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MHD Laminar Boundary Layer Flow and Heat Transfer over a Moving Cylindrical Rod

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ABSTRACT: This paper examines the forced convection flow of incompressible, electrically conducting viscous fluid over a moving cylindrical rod with an applied magnetic field. The system of partial differential equations governing the flow and heat transfer is solved using an efficient implicit finite - difference scheme. Numerical computations are performed for air (Pr = 0.7) and displayed graphically to illustrate the influence of relevant physical parameters on local skin friction and heat transfer coefficients and, also on, velocity and temperature fields. It is observed that the magnetic field increases both the coefficients of skin friction and heat transfer. Further, it reduces the thickness of momentum and thermal boundary layers. Also, the results indicate that the viscous dissipation is found to have a pronounced effect on heat transfer coefficient and thermal transport in the boundary layer region.

Keywords: *Magnetic field, Skin friction coefficient, Viscous dissipation, Heat transfer coefficient*

INTRODUCTION

The study of boundary layer flow on a continuously moving or stretching surface in a quiescent or moving fluid has many important applications in the fields of science and engineering. A number of technical processes concerning polymers involve the cooling of continuous strips or moving rods by drawing them through a quiescent or moving fluid. In these processes, it is very important to control the drag and the heat flux in order to obtain good product quality as the product properties of final product depend to a great extent on the rate of cooling which is governed by the structure of the laminar boundary layer near the moving or the stretching surface. The steady laminar flow induced by the motion of a cylindrical rod issuing from an orifice into a fluid at rest is an example of practical application of a continuous moving cylindrical surface in glass or fiber manufacturing industries. Due to hydrodynamic friction, the fluid adjacent to the rod's surface is carried in the axial direction. At a short distance from the orifice, the flow takes the form of an axisymmetric boundary layer on the surface of the cylinder. The flow will be considered in frames of the laboratory reference system in which the velocity and temperature of the fluid on the rod's surface, equal to the velocity and temperature of the rod which are decreasing in the normal direction to zero. Consequently, the ambient fluid flow and heat transfer has a great influence on the process of fiber formation and needs further investigation.

The study of laminar boundary layer behavior caused by a moving rigid cylindrical surface in a quiescent fluid was first considered by Sakiadis [1] and later the work was extended to the flow on a cylinder moving in a fluid at rest by Crane[2]. Jaffe and Okamura [3] studied the transverse curvature effect on the incompressible laminar boundary layer for a longitudinal flow over a cylinder. Wang [4] reported a similarity reduction of the Navier-Stokes equations describing axisymmetric radial stagnation boundary layer flow normal to the cylinder. Kuiken [5] investigated the cooling of a heat resistance cylinder moving through a fluid. Indeed, in the manufacture of metal and polymer solid cylinders, the material is usually in a molten phase when thrust through an extrusion die and then cools and solidifies some distance away from the die. Experiments by Vleggaar [6] show that the velocity of the material is approximately proportional to the distance, so these systems are often modelled as linear stretching rods or cylinders. Zachara [7] examined the laminar boundary layer on a moving cylindrical rod. Also, Brendan Redmond and David McDonnel [8] applied Hausen integral method to find the rate of heat loss of the fibre during manufacturing of polymer fibres.

In recent years, MHD flow problems have become more important industrially. Magneto-hydrodynamics (MHD) is the branch of continuum mechanics which deals with the study of electrically conducting fluids and electromagnetic forces. The field of MHD was initiated by Swedish physicist, Hannes Alfven for which he received in 1970 the Noble prize. The idea of MHD is that magnetic fields can induce currents in a moving conductive fluid, which create forces on the fluids, and also change the magnetic field itself. MHD problems arise in a wide variety of situations ranging from the explanation of the origin of Earth's magnetic field and the prediction of space weather to the damping of turbulent fluctuations in semiconductor melts during crystal growth and, even in the measurement of the flow rates of beverages in food industry. An interesting application of MHD to metallurgy lies in the purification of molten metals from non-metallic inclusions by the application of a transverse magnetic field. Indeed, MHD laminar boundary layer behavior over a stretching surface is a significant type of flow having considerable practical applications in chemical engineering and polymer processing. Several researchers [9-13] studied laminar boundary layer problems on moving or stretching surfaces in the presence of transverse magnetic field. Recently,Takhar et al. [14] have considered flow and mass transfer on a stretching sheet with a magnetic field and chemically reactive species. To the best of author's knowledge MHD laminar boundary layer flow and heat transfer over a moving cylindrical rod. However, Jayakumar and Eswara [15] studied nonsimilar laminar flow over a moving cylindrical rod with an applied magnetic field considering only momentum transfer inside the boundary layer region.

The objective of this paper is to analyse the steady, nonsimilar laminar incompressible boundary layer flow of an electrically conducting fluid over a moving cylindrical rod along with an applied magnetic field. The partial differential equations, governing the axisymmetric boundary layer flow, with two – point boundary conditions have been solved numerically using Kellar-box [14] method. The present investigation is the extension of the work of [13] to explore heat transfer phenomena.

MATHEMATICAL FORMULATION

Let us consider the steady, axisymmetric laminar boundary layer flow of an incompressible fluid with an applied magnetic field on a continuous cylinder moving from an orifice in an axial direction at constant velocity through a fluid at rest. The radius of the cylinder and its velocity are denoted by "a" and "U" respectively. A magnetic field B_o fixed relative to the fluid is applied in y – direction [See Fig.1(a)] normal to the flow and it is assumed that magnetic Reynolds number is small so that the induced magnetic field can be neglected. The effect of viscous dissipation is included in the analysis. The flow is considered in frames of the boundary layer co – ordinate system in which the x – axis, parallel to the axis of symmetry, is posed along the solid surface and the y – axis is normal to it. The origin of the coordinate system is put in the plane of the orifice.

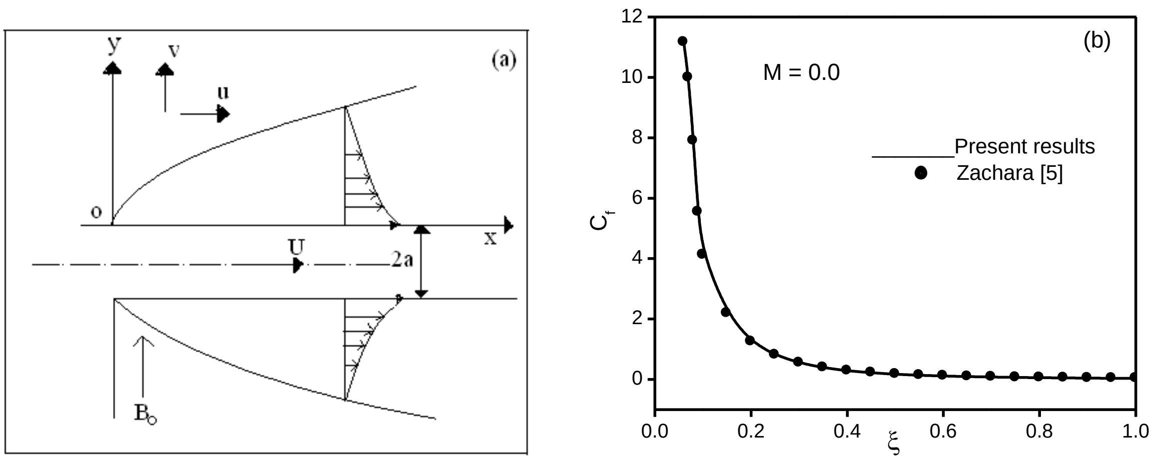

Fig.1 (a) Flow model and co – ordinate system

(b) Comparison of skin friction coefficient (C_f) with that of Zachara [7]

Under the aforementioned assumptions, the boundary layer equations based on conservation of mass, momentum and energy governing the forced convection flow are:

$$\frac{\partial}{\partial x}(ru)+\frac{\partial}{\partial y}(rv)=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\nu}{r}\frac{\partial}{\partial y}\left(r\frac{\partial u}{\partial y}\right)-\frac{\sigma B_0^{\,2}}{\rho}(u-U) \qquad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial x}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma B_0^{\,2}}{\rho}u^2 \qquad (3)$$

where $r(y)=a+y$

The boundary conditions are

$$u(x,0)=U;\quad v(x,0)=0;\quad u(x,\infty)=0;\quad T(x,0)=T_w;\quad T(x,\infty)=T_\infty \quad (4)$$

The Eqn.(2) and (3) are transformed into nondimensional form using the combination of the Mangler and Falkner – Skan transformations :

$$\eta=\left(\frac{U}{\nu x}\right)^{1/2} y\left(1+\frac{y}{2a}\right);\qquad \xi=\left(\frac{4\nu x}{Ua^2}\right)^{1/2};\ \psi(x,y)=a\,(U\nu x)^{1/2} f(\xi,\eta);\ T=T_\infty+(T_w-T_\infty)\,G(\xi,\eta) \quad (5)$$

which satisfies the continuity Eqn.(1). Further, Eqns.(2) - (3) reduce, respectively to

$$(1+\xi\eta)f'''+\xi f''+\frac{ff''}{2}-M(f'-1)=\frac{\xi}{2}\left(f'f'_\xi-f''f_\xi\right) \qquad (6)$$

$$(1+\xi\eta)G''+\frac{\xi G'}{2}+\frac{f\,G'\Pr}{2}+Ec\Pr\left[(1+\xi\eta)f''^2+\frac{Mf'^2}{2}\right]=\frac{\xi\Pr}{2}\left(f'G_\xi-G'f_\xi\right) \qquad (7)$$

where

$$u=Uf';\quad v=-\frac{a}{r}\left(\frac{\nu U}{4x}\right)^{1/2}\{f-\eta f'+\xi f_\xi\};\quad M=\frac{2x\sigma B_0^{\,2}}{\rho U};\ Ec=\frac{U^2}{c_p(T_w-T_\infty)} \qquad (8)$$

The transformed boundary conditions are:

$$f(\xi,0)=0;\quad f'(\xi,0)=1;\qquad f'(\xi,\infty)=0$$
$$G(\xi,0)=1;\qquad G(\xi,\infty)=0 \qquad (9)$$

for $\xi\ge 0$.

The skin friction coefficient and heat transfer coefficient in the form of Nusselt number are expressed as

$$C_f = -\frac{a\,\tau(x,a)}{\mu U} = -\frac{2}{\xi} f''(0) \qquad (10)$$

$$Nu = -\frac{a\left(\frac{\partial T}{\partial y}\right)_{y=0}}{T_w - T_\infty} = -\frac{2}{\xi} G'(0) \qquad (11)$$

Here u and v are the velocity components along x- and y- directions respectively; ξ and η are transformed co – ordinates; ψ and f are the dimensional and dimensionless stream functions, respectively; T and G are the dimension and dimensionless temperatures, respectively; Pr and Ec are Prandtl and Eckert numbers, respectively; M is the nondimensional magnetic parameter; The subscript ξ denote partial derivative with respect to ξ and prime (') denote derivatives with respect to η. Further, the subscripts w and ∞ refers to the conditions at surface and free stream respectively.

It is worth mentioning here that when M = 0.0, Eqn.(6) reduces to

$$(1+\xi\eta)f''' + \xi f'' + \frac{ff''}{2} = \frac{\xi}{2}\left(f'f'_\xi - f''f_\xi\right) \qquad (12)$$

which is exactly same as that of A.Zachara [8] who has studied the laminar momentum boundary layer behavior on a moving cylindrical rod in the absence of magnetic field. On the other hand the Eqn.(6), with an applied magnetic field (M ≠ 0) was solved by Jayakumar and Eswara [13] to obtain the momentum transfer in the boundary layer region on a moving cylindrical rod.

NUMERICAL METHOD

The system of nonlinear partial differential equations (6) and (7) subject to boundary conditions (8) is solved numerically using an implicit finite-difference scheme known as Keller-box method as described in Cebeci and Bradshaw [16]. This method is unconditionally stable and has a second order accuracy with arbitrary spacing.

The method has the following four main steps:

- Reduce (6) and (7) to a system of first order equations;
- Write the difference equations using central differences;
- Linearize the resulting algebraic equations by Newton's method and write them in matrix - vector form;
- Solve the linear system by block-tridiagonal-elimination technique.

To conserve the space, the details of the entire solution procedure of Keller-box method are not presented here. Numerical computations were carried out with the optimized step sizes $\Delta\eta$ in η-direction and $\Delta\xi$ in ξ-direction. Infact, the values of $\Delta\eta$ between 0.1 to 0.2 and the values of $\Delta\xi$ between 0.01 to 0.1 were used so that numerical solutions obtained are independent of $\Delta\eta$ and $\Delta\xi$ chosen, at least to up to three decimal places. . However, a uniform grid $\Delta\eta = 0.2$ and $\Delta\xi = 0.1$ were found to be satisfactory for a convergence criterion of 10^{-4} which gives accuracy to three decimal places. On the otherhand the edge of the boundary layer i.e. $\eta\infty = 8.0$ is used throughout the computations.

RESULTS AND DISCUSSIONS

In order to assess the accuracy of our method of solution, we have compared our skin friction coefficient (C_f) for M = 0.0 (i.e., without magnetic field) with that of Zachara [7] by solving the Eqn.(12) [See Fig.1(b)]. Also, we have compared the numerical results of C_f for $M \neq 0$ by solving the Eqn.(6) with Jayakumar and Eswara [15] [See Fig.2(a)]. Our results are found to be in excellent agreement with the above studies.

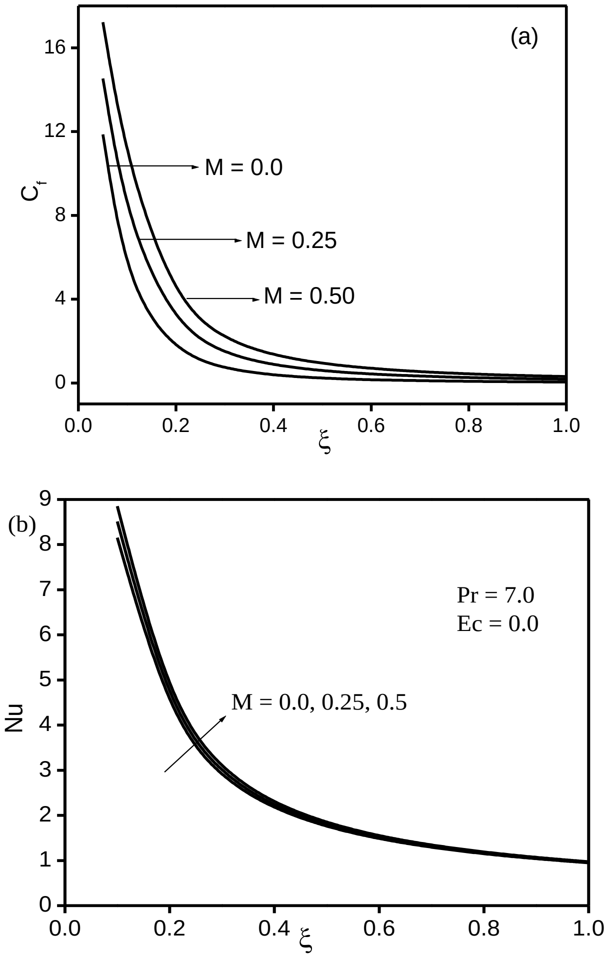

Fig.2 Effect of Magnetic field on (a) skin friction and (b) heat transfer coefficients

The effect of magnetic field (M) on skin friction (C_f) and heat transfer (Nu) coefficients are displayed in Fig.2. As the magnetic field (M) increases the skin friction (C_f) and heat transfer (Nu) coefficients are found to increase for all streamwise locations ξ. The percentage of increase of C_f from M = 0.0 to M = 0.25 is about 38.5 % and it is about 33.8% in the range of M ($0.25 \le M \le 0.50$) at an arbitrary value of ξ (ξ = 0.5). The heat transfer coefficient increases uniformly with the increase of magnetic field.

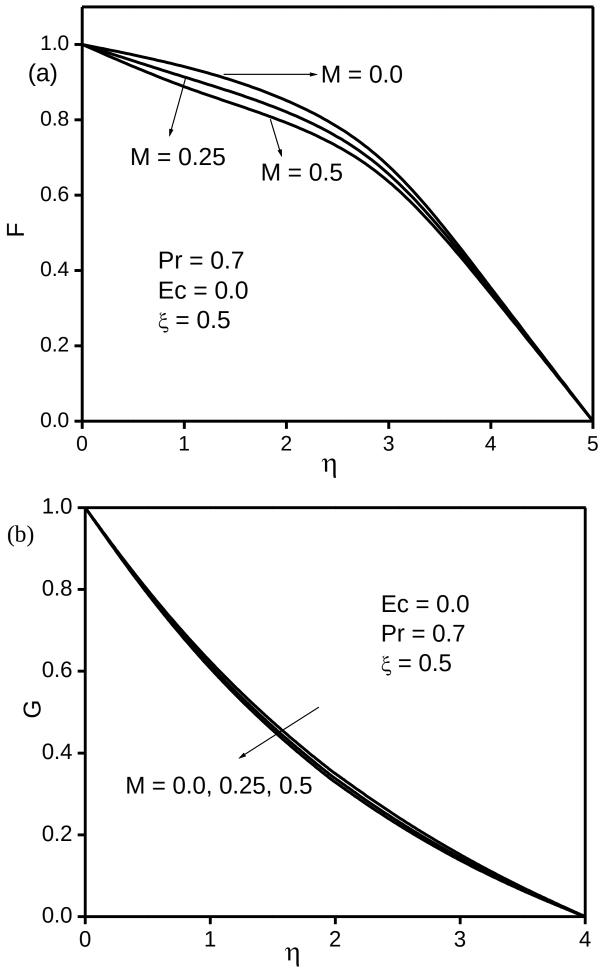

Fig.3 Effect of Magnetic field on (a) Velocity (F) (b) Temperature (G) profiles

Fig.3 shows the effect of magnetic field (M) on velocity (F) and temperature (G) profiles at an arbitrary value of ξ (ξ = 0.5). The magnitude of the velocity and temperature decreases with the increase of magnetic field. It is true for all values of ξ. This results in the reduction of momentum and thermal boundary layer thickness. In fact, the thickness of momentum boundary layer decrease about 3.2% from M = 0.0 to M = 0.25 and is about 2.9% in the range of M ($0.25 \leq M \leq 0.50$) near η = 2.0. Further, the thermal boundary layer thickness decreases uniformly.

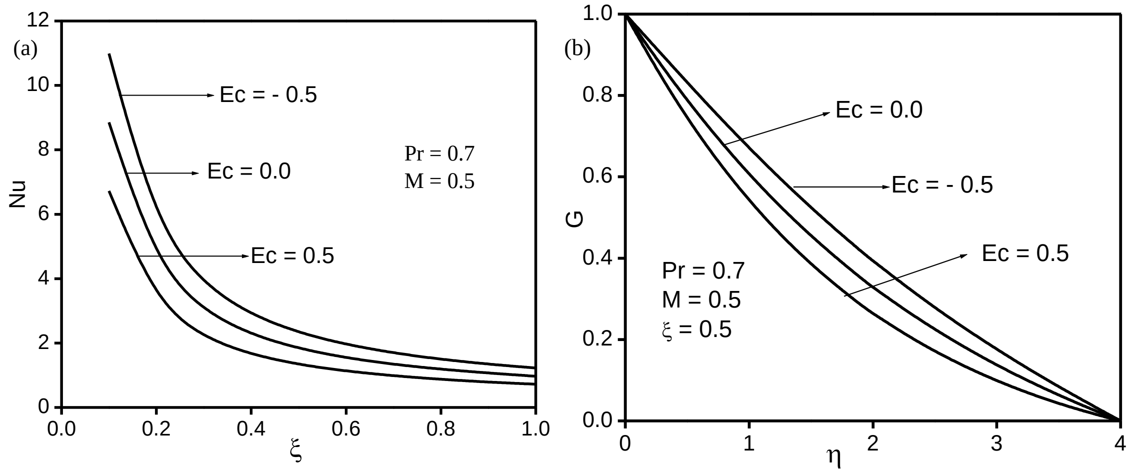

Fig.4. Effect of viscous dissipation on (a) Heat transfer coefficient (b) Temperature profile

Viscous dissipation (*Ec*) is the transformation of energies from internal energy to kinetic energy. Its effect on both heat transfer (*Nu*) and temperature profile (*G*) in the presence of magnetic field (M = 0:5) are shown in Fig.4. It is found that heat transfer decreases with the increase of *Ec* (*Ec* = 0.5) whereas, it increases with the decrease of *Ec* (*Ec* = -0.5) [See Fig.4(a)]. In fact, the percentage of decrease of heat transfer when *Ec* = 0.5 is 49.44% and the percentage of increase in *Nu* when *Ec* = -0.5 is 105.09% at the streamwise location ξ = 0.5. This shows that the heat transfer is strongly depends on viscous dissipation. Also, it is observed that the thermal boundary layer is significantly affected by the viscous dissipation parameter [See Fig.4(b)]. As *Ec* increases the thickness of the thermal boundary layer decreases whereas, the thickness of the thermal boundary layer increases with the decrease of viscous dissipation parameter.

CONCLUSIONS

The steady, laminar incompressible boundary layer flow of an electrically conducting fluid on a moving cylindrical rod with an applied magnetic field has been studied. Nonsimilar solutions of the problem have

been obtained using Keller Box method. The numerical results indicate that, the skin friction and heat transfer

coefficients increases with the increase of magnetic field. The thickness of momentum and thermal boundary layers found to decrease at all axial distances. Further, the heat transfer increases with the decrease of viscous dissipation parameter whereas, it decreases with the increase of viscous dissipation parameter. Also, it is found that the thickness of thermal boundary layer strongly depends on viscous dissipation.

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