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# A Solution of Linear-plus-Linear Fractional Multiobjective Programming Problem

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**Abstract.** In this paper, we proposed a solution procedure for solving multiobjective linear-plus-linear fractional programming problems (MOLLFP).We have positive objective functions for all $x \in X$ by translating objective functions that are negative for some $x \in X$ .Then we have single objective function with quadratic constraints as minimization of the deviations from maximum values of objective functions in the feasible region. So MOLLFP is transformed to goal programming problem.The proposed algorithm always yeilds efficient solution. Three examples are given to illustrate this algorithm.

**Keywords:** Multiobjective Programming, Fractional Programming.

**AMS 2000 Subject Classification:** 90C29, 90C32

## 1.Introduction

The linear fractional programming (LFP) problem, which has been used as an important planning tool for the past four decades, is applied to different deciplines such as financial and corporate sector, inventory management, production planning, marketing and media selection, university planning and student admissions, health care and hospitality, judiciary system, air force maintenance units, banking sector, and others. Fractional progamming is generally used for modeling real life problems with one or more objective(s) such as dept/equity ratio, profit/cost, inventory/sales, actual cost/standard cost, output/employee, student/cost, nurse/patient ratio etc. respect to some constraints.

In literature, different approaches appear to solve different models of linear fractional programming problem (LFPP).

When some of the studies have achieved solution methods [2, 3, 8, 9] others have concentrated on applications [4, 5, 6]. In these studies, the details of LFP is discussed. It is shown that a linear fractional programming problem (LFPP) can be optimized easily. But at the large scale in decision problems, there are more than one objective functions which must be satisfied at the same time as possible. However, most of these are linear fractional objectives. It is difficult to talk about the optimal solutions of these problems. The solutions searched for these problems are weak efficent or strong efficent.

There exist several methodologies to solve multiobjectives linear fractional programming problem (MOLFPP) in literature. Most of these methodologies are computationally burdensome [7]. Kornbluth and Steur [11] have developed an algorithm for solving the MOLFPP for all weak efficent vertices of the feasible region. Nykowski et al. [12] have proposed a compromise procedure for MOLFPP. Choo and Atkins [10] have given an analysis of the bicriteria LFP.

Luhendjula [13] solved MOLFPP using a fuzzy approach. He used linguistic approach to solve MOLFPP by introducing linguistic variables to represent Linguistic aspirations of the decision making (DM). Dutta et al. [14] modified the linguistic approach of Luhandjula [13] to solve MOLFPP. Nuran Guzel and Mustafa Sivri gave Taylor series approach to solve MOLFPP [1] and M. Duran Toksari also developed an algorithm to solve FMOLFPP [ 22 ] by Taylor series approach.

Teterev [18] pointed out that such problems arise when a compromise between absolute and relative terms is to maximized. Major applications of the linear-plus-linear programming problem can be found in transpotation problem, problem of optimizing enterprise capital, the production development fund and the social, cultural and construction fund. Teterev [18] derived an optimality criteria fot (LLFP) using simplex type algorithm. Several authors [17, 19, 20, 21] studied the problem (LLFP) and its variants and have discussed their solution properties. we developed an algorithm for solving multiobjective linear-plus-linear fractional programming problems. We gave three numerical examples to apply proposed algorithm.

## 2. Fractional Programming Problem

In this section, LFP, MOLFP, LLFP, MOLLFP, are described mathematically.

### 2.1. Linear Fractional Programming (LFP)

The genaral format of Linear Fractional Programming may be written as;

$$Max\frac{c^T x+\alpha}{d^T x+\beta}$$

$$s.t.$$

$$Ax=b \qquad (2.1)$$

$$x\geq 0$$

$$x\in R^n, c^T, d^T \in R^n,\ A\in R^{m\times n},\ \alpha,\beta\in R^n, b\in R^m$$

For some values of x, $d^T x+\beta$ may be equal to zero. To avoid such cases, $d^T x+\beta$ is generally set to greater than zero.

### 2.2. Multiobjective Linear Fractional Programming (MOLFP)

A MOLFPP may be written as following;
$Max\, Z(x) = \{Z_1(x), Z_2(x)........ Z_k(x)\}$

$$Z_i(x) = \frac{c_i x + \alpha_i}{d_i x + \beta_i} = \frac{N_i(x)}{D_i(x)} \quad \text{where } D_i(x) > 0, \text{ for all i} \qquad (2.2)$$

s. t.
$x \in X = \{x \in R^n; Ax \le b,\ x \ge 0\}$ with $\alpha_i, \beta_i \in R^n, b \in R^m, A \in R^{m\times n}$
and let the $Max\, Z_i(x) = Z_i^*$.

### 2.3. Linear-plus-Linear Fractional Programming (LLFP)

A general linear- plus linear fractional programming problem has the following form

$$(LLFP) \qquad Max \quad F(x) = (p^T x + \theta) + \frac{c^T x + \alpha}{d^T x + \beta}$$

$$\begin{aligned} & s.\,t. \\ & Ax = b \qquad (2.3) \\ & x \ge 0 \\ & x \in R^n,\ c^T,\ d^T,\ p^T \in R^n,\ A \in R^{m\times n},\ \alpha,\ \beta,\ \theta, \in R^n, b \in R^m \end{aligned}$$

### 2.4. Multiobjective Linear-plus-Linear Fractional Programming (MOLLFP)

A multiobjective linear- plus-linear fractional programming problem may be defined as
$Max / Min\ Z(x) = \{Z_1(x),\ Z_2(x)........ Z_k(x)\}$

$$Z_k(x) = (L_{ik} x + l_{ik}) + \frac{c_{ik} x + \alpha_{ik}}{d_{ik} x + \beta_{ik}} = \frac{N_{ik}}{D_{ik}}, \text{ where } D_{ik}(x) > 0, \text{ for all i} \qquad (2.4)$$

s. t.
$x \in X = \{x \in R^n; Ax \le b,\ x \ge 0\}$ with $b \in R^m, A \in R^{m\times n}$
$c_{ik}, d_{ik}, \alpha_{ik}, \beta_{ik}, L_{ik}, l_{ik} \in R^n$. We restrict ourself for only the values for which denomenator of the fractional part of the objective functions is non-zero.

### 3. Algorithm development

In this paper, we consider the multiobjective linear plus fractional programming problem of the form
$Max \quad Z(x) = \{Z_1(x),\ Z_2(x)........ Z_k(x)\}$
s.t. $\quad x \in X = \{x \in R^n; Ax \le b,\ x \ge 0\}$ with $b \in R^m, A \in R^{m\times n}$

and $Z_i(x) = (L_i x + l_i) + \dfrac{c_i x + \alpha_i}{d_i x + \beta_i} = \dfrac{N_i(x)}{D_i(x)}$, where $D_i(x) > 0$, for all i (3.1)

$c_i, d_i, \alpha_i, \beta_i, L_i, l_i \in R^n$

Let $x_i^*$ be a global maximum point of $Max\, Z_i(x) = (L_i x + l_i) + \dfrac{c_i x + \alpha_i}{d_i x + \beta_i} \ \forall i$

The algorithm for solving MOLLFP is given below :

**Step-1.** Find values of objective functions, $Z^* = \{Z_1^*, Z_2^*, Z_3^*,.....Z_k^*\}$, that maximizes objective functions

$$Max\, Z_i(x) = (L_i x + l_i) + \frac{c_i x + \alpha_i}{d_i x + \beta_i} \quad \forall i \text{ at feasible region.} \qquad (3.2)$$

$$x \in X = \{x \in R^n; Ax \le b, x \ge 0, \ b \in R^m, A \in R^{m\times n}\}$$

**Step-2.** Calculate to $Z_i^{Min} = Min\{(L_i x + l_i) + \frac{c_i x + \alpha_i}{d_i x + \beta_i}\}$. If $Z_i^* < 0$ for some i and do positive $Z_i(x) \ge 0$ using the transformation $Z_i(x) = Z_i(x) - Z_i^{Min}(x)$, then calculate new maximum objective values $Z_i^* = Z_i^*(x) - Z_i^{Min}$.

**Step 3.** Get to $(L_i x + l_i) + \frac{c_i x + \alpha_i}{d_i x + \beta_i} \le (L_i x_i^* + l_i) + \frac{c_i x_i^* + \alpha_i}{d_i x_i^* + \beta_i}$ for i = 1, 2, ..k (3.3)

Where $(L_i x_i^* + l_i) + \frac{c_i x_i^* + \alpha_i}{d_i x_i^* + \beta_i}$ is a quantitative value of the linear- plus linear fractional objective function.

**Step-4.** Write as

$$\{(L_i x + l_i)(d_i x + \beta_i) + (c_i x + \alpha_i)\} - \{(L_i x_i^* + l_i) + \frac{c_i x_i^* + \alpha_i}{d_i x_i^* + \beta_i}\}(d_i x + \beta_i) \le 0 \qquad (3.4)$$

With $(d_i x + \beta_i) > 0$.

**Step-5.** In order to satisfy these targets in step (4). The goal programming model for MOLLFP is proposed as below

$$\{(L_i x + l_i)(d_i x + \beta_i) + (c_i x + \alpha_i)\} - \{(L_i x_i^* + l_i) + \frac{c_i x_i^* + \alpha_i}{d_i x_i^* + \beta_i}\}(d_i x + \beta_i) + \bar{h} = 0 \quad (3.5)$$

for i = 1, 2,..k.

Where $\bar{h}$ is a negative deviation from the i-th goal $\{(L_i x_i^* + l_i) + \frac{c_i x_i^* + \alpha_i}{d_i x_i^* + \beta_i}\}(d_i x + \beta_i)$.

As the linear fractional objectives in problem (1) should be maximized then the total deviation from the goals, $\sum_{i=1}^{k} \bar{h}_i \ge 0$ is should be minimized. Let s = $\min \sum_{i=1}^{k} \bar{h}_i$. Then model with a single objective function and quadratic constraints for MOLLFP may be written as the minimization of the total deviations from the above defined goals as below:

$$s = \underset{x\in X}{MIn} \sum_{i=1}^{k} \bar{h}_i$$

s.t.

$$\{(L_i x + l_i)(d_i x + \beta_i) + (c_i x + \alpha_i)\} - \{(L_i x_i^* + l_i) + \frac{c_i x_i^* + \alpha_i}{d_i x_i^* + \beta_i}\}(d_i x + \beta_i) + \bar{h} = 0 \quad (3.6)$$

for i = 1, 2,..k

**Step-6.** Find optimal solution of the above linear programming problem(3.6)

In problem (3.6), set X is non- empty convex set having feasible points. In fact, the solution of problem (3.6) gives the efficients solutiobn of problem (3.1).

## 4. Numerical Examples

**Example 1.** Let us cosider a MOLPFPP with two objective functions

$$Max\left\{Z_1(x) = (-x_1 - 1) + \frac{-3x_1 + 2x_2}{x_1 + x_2 + 3}, Z_2(x) = (x_2 + 1) + \frac{7x_1 + x_2}{5x_1 + 2x_2 + 1}\right\}$$

$$s.t. \qquad (4.1)$$

$$x_1 - x_2 \geq 1$$
$$2x_1 + 3x_2 \leq 15$$
$$x_1 \geq 3$$
$$x_1, x_2 \geq 0$$

It is observed that $Z_1 < 0, Z_2 \geq 0$, for each x in the feasible region.
$-10.64 \leq Z_1 \leq -4.625$ .and $2.4 \leq Z_2 \leq 4.75$

$Z_1(x) = (-x_1 - 1) + \frac{-3x_1 + 2x_2}{x_1 + x_2 + 3}$ becomes

$$Z_1(x) = (-x_1 - 1) + \frac{-3x_1 + 2x_2}{x_1 + x_2 + 3} + 10.64$$

$$Z_1(x) = \frac{(-x_1^2 - x_1x_2 + 3.64x_1 + 11.64x_2 + 28.92)}{x_1 + x_2 + 3} \qquad (4.2)$$

by transformation $Z_i(x) = Z_i(x) - Z_i^{Min}(x)$ and $0 \leq Z_1 \leq 6.015$

$$\frac{(-x_1^2 - x_1x_2 + 3.64x_1 + 11.64x_2 + 28.92)}{x_1 + x_2 + 3} \leq 6.015 \qquad (4.3)$$

$$(x_2 + 1) + \frac{7x_1 + 2x_2}{5x_1 + 2x_2 + 1} \leq 4.75 \Rightarrow \frac{2x_2^2 + 5x_1x_2 + 12x_1 + 5x_2 + 1}{5x_1 + 2x_2 + 1} \leq 4.75 \qquad (4.5)$$

Then we get

$$x_1^2 + x_1x_2 + 2.38x_1 - 5.62x_2 - \bar{h}_1 = 10.86 \qquad (4.6)$$
$$2x_2^2 + 5x_1x_2 - 11.75x_1 - 4.5x_2 + \bar{h}_2 = 3.75 \qquad (4.7)$$

The given MOLLFP converts to the following non-linear goal programming problem.

$$s = Min(\bar{h}_1 + \bar{h}_2)$$

s.t.

$$x_1^2 + x_1x_2 + 2.38x_1 - 5..62x_2 - \bar{h}_1 = 10.86$$
$$2x_2^2 + 5x_1x_2 - 11.75x_1 - 4.5x_2 + \bar{h}_2 = 3.75 \qquad (4.8)$$
$$x_1 - x_2 \geq 1$$
$$2x_1 + 3x_2 \leq 15$$
$$x_1 \geq 3$$
$$x_1, x_2 \geq 0$$

Now solution to the above non-linear goal programming is obtained as

optimal solution of above problem (4.8) is at (3.6, 2.6) and $\bar{h}_1 = 5.416, \bar{h}_2 = -2.57$ minimum value of s = 2.89. The point (3.6, 2.6) is an efficient solution of the given original problem in the feasible region.
The solution for the original is given by $x_1 = 3.6, \quad x_2 = 2.6, \quad Z_1 = -5.2, \; Z_2(x) = 4.75$

**Example 2.** Let us consider a MOLPFPP with three objective functions

$$Max\left\{ Z_1(x) = (-x_1 - 1) + \frac{-3x_1 + 2x_2}{x_1 + x_2 + 3}, Z_2(x) = (x_2 + 1) + \frac{7x_1 + x_2}{5x_1 + 2x_2 + 1}, Z_3(x) = (x_1 + 1) + \frac{x_1 + 4x_2}{2x_1 + 3x_2 + 2} \right\}$$

$s.t.$

$$x_1 - x_2 \geq 1$$
$$2x_1 + 3x_2 \leq 15$$
$$x_1 + 9x_2 \geq 9$$
$$x_1 \geq 3$$
$$x_1, x_2 \geq 0$$

It is observed that $Z_1 < 0$, $Z_2 \geq 0$, $Z_3 \geq 0$ for each x in the feasible region
$-10.24 \leq Z_1 \leq -4.625 \;\; 2.55 \leq Z_2 \leq 4.75$ and $4.79 \leq Z_3 \leq 8.67$.

Sincce $Z_1 \leq 0$, $Z_1(x) = (-x_1 - 1) + \dfrac{-3x_1 + 2x_2}{x_1 + x_2 + 3}$ becomes

$Z_1(x) = \dfrac{(-x_1^2 - x_1x_2 - 3.24x_1 - 11.24x_2 - 27.72)}{x_1 + x_2 + 3}$ by transformation $Z_i(x) = Z_i(x) - Z_i^{Min}(x)$

and $0 \leq Z_1 \leq 5.62$

$$\frac{(-x_1^2 - x_1x_2 - 3.24x_1 - 11.24x_2 - 27.72)}{x_1 + x_2 + 3} \leq 5.62$$

$$Z_2 = (x_2 + 1) + \frac{7x_1 + 2x_2}{5x_1 + 2x_2 + 1} \leq 4.75 \Rightarrow \frac{2x_2^2 + 5x_1x_2 + 12x_1 + 5x_2 + 1}{5x_1 + 2x_2 + 1} \leq 4.75$$

$$Z_3 = (x_1 + 1) + \frac{x_1 + 4x_2}{2x_1 + 3x_2 + 2} \leq 8.67 \Rightarrow \frac{2x_1^2 + 3x_1x_2 + 5.x_1 + 7x_2 + 2}{2x_1 + 3x_2 + 2} \leq 8.67$$

Then we get

$$x_1^2 + x_1x_2 + 2.38x_1 - 5.62x_2 - \bar{h}_1 = 10.24$$
$$2x_2^2 + 5x_1x_2 - 11.75x_1 - 4.5x_2 + \bar{h}_2 = 3.75$$
$$2x_1^2 + 3x_1x_2 - 12.34x_1 - 19.01x_2 + \bar{h}_3 = 15.34$$

The given MOLLFP converts to the following non-linear goal programming problem.

$$s = Min(\bar{h}_1 + \bar{h}_2 + \bar{h}_3)$$

s.t.

$$x_1^2 + x_1x_2 + 2.38x_1 - 5.62x_2 - \bar{h}_1 = 10.24$$
$$2x_2^2 + 5x_1x_2 - 11.75x_1 - 4.5x_2 + \bar{h}_2 = 3.75$$
$$2x_1^2 + 3x_1x_2 - 12.34x_1 - 19.01x_2 + \bar{h}_3 = 15.34$$

$x_1 - x_2 \geq 1$

$2x_1 + 3x_2 \leq 15$

$x_1 + 9x_2 \geq 9$

$x_1 \geq 3$

$x_1, x_2 \geq 0$

The solution of above programming problem is given by

$x_1 = 3.6, \quad x_2 = 2.6, \quad \bar{h}_1 = 5.416, \quad \bar{h}_2 = -2.57, \quad \bar{h}_3 = 55.19, \quad s = 58.04$

The solution of original problem is obtained as

$x_1 = 3.6, \; x_2 = 2.6, \; Z_1 = -5.20, \; Z_2(x) = 4.75, \; Z_3 = 5.4$

**Example 3.** Let us consider a MOLPFPP with three objective functions that all objectives are negative as below.

$$Max\left\{\begin{array}{l} Z_1(x) = (-x_1 - 1) + \dfrac{-x_1 + x_2 - 4}{6x_1 + x_2 + 3}, Z_2(x) = (-2x_2 + 1) + \dfrac{x_1 - x_2 - 5}{x_2 + 1}, \\ Z_3(x) = (-3x_1 + 1) + \dfrac{3x_1 + x_2 - 17}{-3x_1 + 16} \end{array}\right\}$$

$s.t.$

$x_1 \leq 4$

$x_2 \leq 4$

$x_1 + x_2 \leq 7$

$-x_1 + x_2 \leq 3$

$x_1 - x_2 \leq 3$

$x_1, x_2 \geq 0$

It is observed that $Z_1 < 0$, $Z_2 < 0$, $Z_3 < 0$ for each x in the regions $-5.25 \leq Z_1 \leq -2.08$, $-8.6 \leq Z_2 \leq -2$ and $-12 \leq Z_3 \leq -0.0625$.

$-6x_1^2 - x_1x_2 + 2.48x_1 + 2.08x_2 + \bar{h}_1 = 0.76$

$-2x_2^2 + x_1 + \bar{h}_2 = 2$

$-9x_1^2 - 48.18x_1 + x_2 + \bar{h}_3 = 0.04$

The given MOLLFP converts to the following non-linear goal programming problem.

$s = Min(\bar{h}_1 + \bar{h}_2 + \bar{h}_3)$

s.t.

$-6x_1^2 - x_1x_2 + 2.48x_1 + 2.08x_2 + \bar{h}_1 = 0.76$

$-2x_2^2 + x_1 + \bar{h}_2 = 2$

$-9x_1^2 - 48.18x_1 + x_2 + \bar{h}_3 = 0.04$

$x_1 \le 4$
$x_2 \le 4$
$x_1 + x_2 \le 17$
$-x_1 + x_2 \le 3$
$x_1 - x_2 \le 3$
$x_1, x_2 \ge 0$

The solution of above goal programming problem is given by

$x_1 = 0,\ \ x_2 = 0,\ \ \bar{h}_1 = 0.76,\ \ \bar{h}_2 = 2,\ \ \ \ \bar{h}_3 = 0.04,\ \ \ \ s = 2.8$

The solution of original problem is obtained as

$x_1 = 0,\ \ x_2 = 0\ \ \ Z_1(x) = -2.33,\ \ Z_2(x) = -4\ \ Z_3(x) = -0.0625$ .

## IV. Conclusions:

In this paper, we have proposed a solution procedure for multi objective linear- plus-linear fractional prgramming problem( MOLLFP). We have positive objective functions for all $x \in X$ by translating objective functions that are negative for some $x \in X$ . Then we have single objective function as minimization of the deviations from maximum valus of objective functions in the feasible region. So MOLLFP converts to quadratic goal programing problem. The proposed algorithm always yeilds an efficient solution. The proposed algorithm is applied to three numerical examples.

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