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STEADY MHD MARANGONI CONVECTION FLOW WITH VARIABLE ELECTRICAL CONDUCTIVITY AND EFFECTS OF JOULE HEATING AND VISCOUS DISSIPATION

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Abstract

An analysis is performed to study the effect of temperature dependent electrical conductivity on steady natural convection flow of a viscous incompressible low Prandtl (Pr < < 1) electrically conducting fluid along an isothermal vertical non-conducting plate in the presence of transverse magnetic field and effects of Joule heating and viscous dissipation. The governing equations of continuity, momentum and energy are transformed into ordinary differential equations via the similarity transformations. The resulting coupled non-linear ordinary differential equations are solved using Cranck-Nicolson method. The velocity and temperature distributions are discussed numerically and presented through graphs.

Keywords: Steady, MHD, Marangoni Convection, boundary layer, variable electrical conductivity, joule heating, viscous dissipation.

Introduction

In recent years, many researchers have investigated the Marangoni convection and applied such convection into their problems. Marangoni convection is induced by the variations of the surface tension gradients at the surface of immiscible fluids Golia and Viviani[1]. Kang and Kashiwagi[2] have studied the effect of the Marangoni convection in the ammonia-water absorption process. Tan[3] investigated the gas diffusion in liquids that can cause the Marangoni convection. Further, Christopher and Wang[4] studied the Prandtl number effect on the Marangoni convection.

On the other hand, the study of Magnetohydrodynamics (MHD) is important in the heat and mass transport process. The study of heat transfer is integral part of natural convection flow and belongs to the class of problems in boundary layer theory. The quantity of heat transferred is highly dependent upon the fluid motion within the boundary layer. A large number of physical phenomena involve natural convection was studied by Jaluria[5], which are enhanced and driven by internal heat generation. In such flows the buoyancy force is incremented due to heat generation resulting in modification of heat transfer characteristic. The effect of internal heat generation is especially pronounced for low Prandtl number fluid. MHD is the study of motion of electrically conducting incompressible fluid in the presence of magnetic field i.e. an electromagnetic field interacting with the velocity field of an electrically conducting fluid. Hydromagnetic flows have become important due to industrial applications, for instance it is used to deal with the problem of cooling of nuclear reactor by fluid having very low Prandtl number was studied by Michiyoshi et. al[6] and Fumizawa[7].

The effect of viscous dissipation and Joule heating are usually characterized by the Eckert number and the product of the Eckert number and the magnetic parameter, respectively, and both effects are important in nuclear engineering was studied by Alim et. al[8]. Duwairi[9] had presented the effects of Joule heating and viscous dissipation on the forced convection flow in the present of thermal radiation. Aissa and Mohammadein[10] have analyzed the effects of the magnetic parameter, Joule heating, viscous dissipation and heat generation on ht MHD micropolar fluids that past through a stretching sheet.

Aim of the present study is to investigate the effects of Joule heating and viscous dissipation on the MHD Marangoni convection with varying electrical conductivity.

Formulation of the Problem

We consider the steady, two-dimensional, laminar boundary layer flow of a viscous incompressible fluid of an variable electrically-conducting fluid over a flat surface. The surface is assumed to be in the presence of surface tension due to temperature. Further, a strong magnetic field of strength B_0 is applied normal to the surface which then produces the magnetic forces along the surface. Under the above assumptions and with the usual boundary layer approximations the governing equations are as follows

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \ldots(1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = v\frac{\partial^2 v}{\partial y^2} + g\beta\,(T - T_\infty) + g\beta^*\,(C - C_\infty) - \frac{\sigma^* B_0^2 u}{\rho} - \frac{vu}{k} \qquad \ldots(2)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\sigma^* B_0^2 u^2}{\rho c_p} + \frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 \quad \text{...(3)}$$

where electrical conductivity σ^* is variable with temperature as given below

$$\sigma^* = \frac{\sigma}{1+\varepsilon\theta} \quad \text{...(4)}$$

The boundary conditions are

$$y = 0 : u = 0,\ v = 0,\ T = T_w$$

$$y \to \infty : u \to 0,\ v \to 0,\ T \to T_\infty \quad \text{...(5)}$$

Method of Solution

Introducing the stream function $\psi(x, y)$ such that $u = \frac{\partial \psi}{\partial y}$ and $v = \frac{\partial \psi}{\partial x}$ ...(6)

where $\psi(x, y) = 4\nu f(\eta)\left(\frac{Gr}{4}\right)^{1/4}$ and the similarity variable $\eta = \frac{y}{x}\left(\frac{Gr}{4}\right)^{1/4}$ ...(7)

Here, Magnetic field intensity B_0 must be propotional to $x^{-1/4}$ (Chen[11]), in order to eliminate the dependency of M on x.

It is observed that the equation (1) is identical satisfied with equation (6). Substituting equations (7) and into the equations (2) and (3), along with the quation (4), the resulting coupled non-linear ordinary differential equations are

$$f''' - 2f'^2 + 3ff'' + \theta - \frac{M}{1+\varepsilon\theta} f' = 0 \quad \text{...(8)}$$

and

$$\theta'' + 3\mathrm{Pr}\theta' f + \mathrm{Ec}[\mathrm{M}^2(f')^2 + (f'')^2] = 0 \quad \text{...(9)}$$

The boundary conditions are reduced to

$$f(0) = 0,\ f'(0) = 0,\ f'(\infty) = 0,\ \theta(0) = 1 \text{ and } \theta(\infty) = 0 \qquad \ldots(10)$$

The governing boundary layer equations (8) and (9) with boundary conditions (10) are solved using Cranck-Nicolson Implicit Scheme.

Results and Discussion

It is observed from the Figure 1 that the variations of the surface temperature gradient with the Eckert number for different values of the Prandtl number i.e. Pr = 0.7, 7 and 100. It is worth mentioning that Ec = 0 means that there are no effects of Joule heat and viscous dissipation while Ec>0 indicates the combined effects of Joule heating and viscous dissipation. The figure shows that as Ec increases, the surface temperature gradient decreases. Further, when we consider higher Prandtl number, we notice further further reduction of the surface temperature gradients.

Meanwhile, as seen in the Figure2, the parameter M tends to increase the temperature profiles. However, the temperature profile can be reduced using higher Prandtl number. Further, Figure 3 and 4 present the combined effects of the Joule heating and viscous dissipation on the temperature profiles when M = 1 for Pr = 0.7 and 7 respectively. It is noted that increasing the values of the Eckert number will produce an increase in the temperature profiles for Prandtl number considered.

It is observed from figure 5 that the effect of electrical conductivity parameter ε on the velocity profiles. The fluid with high electrical conductivity experience more retarding force in the presence of transverse magnetic field and therefore the fluid velocity decreases, but as the parameter ε increases (which mean that electrical conductivity of fluid decreases) the retarding force acting on the fluid decline and hence the fluid velocity increases. Thus the increase in parameter ε has a kind of nullifying effect on magnetic parameter. The effect of ε is even more for higher value of magnetic parameter as can be seen from apartness in velocity profiles at higher value of magnetic parameter.

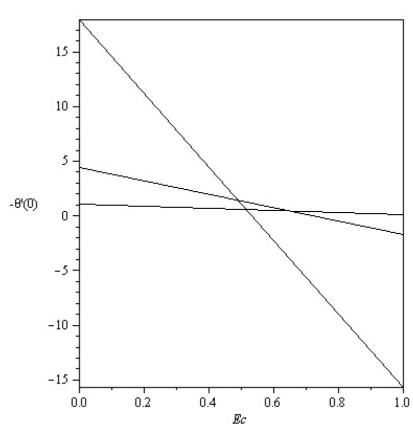

Fig. 1 Variations of the surface temperature gradients, -θ(0) with the Eckert number, Ec for different Prandtl number Pr

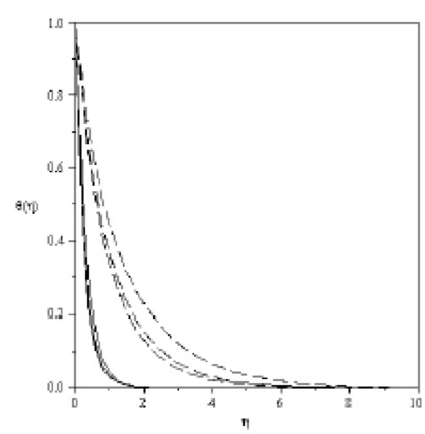

Fig. 2 Effects of M on the temperature profiles when Ec= 0.2 for different Pr.

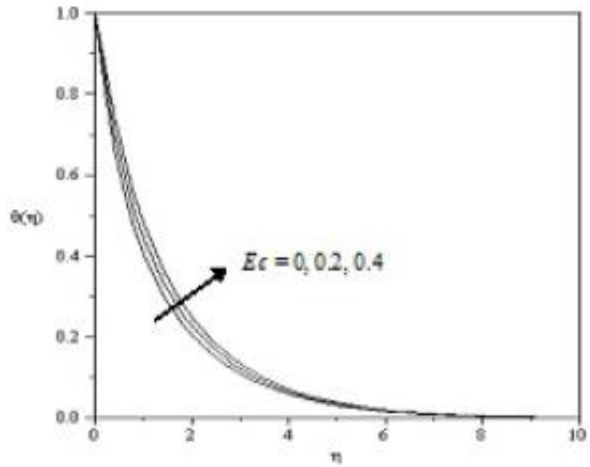

Fig. 3 Effects of Ec on the temperature profiles when Pr = 0.7, M = 1.

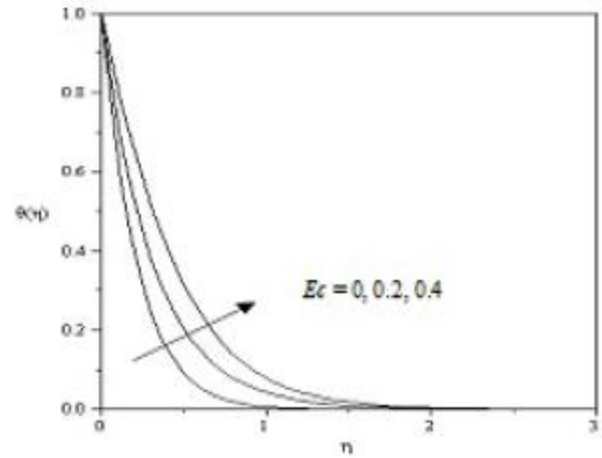

Fig. 4 Effects of Ec on the temperature profiles when Pr= 7, M =1.

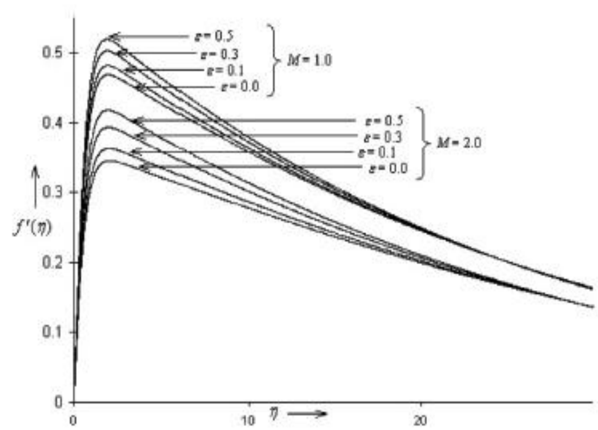

Fig. 5 Velocity distribution versus η when Pr = 0.001.

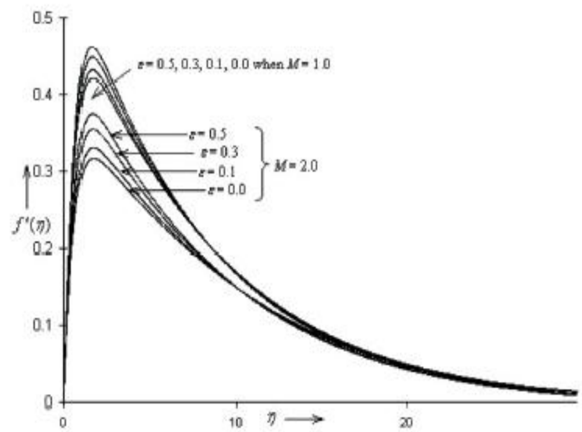

Fig. 4 Velocity distribution versus η when Pr = 0.01

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