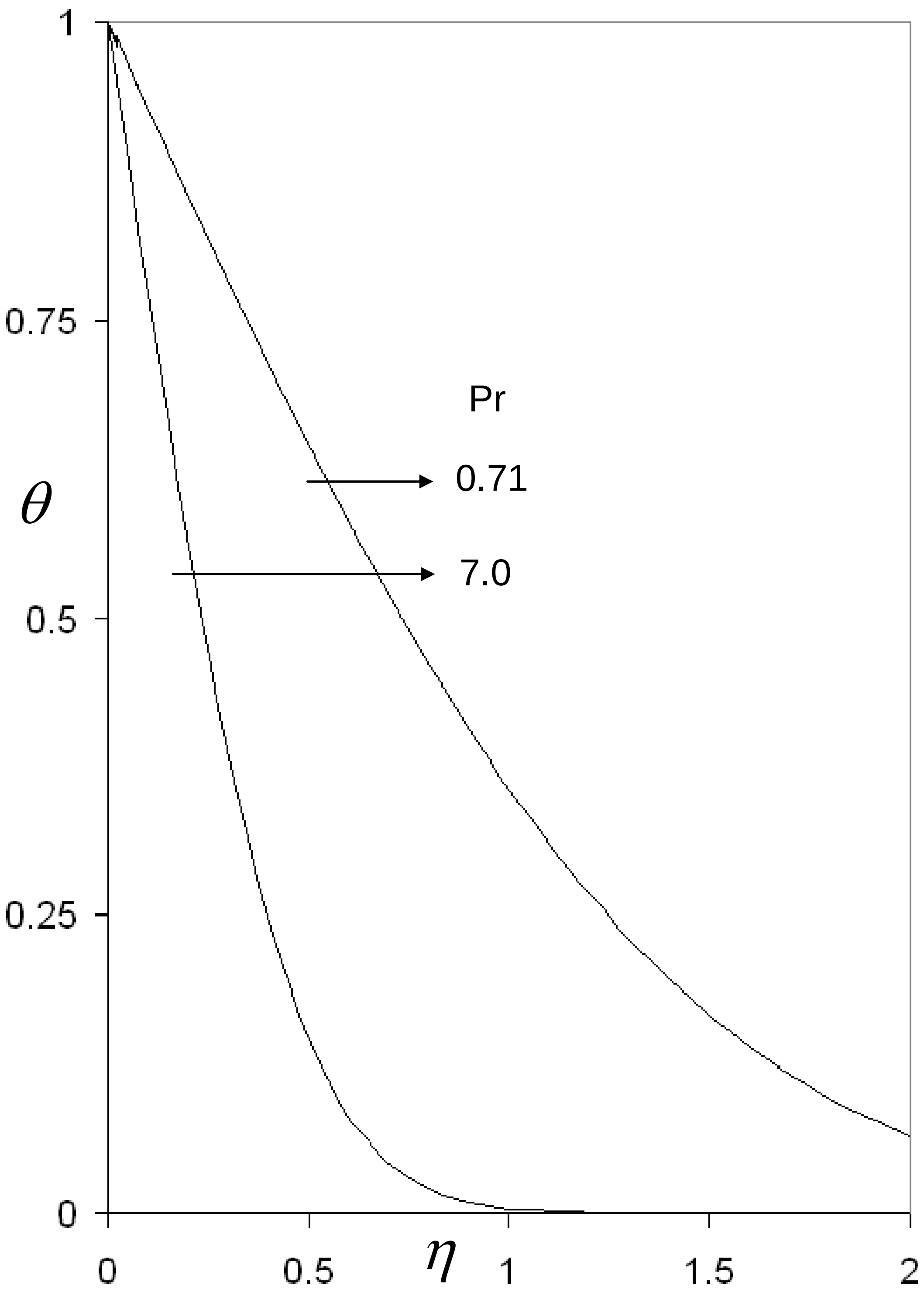

Figure 4: Temperature profiles for different Pr

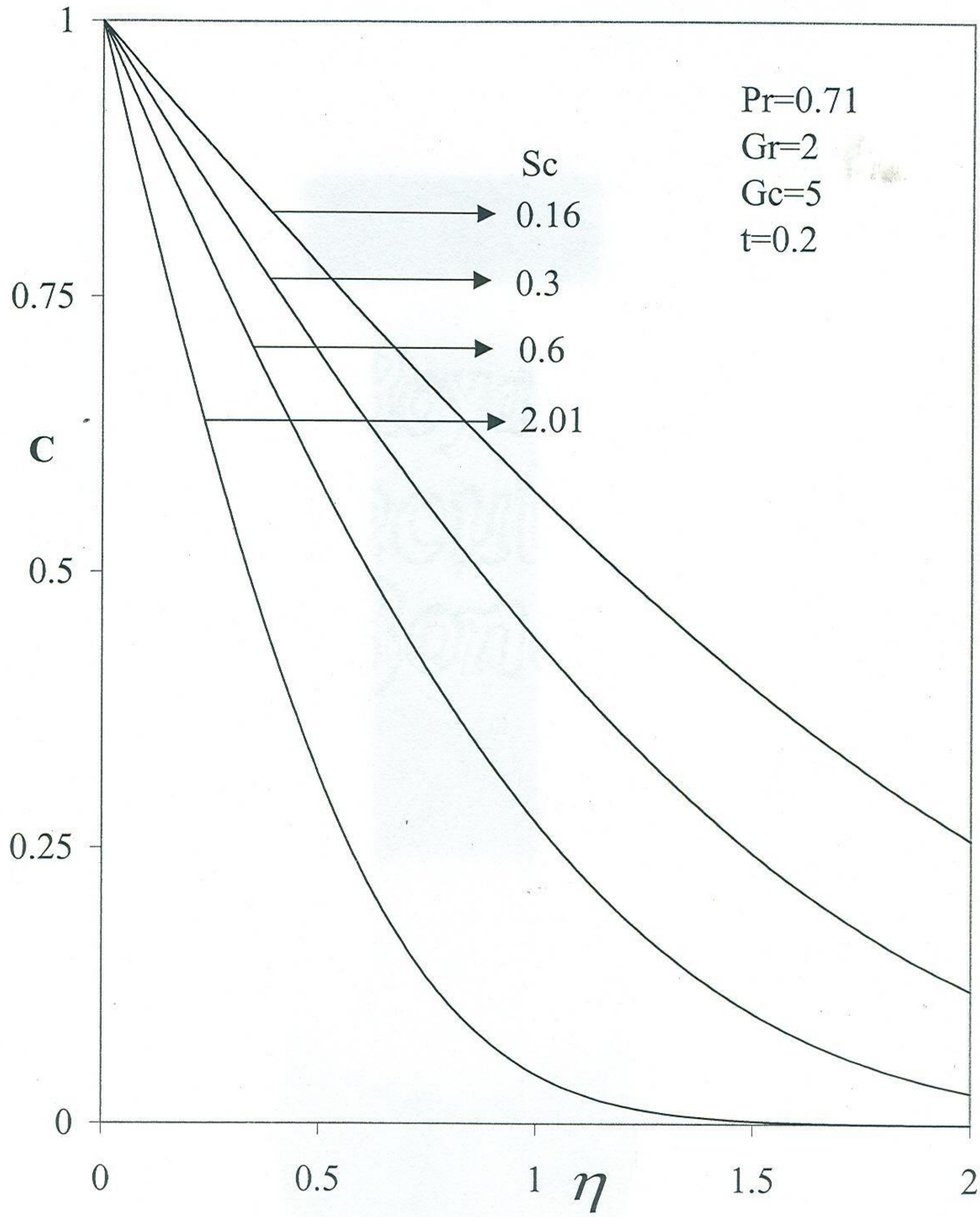

Figure 5 Concentration profiles for different Sc