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A Two Level and Four Species Ecological Ammensalism-Global Stability Analysis

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ABSTRACT

The paper purposes to look into the global stability of a mathematical model of two level Ammensalism with Four Species (A,P,E,H) Eco-System of Ammensal-Prey(A),Predator-Ammensal(P),Enemy-Ammensal(E) and malice(M) species. The two levels of Ammensalism are considered in the pairs of (P, H) and (E,H). The model is launched with a set of first order non-linear four ordinary differential coupled equations. Global stability of the model-*“A Two Level and Four species Ecological Ammensalism”* is investigated in the coexistent case.

Keywords: Ammensal,Prey,Predator,Enemy,Malice,Eco-System,Equillibrium point, stable.

AMS Classification: 92D25, 92D40

1. INTRODUCTION:

Lyapunov introduced mainly two methods to investigate the global stability of a dynamic system. He discussed various types of stability with effective methods. Those methods are often used in different fields of research. The idea of Lyapunov stability is now carried and applied to discuss at infinite dimensional manifolds. After linearization of the basic equations in non linear systems, the stability analysis will be studied at normal steady state with the help of second method of Lyapunov. The first method was developed by Lyapunov for the solution in a series which was then proved convergent within limits.

In the second method, constructing a Lyapunov function V(x) for a system which has a point of equilibrium at x=0. Consider a function $V(x): R^n \to R$ such that

$(i)\ V(x) \geq 0$ with equality if and only if x= o (positive definite)

$(ii)\ \dot{V}(x) = \frac{d}{dt}V(x) \leq 0$ with equality if and only if x= 0 (negative definite).

Then the system is asymptotically stable in the sense of Lyapunov.

Mathematical modeling of ecosystems was pioneered by Lotka [12] in 1925 and by Volterra [14] in 1931.], Kapur [10,11] and several authors. N.C. Srinivas [15] studied the competitive ecosystems of two, three or four species with limited and unlimited resources. The basic concepts of modeling have been presented in the treatises of Meyer [13].Recently Acharyulu et al. [1-9] obtained some noteworthy results "on the stability of many ecological Ammensal models.

In this analytical study, the article aims to investigate a two level Ammensalism model with Four Species (A,P,E,H) Syn Eco-System of Ammensal-Prey(A),Predator-Ammensal (P),Enemy-Ammensal(E) and malice(M) species.The System contains Ammensal –prey(A) , a Predator –Ammensal(P), enemy-Ammensal(E) and malice(M). The two levels of Ammensalism are considered in the pairs of (P, H) and (E,H). The model is launched with a set of first order non-linear four ordinary differential coupled equations. Sixteen equilibrium points are obtained. Global stability of this model is investigated.
A Schematic diagram of the model can be shown as in Fig.1.

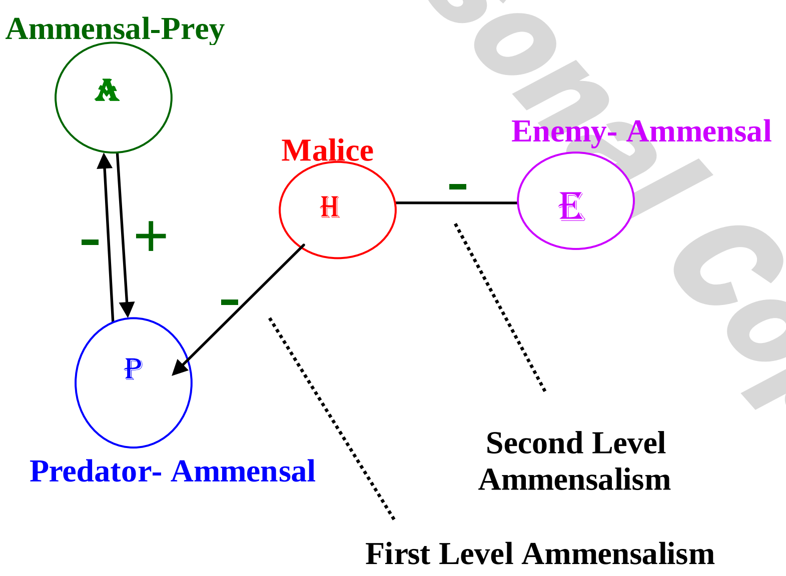

Fig. 1 Schematic diagram

2. NOTATIONS :

A : Prey-Ammensal:Prey for P and Ammensal for E.
P : Predator-Ammensal:Predator surviving upon A and Ammensal for M.
E : Enemy-Ammensal:Enemy for the Ammensal-Prey (A) and Ammensal for M.

M	: Malice of the Predator-Ammensal (P) & Enemy-Ammensal (E)
A,P,E,M	: The Population growth rates of A,P,E&M at time t.
t	: Time instant
n_i	: Natural growth rates of A,P,E&M, i=a,p,e & m respectively.
n_{ii}	: Self inhibition coefficient of A,P,E&M, i=a,p,e & m respectively.
n_{ap}, n_{pa}	: Interaction coefficients of A due to P and P due to A.
n_{ae}, n_{pm}, n_{em}	: Inhibition coefficients for A due to the enemy E, P due to the malice M, E due to malice M respectively
$K_i = \frac{n_i}{n_{ii}}$	: Carrying capacities of A,P,E&M, i=a,p,e & m respectively
$\alpha_{ij} = \frac{n_{ij}}{n_{ii}}$	: Ammensal coefficient of A,P,E&M, i,j=a,p,e & m(i≠j)
β_{ab}, β_{ba}	: Interaction (A-P) coefficients of A due to P and P due to A

Further the variables A,P,E&M are non-negative and the model parameters n_a, n_p, n_e, n_m; n_{aa}, n_{pp}, n_{ee}, n_{mm}; n_{ap}, n_{pa}, n_{ae}, n_{pm}, n_{em} are assumed to be non-negative constants.

3. BASIC EQUATIONS:

The model equations of two level and four species (A,P,E,H) ecological Ammensalism are constituted by the following system of first order non-linear ordinary differential equations

The model equations for the growth rates of A,P,E,M are

$$(i)\quad \frac{dA}{dt} = n_{aa}(K_a A - A^2 - \beta_{ap} AP) \qquad (1)$$

$$(ii)\quad \frac{dP}{dt} = n_{pp}(K_p P - P^2 + \beta_{pa} PA - \alpha_{pm} PM) \qquad (2)$$

$$(iii)\quad \frac{dE}{dt} = n_{ee}(K_e E - E^2 - \alpha_{em} EM) \qquad (3)$$

$$(iv)\quad \frac{dM}{dt} = n_{mm}(K_m M - M^2) \qquad (4)$$

4. EQUILIBRIUM STATES:

The system under this investigation has sixteen equilibrium states defined by

$$\frac{dS}{dt} = 0,\ where\ S = \mathrm{A, P, E, M}$$

A.States in which none of the four species is washed out.

(i). The co-existent state (or)Normal steady state

$$\overline{A}=n_{pp}\frac{n_a}{n_{aa}n_{pp}+n_{ap}n_{pa}}-n_{ap}\frac{n_pn_{mm}+n_{pm}n_m}{n_{mm}\ \ n_{aa}n_{pp}+n_{ap}n_{pa}},$$

$$\overline{P}=n_{aa}\frac{n_pn_{mm}-n_{pm}n_m}{n_{mm}\ \ n_{aa}n_{pp}+n_{ap}n_{pa}}+\frac{n_{pa}}{n_{aa}n_{pp}+n_{ap}n_{pa}},$$

$$\overline{E}=\frac{n_en_{mm}-n_mn_{em}}{n_{ee}n_{mm}}\ and\ \overline{M}=\frac{n_m}{n_{mm}}$$

It will exist only when $n_en_{mm}>n_mn_{em,}n_pn_{mm}>n_{pm}n_m$,
and $n_{ee}n_{mm}>0$

B.States in which one of the four species is washed out and the other three are not:

(ii). Only the Malice (M) is washed out

$$\overline{A}=\frac{n_{pp}n_an_{ee}-n_pn_{ap}n_{ee}}{n_{ee}\ \ n_{aa}n_{pp}+n_{ap}n_{pa}},\overline{P}=\frac{n_{pa}n_an_{ee}+n_pn_{aa}n_{ee}}{n_{ee}\ \ n_{aa}n_{pp}+n_{ap}n_{pa}},$$

$$\overline{E}=\frac{n_e}{n_{ee}},\overline{M}=0.$$

(iiii). Only the predator-Ammensal (P) is washed out

$$\overline{A}=\frac{n_a}{n_{aa}},\overline{P}=0,$$

$$\overline{E}=\frac{n_en_{mm}-n_mn_{em,}}{n_{ee}n_{mm}}\ and\ \ \overline{M}=\frac{n_m}{n_{mm}}$$

It will exist only when $n_en_{mm}>n_mn_{em.}$

(iv).Only the Prey-Ammensal (A) is washed out

$$\overline{A}=0,\overline{P}=\frac{n_pn_{mm}-n_{pm}n_m}{n_{pp}n_{mm}},\overline{E}=\frac{n_en_{mm}-n_mn_{em,}}{n_{ee}n_{mm}},\overline{M}=\frac{n_m}{n_{mm}}$$

It will exist only when $n_pn_{mm}>n_{pm}n_m,n_en_{mm}>n_mn_{em}.$

(v). Only the Enemy-Ammensal (E) is washed out:

$$\overline{A}=\frac{n_an_{pp}n_{mm}-n_{ap}(n_pn_{mm}-n_mn_{pm})}{n_{mm}(n_{aa}n_{pp}+n_{ap}n_{pa})},\overline{P}=\frac{n_an_{pa}n_{ee}+n_{aa}(n_pn_{mm}-n_mn_{pm})}{n_{44}(n_{aa}n_{pp}+n_{ap}n_{pa})},$$

$$\overline{E}=0,\overline{M}=\frac{n_m}{n_{mm}}$$

It will exist only when $n_an_{pp}n_{mm}>n_{ap}(n_pn_{mm}-n_mn_{pm})$
and $n_an_{pa}n_{ee}>n_{aa}(n_pn_{mm}-n_mn_{pm})$

C.States in which two of the four species are washed out and the other two are not:

(vi). Ammensal-Prey (A) and Enemy-Ammensal (E) are washed out:

$$\overline{A}=0,\overline{P}=\frac{n_p n_{mm}-n_m n_{pm}}{n_{pp}n_{mm}},\overline{E}=0,\overline{M}=\frac{n_m}{n_{mm}}$$

It will exist only when $n_p n_{mm}-n_m n_{pm}>0$

(vii). Ammensal-Prey (S_1) and Predator-Ammensal (S_2) are washed out:

$$\overline{A}=0,\overline{P}=0,\overline{E}=\frac{n_e n_{mm}-n_m n_{em}}{n_{ee}n_{mm}},\overline{M}=\frac{n_m}{n_{mm}}$$

It will exist only when $n_e n_{mm}-n_m n_{em}>0$

(viii). Predator-Ammensal (P) and Enemy-Ammensal (E) are washed out:

$$\overline{A}=\frac{n_a}{n_{aa}},\overline{P}=0,\overline{E}=0,\overline{M}=\frac{n_m}{n_{mm}}$$

(ix). Predator -Ammensal(P) and Malice (M) are washed out:

$$\overline{A}=\frac{n_a}{n_{aa}},\overline{P}=0,\overline{E}=\frac{n_e}{n_{ee}},\overline{M}=0,$$

(x). Ammensal-Prey (A) and Malice (M) are washed out:

$$\overline{A}=0,\overline{P}=\frac{n_p}{n_{pp}},\overline{E}=\frac{n_e}{n_{ee}},\overline{M}=0$$

(xi). Ammensal-Prey (A) and Predator-Ammensal (P) survive:

$$\overline{A}=\frac{n_a n_{pp}-n_p n_{ap}}{n_{aa}n_{pp}+n_{ap}n_{pa}},\overline{P}=\frac{n_a n_{pa}+n_p n_{aa}}{n_{aa}n_{pp}+n_{ap}n_{pa}},\overline{E}=0,\overline{M}=0$$

It will exist only when $n_a n_{pp}>n_p n_{ap}$

D.States in which three of the four species are washed out and the fourth is not.

(xii). Only the Predator-Ammensal (P) endures: $\overline{A}=0,\overline{P}=\frac{n_p}{n_{pp}},\overline{E}=0,\overline{M}=0$

(xiii). Only the Malice (M) of P and E survives: $\overline{A}=0,\overline{P}=0,\overline{E}=0,\overline{M}=\frac{n_m}{n_{mm}}$

(xiv).Only the Enemy-Ammensal(E) of A exists: $\overline{A}=0,\overline{P}=0,\overline{E}=\frac{n_e}{n_{ee}},\overline{M}=0$

(xv). Only the Ammensal-Prey (A) survives: $\overline{A}=\frac{n_a}{n_{aa}},\overline{P}=0,\overline{E}=0,\overline{M}=0$

E. Fully washed out state:

$$(xvi)\ \overline{A}=0,\overline{P}=0,\overline{E}=0,\overline{M}=0$$

5. Liapunov's Function for Global Stability:

By using Liapunov's technique, we can establish the global stability in this case. The Liapunov's fuction can be constructed in two level- four species Ammensalism as below. global stability is discussed at normal state.

It is noticed that this model is stable at normal steady state.

$$i.e\quad \overline{A}=\frac{n_{pp}n_a}{n_{aa}n_{pp}+n_{ap}n_{pa}}-n_{ap}\frac{n_p n_{mm}+n_{pm}n_m}{n_{mm}\ \ n_{aa}n_{pp}+n_{ap}n_{pa}},$$

$$\overline{P}=\frac{n_{aa}\ \ n_p n_{mm}-n_{pm}n_m}{n_{mm}\ \ n_{aa}n_{pp}+n_{ap}n_{pa}}+\frac{n_{pa}}{n_{aa}n_{pp}+n_{ap}n_{pa}},$$

$$\overline{E}=\frac{n_e n_{mm}-n_m n_{em}}{n_{ee}n_{mm}}\ and\ \overline{M}=\frac{n_m}{n_{mm}}$$

$$It\ will\ exist\ only\ when\ \ n_e n_{mm}>n_m n_{em,}n_p n_{mm}>n_{pm}n_m\ ,$$

$$and\ n_{ee}n_{mm}>0$$

Consider Liapunov function

$$L(A,P,E,M)=A\text{-}\overline{A}+\overline{A}[\log\overline{A}\text{-}\log A]+\lambda_1\ \ P\text{-}\overline{P}+\overline{P}[\log\overline{P}\text{-}\log P]$$

$$+\lambda_2\ \ E\text{-}\overline{E}+\overline{E}[\log\overline{E}\text{-}\log E\ \ +\lambda_3\ \ M\text{-}\overline{M}+\overline{P}[\log\overline{M}\text{-}\log M$$

where λ_1,λ_2 *and* λ_3 are scalars which protect the nature of Liapunov's function.

The change in Liapunov's function can be derived as below

$$\frac{dL}{dt}=\left(\frac{A-\overline{A}}{A}\right)\frac{dA}{dt}+\lambda_1\left(\frac{P-\overline{P}}{P}\right)\frac{dN_2}{dt}+\lambda_2\left(\frac{E-\overline{E}}{E}\right)\frac{dE}{dt}+\lambda_3\left(\frac{M-\overline{M}}{M}\right)\frac{dM}{dt}$$

$$\frac{dL}{dt}=\left(1-\frac{\overline{A}}{A}\right)\frac{dA}{dt}+\lambda_1\left(1-\frac{\overline{P}}{P}\right)\frac{dP}{dt}+\lambda_2\left(1-\frac{\overline{E}}{E}\right)\frac{dE}{dt}+\lambda_3\left(1-\frac{\overline{M}}{M}\right)\frac{dM}{dt}$$

$$\frac{dL}{dt}=\left(1-\frac{\overline{A}}{A}\right)\ A\ \ n_{aa}K_a-n_{aa}A-n_{aa}\beta_{ap}P$$

$$+\lambda_1\left(1-\frac{\overline{P}}{P}\right)P(n_{pp}K_p-n_{pp}P+n_{pp}\beta_{pa}A-n_{pp}\alpha_{pm}M)$$

$$+\lambda_2\left(1-\frac{\overline{E}}{E}\right)\ E\ \ n_{ee}K_e-n_{ee}E-n_{ee}\alpha_{em}M$$

$$+\lambda_3\left(1-\frac{\overline{M}}{M}\right)\ M\ \ n_{mm}K_m-n_{mm}M$$

$$= A-\overline{A} \quad n_{aa}K_a - n_{aa}A - n_{aa}\beta_{ap}P$$

$$+ \lambda_1 \; P-\overline{P} \; \{n_{pp}K_p - n_{pp}P + n_{pp}\beta_{pa}A - n_{pp}\alpha_{pm}M\}$$

$$+\lambda_2 \; E-\overline{E} \quad n_{ee}K_e - n_{ee}E - n_{ee}\alpha_{em}M$$

$$+\lambda_3 \; M-\overline{M} \quad n_{mm}K_m - n_{mm}M$$

$$\frac{dL}{dt} = A-\overline{A} \quad n_{aa}\overline{A} + n_{aa}\beta_{ap}\overline{P} + n_{aa}\alpha_{ae}\overline{E} - n_{aa}A - n_{aa}\beta_{ap}P$$

$$+\lambda_1 \; P-\overline{P} \quad n_{pp}\overline{P} - n_{pp}\beta_{pa}\overline{A} + n_{pp}\alpha_{pm}\overline{M} - n_{pp}P + n_{aa}\beta_{ap}A - n_{pp}\alpha_{pm}M$$

$$+\lambda_2 \; E-\overline{E} \quad n_{ee}\overline{E} + n_{ee}\alpha_{em}\overline{M} - n_{ee}E - n_{ee}\alpha_{em}M$$

$$+\lambda_3 \; M-\overline{M} \quad n_{mm}K_m\overline{M} - n_{mm}M \quad)$$

$$= A-\overline{A} \quad -n_{aa} \; A-\overline{A} \; -n_{aa}\beta_{ap} \; P-\overline{P}$$

$$+\lambda_1 \; P-\overline{P} \quad -n_{pp} \; P-\overline{P} + n_{pp}\beta_{pa} \; A-\overline{A} \; - n_{pp}\alpha_{pm} \; M-\overline{M}$$

$$+\lambda_2 \; E-\overline{E} \quad -n_{ee} \; E-\overline{E} \; - n_{ee}\alpha_{em} \; M-\overline{M}$$

$$+ \lambda_3 \; M-\overline{M} \quad -n_{mm} \; M-\overline{M} \qquad (5)$$

$$\frac{dL}{dt} = -n_{aa} \; A-\overline{A}^{\;2} - n_{aa}\beta_{ap} \; A-\overline{A} \quad P-\overline{P}$$

$$+\lambda_1 \; (-n_{pp}) \; P-\overline{P}^{\;2} + n_{pp}\beta_{pa} \; A-\overline{A} \; P-\overline{P} \; - n_{pp}\alpha_{pm} \; M-\overline{M} \quad P-\overline{P}$$

$$+\lambda_2 \quad -n_{ee} \quad E-\overline{E}^{\;2} - n_{ee}\alpha_{em} \; E-\overline{E} \quad M-\overline{M}$$

$$+ \lambda_3 \quad -n_{mm} \quad M-\overline{M}^{\;2} \qquad (6)$$

$selecting \; \lambda_1 = \dfrac{n_{aa}\beta_{ap}}{n_{pp}\beta_{pa}}, \lambda_2 \; and \; \lambda_3$ are any positive constants, (6) becomes,

$$\frac{dL}{dt} = -n_{aa} \; A-\overline{A}^{\;2} - n_{aa}\beta_{ap} \; P-\overline{P}^{\;2} - \frac{n_{aa}\beta_{ap}\alpha_{pm}}{\beta_{pa}} \; P-\overline{P} \quad M-\overline{M}$$

$$-\lambda_2 n_{ee} \; E-\overline{E}^{\;2} - \lambda_2 n_{ee}\alpha_{em} \; E-\overline{E} \quad M-\overline{M} \; - \lambda_3 n_{mm} \; M-\overline{M}^{\;2}$$

$$Let \;\; N_1 = \frac{n_{aa}\beta_{ap}}{\beta_{pa}}, N_2 = \frac{n_{aa}\beta_{ap}\alpha_{pm}}{\beta_{pa}}, N_3 = \lambda_2 n_{ee}, N_4 = \lambda_2 n_{ee}\alpha_{em} \;\&\; N_5 = \lambda_3 n_{mm}$$

$$\frac{dL}{dt} = -n_{aa} \; A-\overline{A}^{\;2} - N_1 \; P-\overline{P}^{\;2} - N_2 \; P-\overline{P} \quad M-\overline{M}$$

$$-N_3 \; E-\overline{E}^{\;2} - N_4 \; E-\overline{E} \quad M-\overline{M} \; - N_5 \; M-\overline{M}^{\;2}$$

$$\frac{dL}{dt} = \quad -n_{aa}K_a \; A-\overline{A}^{\;2} - N_1 \; P-\overline{P}^{\;2} - N_3 \; E-\overline{E}^{\;2} - N_5 \; M-\overline{M}^{\;2}$$

$$+EN_4\overline{M} - M(N_2\overline{P} - N_4(E-\overline{E}) - \overline{M}(N_2(P-\overline{P}) + N_4E_3)$$

$$\frac{dL}{dt} = -n_{aa}\ \ A-\overline{A}\ ^2 - N_1\ \ P-\overline{P}\ ^2 - N_3\ \ E-\overline{E}\ ^2 - N_5\ \ M-\overline{M}\ ^2$$

$$-M(N_2\overline{P} - 2N_4E + N_4\overline{E}(1-\frac{\overline{M}}{M})) - \overline{N_4}(N_2P - N_2\overline{A} + N_4E)$$

$$\therefore \frac{dL}{dt} < 0 \quad provided \quad N_2\overline{P} - 2N_4E + N_4\overline{E}(1-\frac{\overline{M}}{M}) \geq 0,\ N_2P - N_2\overline{A} + N_4E \geq 0$$

Hence the co-existence state is globally stable.

CONCLUSION:

Global stability of **Two Level –Four Species Ecological Ammensalism** is established at coexistence state.

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