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CHEMICAL REACTION EFFECTS ON MHD FLOW PAST AN IMPULSIVELY STARTED ISOTHERMAL VERTICAL PLATE WITH UNIFORM MASS DIFFUSION

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Numerical technique is employed to derive a solution to the transient natural convection flow of an incompressible viscous fluid past an impulsively started Infinite isothermal vertical plate with uniform mass diffusion in the presence of magnetic field and homogeneous chemical reaction of first order. The governing equations are solved using implicit finite-difference method. The effect of velocity, temperature and concentration for different parameters like magnetic field parameter, chemical reaction parameter, Prandtl number, Schmidt number, thermal Grashof number and mass Grashof number are studied. It is observed that the fluid velocity decreases with increasing chemical reaction parameter and magnetic field parameter.

key words : chemical reaction, magnetic field, infinite vertical plate, finite-difference.

1. Introduction

Chemical reactions can be codified as either heterogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. Many transport processes exist in nature and in industrial applications in which the simultaneous heat and mass transfer as a result of combined buoyancy effects of thermal diffusion and diffusion of chemical species. However, in nature, along with free-convection currents caused by temperature differences, the flow is also affected by the differences in concentration. Such a study is found useful in chemical processing industries such as fibre drawing, crystal pulling from the the melt and polymer production.

Magnetoconvection plays an important role in various industrial applications. Examples include magnetic control of molten iron flow in the steel industry, liquid metal cooling in nuclear reactors and magnetic suppression of molten semi-conducting materials. It is of importance in connection with many engineering problems, such as sustained plasma confinement for controlled thermonuclear fusion and electromagnetic casting of metals. MHD finds applications in electromagnetic pumps, controlled fusion research, crystal growing, MHD couples and bearings, plasma jets and chemical synthesis.

Chambre and Young (1958) have analyzed a first order chemical reaction in the neighborhood of a horizontal plate. The effects of transversely applied magnetic field, on the flow of an electrically conducting fluid past an impulsively started infinite isothermal vertical plate was studied by Soundalgekar et al (1979). The dimensionless governing equations were solved using Laplace transform technique.

Kumari and Nath (1999) studied the development of the asymmetric flow of a viscous electrically conducting fluid in the forward stagnation point region of a two-dimensional body and over a stretching surface with an applied magnetic field, when the external stream or the stretching surface was set into impulsive motion from the rest. The governing equations were tackled using implicit finite difference scheme. Das et al (1999) studied an exact solution to the flow of a viscous incompressible fluid past an impulsively started infinite isothermal vertical plate in the presence of mass diffusion and first order chemical reaction. In all above studies, the dimensionless governing equations are solved using Laplace transform technique.

The problem of unsteady natural convection flow past an impulsively started infinite isothermal vertical plate with mass diffusion in the presence of chemical reaction and magnetic field has not received attention of any researcher. Hence, the present study is to investigate the MHD flow past an impulsively started infinite isothermal vertical plate with homogeneous first order chemical reaction by an implicit finite-difference scheme of Crank-Nicolson type.

2. Formulation of the problem

A transient, laminar, unsteady natural convection MHD flow of a viscous incompressible fluid past an impulsively started infinite isothermal vertical plate with uniform mass diffusion is considered. It is assumed that there is a first order chemical reaction between the diffusing species and the fluid. Here, the x-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to the plate. Initially, it is assumed that the plate and the fluid are of the same temperature and concentration. The plate starts moving impulsively in the vertical direction with constant velocity u_0 against gravitational filed. The temperature of the plate is raised uniformly and the concentration level near the plate is also raised at an uniform rate. They are maintained at the same level for all time at time $t' > 0$. A transverse magnetic field of uniform strength B_0 is assumed to be applied normal to the plate. The induced magnetic field and viscous dissipation is assumed to be negligible. Then under the above assumptions, the governing boundary layer equations of mass, momentum and concentration for free convective flow with usual Boussinesq's approximation are as follows:

$$\frac{\partial u}{\partial t'} = g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0{}^2}{\rho}u \tag{2.1}$$

$$\rho C_p \frac{\partial T'}{\partial t'} = \frac{\partial^2 T'}{\partial y^2} \tag{2.2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y^2} - K_l C' \tag{2.3}$$

The initial and boundary conditions are

$$\begin{aligned}
&t' \le 0: \quad u = 0, \quad T' = T'_\infty \quad C' = C'_\infty \\
&t' > 0: \quad u = u_0, \quad T' = T'_w \quad C' = C'_w \qquad at\ y = 0 \\
&\qquad\quad u = 0, \quad T' = T'_\infty, \quad C' = C'_\infty \qquad at\ x = 0 \\
&\qquad\quad u \to 0, \quad T' \to T'_\infty \quad C' \to C'_\infty \qquad as\ y \to \infty
\end{aligned} \tag{2.4}$$

On introducing the following non-dimensional quantities

$$\begin{aligned}
&X = \frac{xu_0}{\nu},\ Y = \frac{yu_0}{\nu},\ U = \frac{u}{u_0},\ t = \frac{t' u_0{}^2}{\nu} \\
&T = \frac{T' - T'_\infty}{T'_w - T'_\infty},\ Gr = \frac{\nu g\beta(T'_w - T'_\infty)}{u_0{}^3},\ C = \frac{C' - C'_\infty}{C'_w - C'_\infty},\ Gc = \frac{\nu g\beta(C'_w - C'_\infty)}{u_0{}^3} \\
&\Pr = \frac{\nu}{\alpha},\ Sc = \frac{\nu}{D},\ K = \frac{\nu K_l}{u_0^2},\ M = \frac{\sigma B_0^2 \nu}{\rho u_0^2}
\end{aligned} \tag{2.5}$$

Equations (2.1) to (2.3) are reduced to the following non-dimensional form

$$\frac{\partial U}{\partial t} = GrT + GcC + \frac{\partial^2 U}{\partial Y^2} - MU \tag{2.6}$$

$$\frac{\partial T}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 T}{\partial Y^2} \tag{2.7}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} - KC \tag{2.8}$$

The corresponding initial and boundary conditions in non-dimensional quantities are

$$\begin{aligned}
&t \le 0: \quad U = 0, \quad T = 0, \quad C = 0 \\
&t > 0: \quad U = 1, \quad T = 1, \quad C = 1 \qquad at\ Y = 0 \\
&\qquad\quad U = 0, \quad T = 0, \quad C = 0 \qquad at\ X = 0 \\
&\qquad\quad U \to 0, \quad T \to 0, \quad C \to 0 \qquad as\ Y \to \infty
\end{aligned} \tag{2.9}$$

3. Numerical Technique

In order to solve these unsteady, non-linear coupled equations (2.6) to (2.8) under the conditions (2.9), an implicit finite difference scheme of Crank-Nicolson type has been employed. The finite difference equations corresponding to equations (2.6) to (2.8) are as follows:

$$\frac{\left[U_{i,j}^{n+1}-U_{i,j}^{n}\right]}{\Delta t}=\frac{Gr}{2}\left[T_{i,j}^{n+1}-T_{i,j}^{n}\right]+\frac{Gc}{2}\left[C_{i,j}^{n+1}-C_{i,j}^{n}\right]-\frac{M}{2}\left[U_{i,j}^{n+1}+U_{i,j}^{n}\right]+\frac{\left[U_{i,j-1}^{n+1}-2U_{i,j}^{n+1}+U_{i,j+1}^{n+1}+U_{i,j-1}^{n}-2U_{i,j}^{n}+U_{i,j+1}^{n}\right]}{2\left(\Delta Y\right)^{2}} \quad (3.1)$$

$$\frac{\left[T_{i,j}^{n+1}-T_{i,j}^{n}\right]}{\Delta t}=\frac{1}{\Pr}\frac{\left[T_{i,j-1}^{n+1}-2T_{i,j}^{n+1}+T_{i,j+1}^{n+1}+T_{i,j-1}^{n}-2T_{i,j}^{n}+T_{i,j+1}^{n}\right]}{2\left(\Delta Y\right)^{2}} \quad (3.2)$$

$$\frac{\left[C_{i,j}^{n+1}-C_{i,j}^{n}\right]}{\Delta t}=\frac{1}{Sc}\frac{\left[C_{i,j-1}^{n+1}-2C_{i,j}^{n+1}+C_{i,j+1}^{n+1}+C_{i,j-1}^{n}-2C_{i,j}^{n}+C_{i,j+1}^{n}\right]}{2\left(\Delta Y\right)^{2}}-\frac{K}{2}(C_{i,j}^{n+1}+C_{i,j}^{n}) \quad (3.3)$$

Here the region of integration is considered as a rectangle with sides *Xmax* (=1) and Y_{max}(=14), where Y_{max} corresponds to $Y = \infty$ which lies very well outside both the momentum and energy boundary layers. The maximum of *Y* was chosen as 14 after some preliminary investigations so that the last two of the boundary conditions (2.9) are satisfied with in the tolerance limit 10^{-5}.

After experimenting with a few set of mesh sizes, the mesh sizes have been fixed at the level $\Delta X = 0.05$, $\Delta Y = 0.25$ with time step $\Delta t = 0.01$. In this case, the spatial mesh sizes are reduced by 50% in one direction, and later in both directions, and the results are compared. It is observed that, when the mesh size is reduced by 50% in the *Y*-direction, the results differ in the fifth decimal place while the mesh sizes are reduced by 50% in *X*-direction or in both directions, the results are comparable to three decimal places. Hence, the above mesh sizes have been considered as appropriate for calculation. The coefficient $U_{i,j}^{n}$ appearing in the finite-difference equations are treated as constants in any one time step. Here *i*-designates the grid point along the *X*-direction, *j* along the *Y*-direction and the superscript n along the t-direction. The values of *U, V* and T are known at all grid points at $t = 0$ from the initial conditions.

The computations of *U, T* and *C* at time level (n +1) using the values at previous time level (n) are carried out as follows: The finite-difference equation (3.3) at every internal nodal point on a particular *i*-level constitute a tridiagonal system of equations. Such a system of equations are solved by using Thomas algorithm as discusses in Carnahan et al (1969).Thus, the values of *C* are found at every nodal point for a particular *i* at $(n+1)^{th}$ time level. Similarly, the values of *T* and *U* are calculated from equations (3.2) and (3.1) respectively. This process is repeated for various *i*-levels.Thus the values of *C, T,* and *U* are known, at all grid points in the rectangle region at $(n+1)^{th}$ time level.

4. Results and Discussion

The velocity profiles for different magnetic parameter are shown in Fig.1. The velocity profiles presented are those at $X = 1.0$. It is observed that for (M = 0, 2, 5, 10), $K = 2$, Gr =2, Gc =5, Pr = 0.71, and Sc=0.6, the velocity decreases in the presence of magnetic field than its absence. This shows that the increase in the magnetic field parameter leads to a fall in the velocity. This agrees with the expectations, since the magnetic field exerts a retarding force on the free convective flow.

The effect of velocity for different chemical reaction parameter (K = 2, 5, 10), M = 2, Gr = 2, Gc = 5, Pr = 0.71 and Sc = 0.6 are shown in Fig.2. It is observed that the velocity increases with decreasing chemical reaction parameter.

The temperature profiles for different values of the Prandtl number and chemical reaction parameter are shown in Fig.3. The temperature increases with increasing chemical reaction parameter and decreases with increasing Prandtl number. This shows that the buoyancy effect on the temperature distribution is very significant in air (Pr = 0.71) compared to water (Pr = 7.0). It is known that the Prandtl number plays an important role in flow phenomena, because it is a measure of the relative magnitude of viscous boundary layer thickness to the thermal boundary layer thickness.

The effect of the chemical reaction parameter and the Schmidt number is very important for concentration profiles. The steady-state concentration profiles for different values of the chemical reaction parameter and Schmidt number are shown in Fig.4. There is a fall in concentration due to increasing the values of the chemical reaction parameter or Schmidt number.

5. Conclusions

A detailed numerical study of the incompressible viscous fluid MHD flow past an impulsively started infinite isothermal vertical plate with uniform mass diffusion in the presence of homogeneous chemical reaction of first order, is presented in the paper. Dimensionless governing equations are solved using the implicit finite-difference scheme of the Crank-Nicolson type. The fluids considered in this study are air and water. The study performed allows the following conclusions.

1. The velocity and concentration increases with decreasing chemical reaction parameter.
2. The temperature increases with increasing chemical reaction parameter and decreases with increasing Prandtl number.

Nomenclature

a^* - absorption coefficient
B_0 - magnetic field strength
g - accelaration due to gravity
D - mass diffusion coefficient
Gr - thermal Grashof number
Gc- mass Grashof number
M - magnetic field parameter
Nu_x- dimensionless local Nusselt number
$\overline{\mathrm{Nu}}$ - dimensionless average Nusselt number
Pr - Prandtl number
K_l - dimensionless chemical reaction parameter
K - chemical reaction parameter

T' - temperature
T - dimensionless temperature
C' - concentration
C - dimensionless concentration
t' - time
t - dimensionless time
u_0 - velocity of the plate
u, v - velocity components in x, y-directions respectively
U, V - dimensionless velocity components in X, Y-directions respectively
x - spatial coordinate along the plate
X - dimensionless spatial coordinate along the plate
y - spatial coordinate normal to the plate
Y - dimensionless spatial coordinate normal to the plate

Greek symbols

α - thermal diffusivity
β - coefficient of volume expansion
β^* - volumetric coefficient of expansion with concentration
μ - coefficient of viscosity
ν - kinematic viscosity
σ - Stefan-Boltzmann constant
τ_x - dimensionless local skin-friction
$\bar{\tau}$ - dimensionless average skin-friction

Subscripts

w - conditions at the wall
∞ - conditions in the free stream
i - grid point along the *X*-direction
j - grid point along the *Y*-direction

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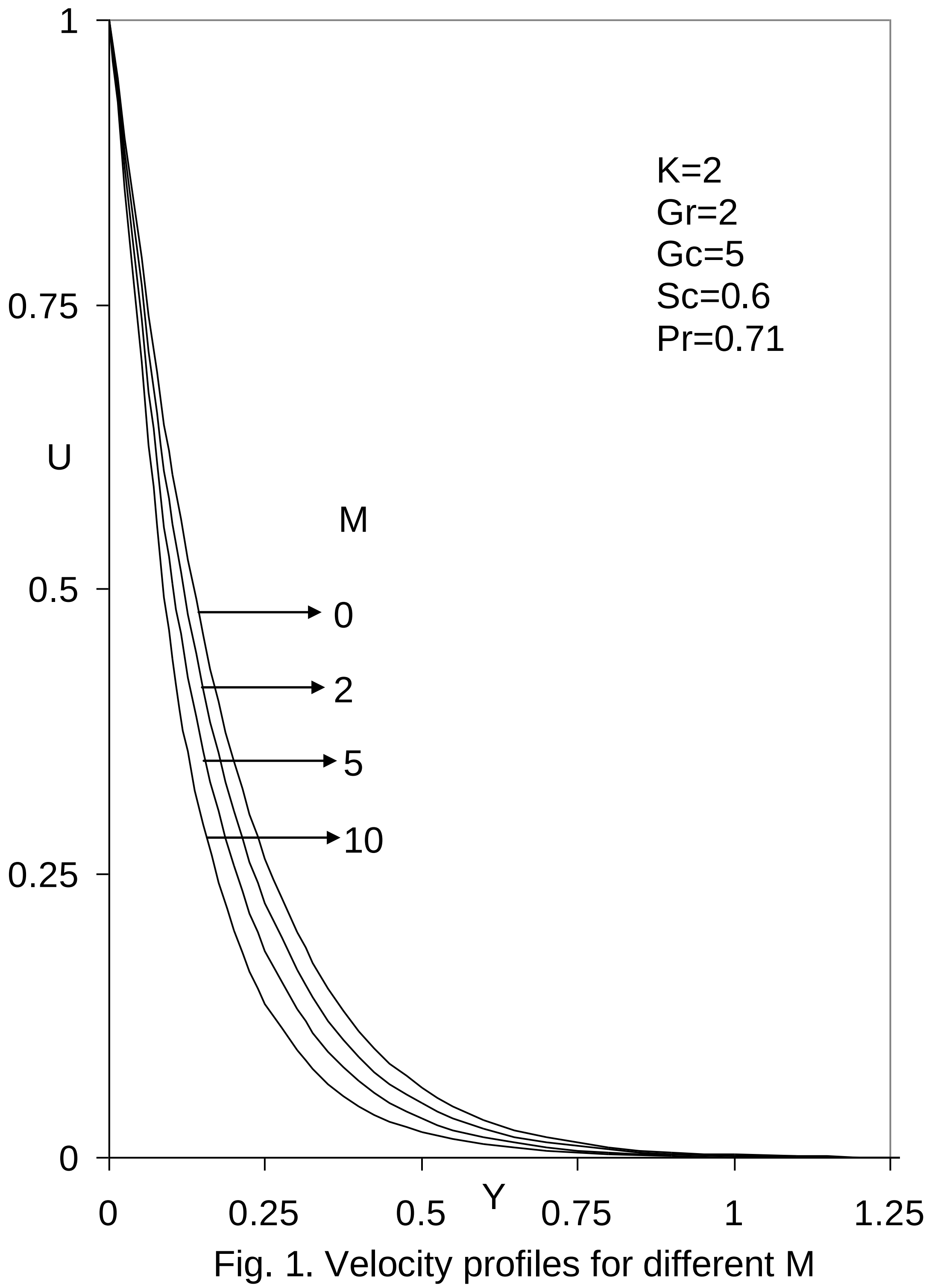

Fig. 1. Velocity profiles for different M

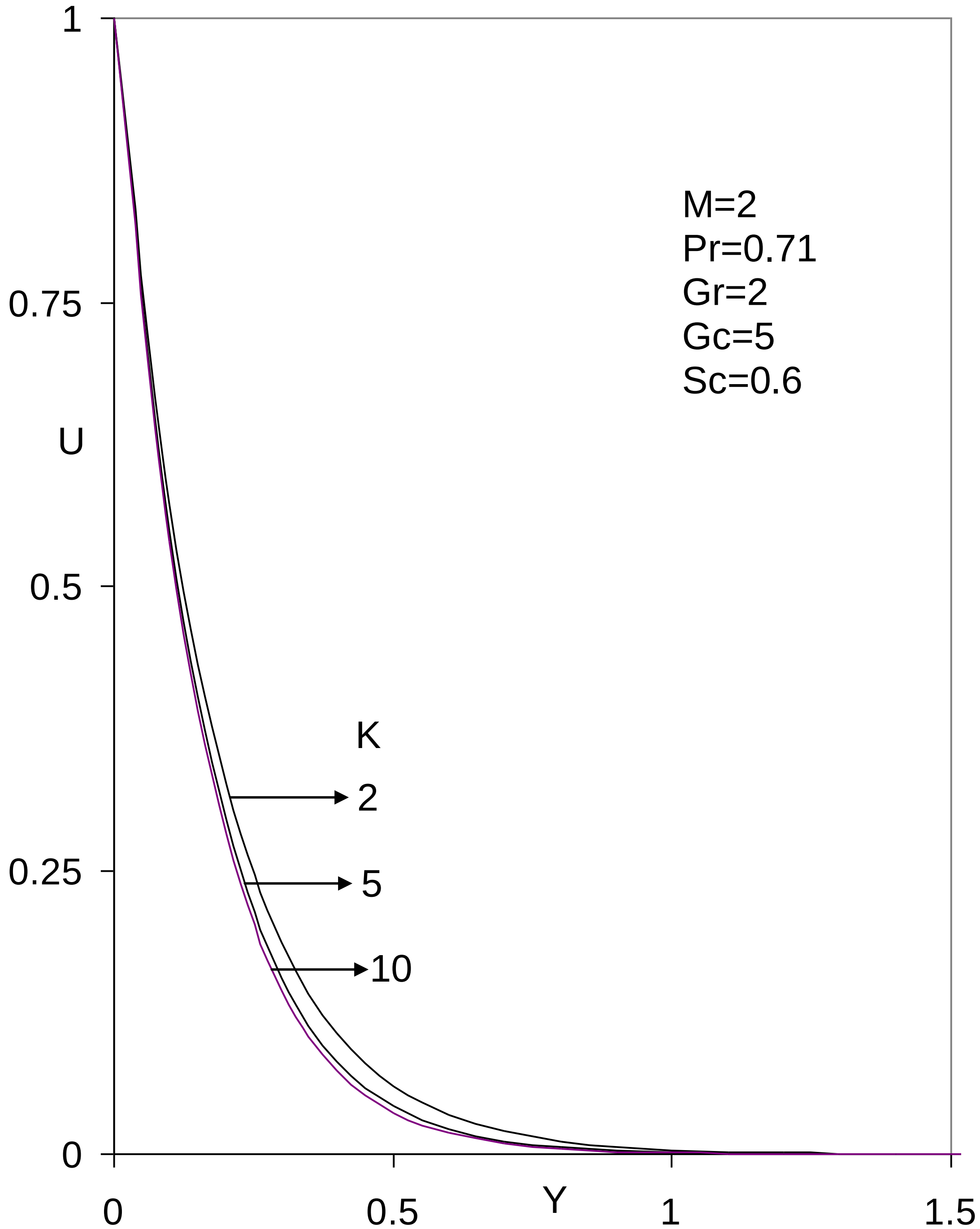

Fig. 2. Velocity profiles for different K

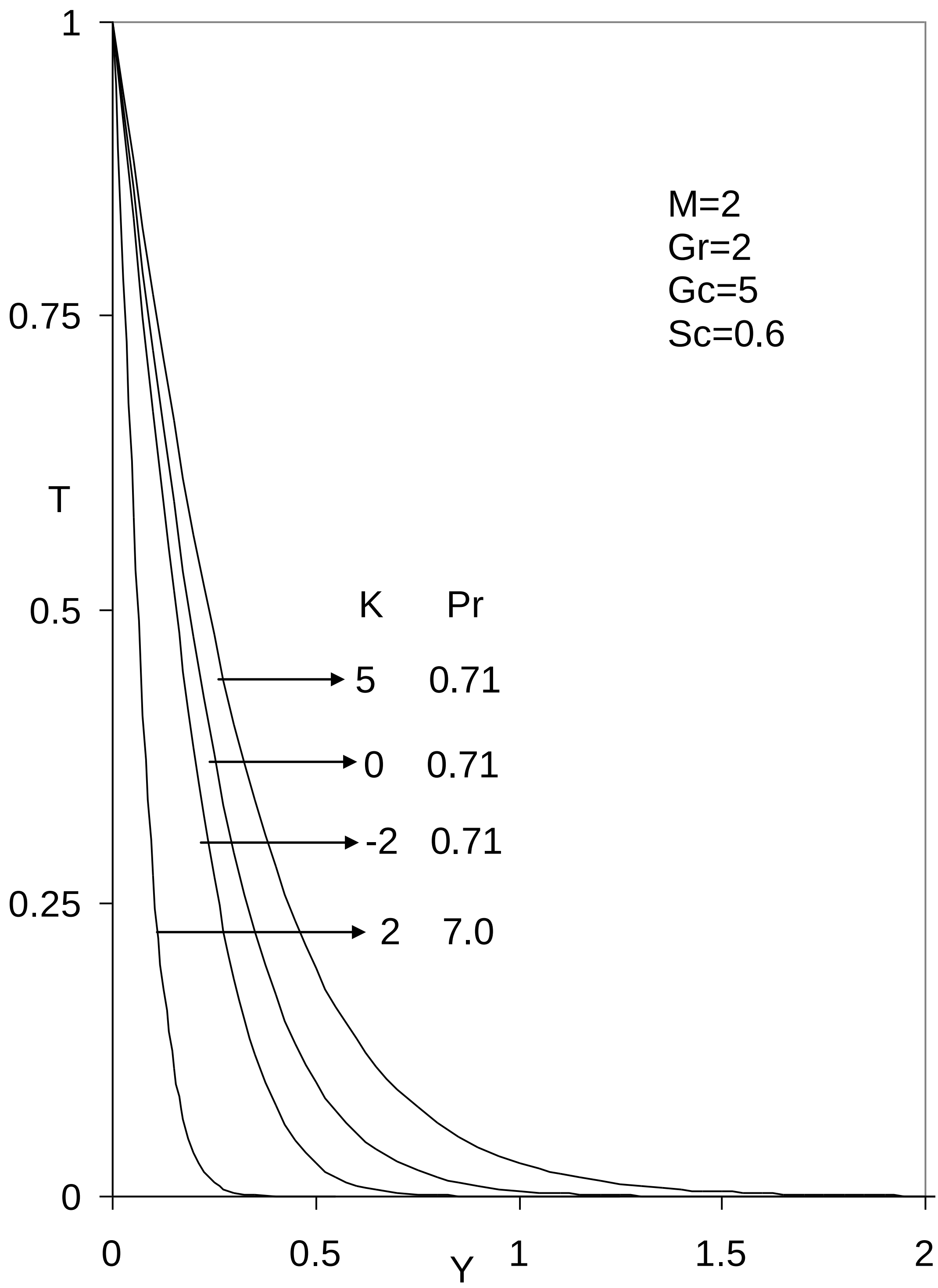

Fig. 3. Temperature profiles for different K and Pr

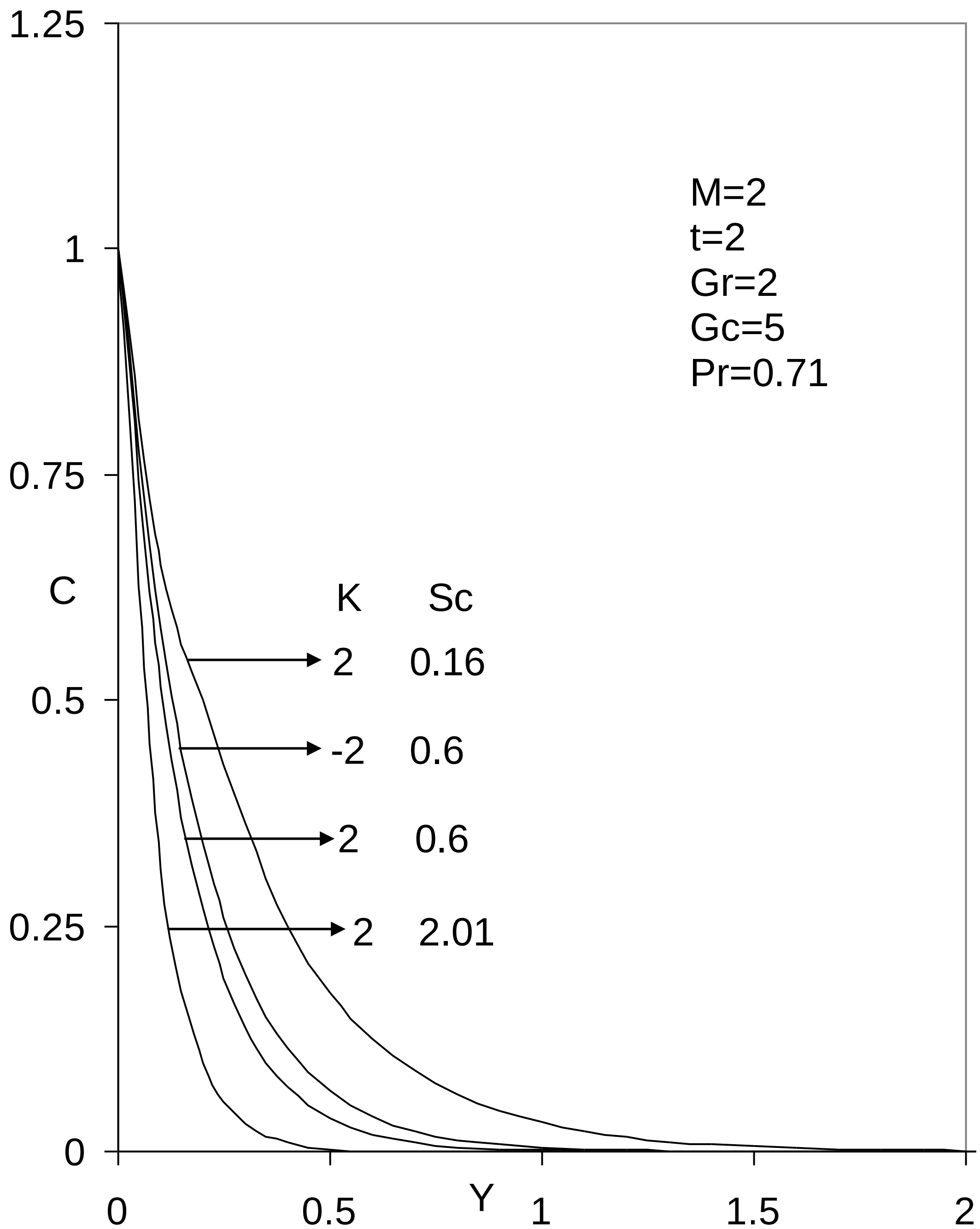

Fig. 4. Concentration profiles for different K and Sc