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IDEALS IN TERNARY SEMIGROUPS

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ABSTRACT

In this paper the terms ideal, trivial ideal, proper ideal, maximal ideal are introduced. It is proved that the union and intersection of any family of ideals of ternary semigroup T is an ideal of T. It is also proved that union of all proper ideals of ternary semigroup T is the unique maximal ideal of T. The terms ideal of ternary semigroup T generated by A, principal ideal generated by an element are introduced. It is proved that the ideal of a ternary semigroup T generated by a non-empty subset A is the intersection of all ideals of T containing A. It is also proved that T is a ternary semigroup and $a \in T$ then $J(a) = a \cup aTT \cup TTa \cup TaT \cup TTaTT$. The terms, simple ternary semigroup, globally idempotent ideal are introduced. In any ternary semigroup T, principal ideals of T form a chain and ideals of T form a chain are equivalent. It is proved that a ternary semigroup T is simple ternary semigroup if and only if TTaTT = T for all $a \in T$. It is also proved that if T is a globally idempotent ternary semigroup having maximal ideals then T contains semisimple elements.

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Key words: Ideal, trivial ideal, proper ideal, chain, ideal of ternary semigroup T generated by A, principal ideal generated by *a*, simple ternary semigroup T.

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1. INTRODUCTION:

The theory of ternary algebraic system was introduced by Lehmer [13] in 1932, but earlier such structures were studied by Kasner [10] who gave the idea of n-ary algebras . Ternary semigroups are universal algebras with one associative ternary operation.

Anjaneyulu.A[1], [2] initiated the study of ideals in semigroups. S.Kar and B.K.Maity [9]initiated the study of some ideals of ternary semigroups. Sioson. F. M [18] studied about ideal theory in ternary semigroups. Santiago. M. L. and Bala S.S [16] developed the theory of ternary semigroups. Iampan. A [7] gave the idea of Lateral ideals of ternary semigroups.

2. PRELIMINARIES :

DEFINITION 2.1 : Let T be a non-empty set. Then T is said to be a ***ternary semigroup*** if there exist a mapping from T×T×T to T which maps $(x_1, x_2, x_3) \to x_1x_2x_3$ satisfying the condition : $[x_1x_2x_3\ x_4x_5] = [x_1\ x_2x_3x_4\ x_5] = [x_1x_2\ x_3x_4x_5]\ \forall\ x_i \in T,\ 1 \le i \le 5$.

DEFINITION 2.2 : A ternary semigroup T is said to be ***commutative*** provided for all $a,b,c \in$ T, we have $abc = bca = cab = bac = cba = acb$.

DEFINITION 2.3 : Let T be ternary semigroup. A non empty subset S of T is said to be a ***ternary subsemigroup*** of T if $abc \in$ S for all $a,b,c \in$ S.

NOTE 2.4 : A non empty subset S of a ternary semigroup T is a ternary subsemigroup if and only if SSS $\subseteq$ S.

3. <u>IDEALS</u> :

DEFINITION 3.1 : A nonempty subset A of a ternary semigroup T is said to be ***left ideal*** of T if $b, c \in$ T, $a \in$ A implies $bca \in$ A.

NOTE 3.2 : A nonempty subset A of a ternary semigroup T is said to be a left ideal of T if and only if TTA $\subseteq$ A.

THEOREM 3.3 : The nonempty intersection of any two left ideals of a ternary semigroup T is a left ideal of T.

Proof : Let A , B be two left ideals of T. Let $a \in A \cap B$ and $b,c \in$ T
$a \in A \cap B \Rightarrow a \in$ A and $a \in$ B

$a \in$ A ; $b,c \in$ T , a is a left ideal of T $\Rightarrow bca \in$ A.
$a \in$ B ; $b,c \in$ T , b is a left ideal of T $\Rightarrow bca \in$ B.

$bca \in$ A, bca $\in$ B $\Rightarrow bca \in A \cap B$. Therefore $A \cap B$ is a left ideal of T.

THEOREM 3.4: The nonempty intersection of any family of left ideals of a ternary semigroup T is a left ideal of T.

proof : Let $A_\alpha\ _{\alpha \in \Delta}$ be a family of left ideals of T and let A = $\bigcap_{\alpha \in \Delta} A_\alpha$

Let $a \in$ A ; $b , c \in$ T. Now $a \in$ A, $a \in \bigcap_{\alpha \in \Delta} A_\alpha \Rightarrow a \in A_\alpha$ for each $\alpha \in \Delta$.

$a \in A_\alpha$, $b , c \in$ T, A_α is a left ideal of T $\Rightarrow bca \in A_\alpha$

$bca \in A_\alpha$ for all $\alpha \in \Delta \Rightarrow bca \in \bigcap_{\alpha\in\Delta} A_\alpha \Rightarrow bca \in$ A. Therefore A is a left ideal of T.

THEOREM 3.5 : The union of any two left ideals of a ternary semigroup T is a left ideal of T.

Proof : Let A_1, A_2 be two left ideals of a ternary semigroup T..

Let A = $A_1 \bigcup A_2$. Clearly A is a nonempty subset of T.

Let $a \in$ A, $b,c \in$ T. Now $a \in$ A $\Rightarrow a \in A_1 \bigcup A_2 \Rightarrow a \in A_1$ or $a \in A_2$.

Suppose $a \in A_1$. So a $\in A_1$; b , $c \in$ T; A_1 is a left ideal of T $\Rightarrow bca \in A_1 \subseteq A_1 \bigcup A_2$ = A $\Rightarrow bca \in$ A

Suppose $a \in A_2$. So $a \in A_2$; b , $c \in$ T; A_2 is a left ideal of T $\Rightarrow bca \in A_2 \subseteq A_1 \bigcup A_2$ = A. $\Rightarrow bca \in$ A.

Therefore $a \in$ A, b, $c \in$ T $\Rightarrow bca \in$ A and hence A is a left ideal of T.

THEOREM 3.6 : The union of any family of left ideals of a ternary semigroup T is a left ideal of T.

Proof : Let $A_\alpha \;{}_{\alpha\in\Delta}$ be a family of left ideals of a ternary semigroup T.

Let A = $\bigcup_{\alpha\in\Delta} A_\alpha$. Clearly A is a non-empty subset of T.

Let $a \in$ A ; b , $c \in$ T.

$a \in$ A $\Rightarrow a \in \bigcup_{\alpha\in\Delta} A_\alpha \Rightarrow a \in A_\alpha$ for some $\alpha \in \Delta$.

$a \in A_\alpha$, b , $c \in$ T, A_α is a left ideal of T $\Rightarrow bca \in A_\alpha \subseteq \bigcup_{\alpha\in\Delta} A_\alpha$ = A

$\Rightarrow bca \in$ A. There fore A is a left ideal of T.

DEFINITION 3.7 : A nonempty subset of a ternary semigroup T is said to be a ***lateral ideal*** of T if b, $c \in$ T , $a \in$ A implies $bac \in$ A.

NOTE 3.8 : A nonempty subset of A of a ternary semigroup T is a lateral ideal of T if and only if TAT $\subseteq$ A.

THEOREM 3.9 : The nonempty intersection of any two lateral ideals of a ternary semigroup T is a lateral ideal of T.

Proof : Let A, B be two lateral ideals of T. Let $a \in$ A $\bigcap$ B and b, $c \in$ T.
$a \in$ A $\bigcap$ B $\Rightarrow a \in$ A and $a \in$ B.

$a \in$ A ; b, $c \in$ T , A is a lateral ideal of T $\Rightarrow bac \in$ A.

$a \in$ B ; b, $c \in$ T , B is a lateral ideal of T $\Rightarrow bac \in$ B.

$bac \in$ A, $bac \in$ B $\Rightarrow bac \in$ A $\bigcap$ B . Therefore A $\bigcap$ B is a lateral ideal of T.

THEOREM 3.10 : The nonempty intersection of any family of lateral ideals of a ternary semigroup T is a lateral ideal of T.

Proof: Let $A_{\alpha}\ {}_{\alpha\in\Delta}$ be a family of lateral ideals of T and let A = $\bigcap_{\alpha\in\Delta} A_{\alpha}$

Let $a \in$ A ; b , $c \in$T. $a \in$ A $a\in \bigcap_{\alpha\in\Delta} A_{\alpha} \Rightarrow a \in A_{\alpha}$ for each $\alpha \in \Delta$.

$a \in A_{\alpha}$, b , $c \in$T, A_{α} is a lateral ideal of T $\Rightarrow bac \in A_{\alpha}$

$bac \in A_{\alpha}$ for all $\alpha \in \Delta \Rightarrow bac \in \bigcap_{\alpha\in\Delta} A_{\alpha} \Rightarrow bac \in$ A. Therefore A is a lateral ideal of T.

THEOREM 3.11 : The union of any two lateral ideals of a ternary semigroup T is a lateral ideal of T.

Proof : Let $A_{1,} A_2$ be two lateral ideals of a ternary semigroup T.
Let A = $A_1 \cup A_2$. Clearly A is a non-empty subset of T.

Let $a \in$A; b,$c \in$T. a $\in$A $\Rightarrow a \in A_1 \cup A_2 \Rightarrow a \in A_1$ (or) $a \in A_2$

Suppose $a \in A_1$. So $a \in A_1$; b, $c \in$T; A_1 is a lateral ideal of T $\Rightarrow$ $bac \in A_1 \subseteq A_1 \cup A_2$ = A.

$\Rightarrow bac \in$ A.

Suppose $a \in A_2$. So $a \in A_2$; b , $c \in$T; A_2 is a lateral ideal of T$\Rightarrow bac \in A_2 \subseteq A_1 \cup A_2$ = A

$\Rightarrow bac \in$ A.

Therefore $a \in$A, b, $c \in$T $\Rightarrow$ $bac \in$ A and hence A is a lateral ideal of T.

THEOREM 3.12 : The union of any family of lateral ideals of a ternary semigroup T is a lateral ideal of T.

Proof : Let $A_{\alpha}\ {}_{\alpha\in\Delta}$ be a family of lateral ideals of a ternary semigroup T.

Let A = $\bigcup_{\alpha\in\Delta} A_{\alpha}$. Clearly A is a non-empty subset of T.

Let $a \in$ A ; b,$c \in$T. $a \in$A $\Rightarrow a \in \bigcup_{\alpha\in\Delta} A_{\alpha} \Rightarrow a \in A_{\alpha}$ for some $\alpha \in \Delta$.

$a \in A_{\alpha}$, b , $c \in$T, A_{α} is a lateral ideal of T $\Rightarrow bac \in A_{\alpha} \subseteq \bigcup_{\alpha\in\Delta} A_{\alpha}$ = A

$\Rightarrow bac \in$ A. Therefore A is a lateral ideal of T.

DEFINITION 3.13 : A nonempty subset A of a ternary semigroup T is a ***right ideal*** of T if b, $c \in$ T , $a \in$ A implies $abc \in$ A

NOTE 3.14 : A nonempty subset A of a ternary semigroup T is a right ideal of T if and only if ATT $\subseteq$ A.

THEOREM 3.15 : The nonempty intersection of any two right ideals of a ternary semigroup T is a right ideal of T.

Proof : Let A , B be two right ideals of T. $a \in A \cap B$ and $b, c \in T$

$a \in A \cap B \Rightarrow a \in A$ and $a \in B$

$a \in A$; $b, c \in T$, A is a right ideal of T $\Rightarrow abc \in A$.

$a \in B$; $b, c \in T$, B is a right ideal of T $\Rightarrow abc \in B$.

$abc \in A$, $abc \in B \Rightarrow abc \in A \cap B$. Therefore $A \cap B$ is a right ideal of T.

THEOREM 3.16 : The nonempty intersection of any family of right ideals of a ternary semigroup T is a right ideal of T.

Proof : Let $\{A_\alpha\}_{\alpha \in \Delta}$ be a family of right ideals of T and let A = $\bigcap_{\alpha \in \Delta} A_\alpha$

Let $a \in A$; $b , c \in T$. $a \in A \Rightarrow a \in \bigcap_{\alpha \in \Delta} A_\alpha \Rightarrow a \in A_\alpha$ for each $\alpha \in \Delta$.

$a \in A_\alpha$, $b , c \in T$, A_α is a right ideal of T $\Rightarrow abc \in A_\alpha$

$abc \in A_\alpha$ for all $\alpha \in \Delta \Rightarrow abc \in \bigcap_{\alpha \in \Delta} A_\alpha \Rightarrow abc \in A$. Therefore A is a right ideal of T.

THEOREM 3.17 : The union of any two right ideals of a ternary semigroup T is a right ideal of T.

Proof : Let A_1, A_2 be two right ideals of a ternary semigroup T.

Let A = $A_1 \cup A_2$. Clearly A is a nonempty subset of T. Let $a \in A$; $b,c \in T$.

$a \in A \Rightarrow a \in A_1 \cup A_2 \Rightarrow a \in A_1$ (or) $a \in A_2$.

Suppose $a \in A_1$. So $a \in A_1$; $b, c \in T$; A_1 is a right ideal of T $\Rightarrow abc \in A_1 \subseteq A_1 \cup A_2$ = A $\Rightarrow abc \in A$.

Suppose $a \in A_2$. So $a \in A_2$; $b, c \in T$; A_2 is a right ideal of T $\Rightarrow abc \in A_2 \subseteq A_1 \cup A_2$ = A $\Rightarrow$ $abc \in A$.

Therefore $a \in A$, b, c $\in T \Rightarrow abc \in A$ and hence A is a right ideal of T.

THEOREM 3.18 : The union of any family of right ideals of a ternary semigroup T is a right ideal of T.

Proof : Let $\{A_\alpha\}_{\alpha \in \Delta}$ be a family of right ideals of a ternary semigroup T.

Let A = $\bigcup_{\alpha \in \Delta} A_\alpha$. Clearly A is a non-empty subset of T.

Let $a \in$ A ; b , $c \in$ T. $a \in$ A $\Rightarrow a \in \bigcup_{\alpha \in \Delta} A_\alpha \Rightarrow a \in A_\alpha$ for some $\alpha \in \Delta$.

$a \in A_\alpha$, b , $c \in$ T, A_α is a right ideal of T $\Rightarrow abc \in A_\alpha \subseteq \bigcup_{\alpha \in \Delta} A_\alpha$ = A

$\Rightarrow abc \in$ A. Therefore A is a right ideal of T.

DEFINITION 3.19 : A nonempty subset A of a ternary semigroup T is a ***two sided ideal*** of T if $b, c \in$ T , $a \in$ A implies $bca \in$ A, $abc \in$ A.

NOTE 3.20 : A nonempty subset A of a ternary semigroup T is a two sided ideal of T if and only if it is both a left ideal and a right ideal of T .

THEOREM 3.21 : The nonempty intersection of any two sided ideals of a ternary semigroup T is a two sided ideal of T.

Proof: Let A , B be two two sided ideals of T.

$a \in$ A $\cap$ B and $b, c \in$ T . $a \in$ A $\cap$ B $\Rightarrow a \in$ A and $a \in$ B.
$a \in$ A ; $b, c \in$ T , A is a two sided ideal of T $\Rightarrow bca$, $abc \in$ A.

$a \in$ B ; $b, c \in$ T , B is a two sided ideal of T $\Rightarrow bca, abc \in$ B.

Therefore bca , $abc \in$ A, $bca, abc \in$ B $\Rightarrow bca, abc \in$ A $\cap$ B .

Therefore A $\cap$ B is a two sided ideal of T.

THEOREM 3.22 : The nonempty intersection of any family of two sided ideals of a ternary semigroup T is a two sided ideal of T.

Proof : Let A_α $_{\alpha \in \Delta}$ be a family of two sided ideals of T and let A = $\bigcap_{\alpha \in \Delta} A_\alpha$

Let $a \in$ A ; b , $c \in$ T. $a \in$ A $\Rightarrow a \in \bigcap_{\alpha \in \Delta} A_\alpha \Rightarrow a \in A_\alpha$ for each $\alpha \in \Delta$.

$a \in A_\alpha$, b , $c \in$ T, A_α is a two sided ideal of T $\Rightarrow bca, abc \in A_\alpha$

$bca, abc \in A_\alpha$ for all $\alpha \in \Delta \Rightarrow bca, abc \in \bigcap_{\alpha \in \Delta} A_\alpha \Rightarrow bca$, $abc \in$ A.

Therefore A is a two sided ideal of T.

THEOREM 3.23 : The union of any two two sided ideals of a ternary semigroup T is a two sided ideal of T.

Proof : Let A_1, A_2 be two, two sided ideals of a ternary semigroup T.

Let A = $A_1 \cup A_2$. Clearly A is a non-empty subset of T.

Let $a \in$ A; $b,c \in$ T. $a \in$ A $\Rightarrow a \in A_1 \cup A_2 \Rightarrow a \in A_1$ (or) $a \in A_2$

Suppose $a \in A_1$. So $a \in A_1$; $b,c \in$ T; A_1 is a two sided ideal of T

$\Rightarrow$bca, $abc \in A_1 \subseteq A_1 \cup A_2$ = A $\Rightarrow$ bca , $abc \in$ A.

Suppose $a \in A_2$. So a $\in A_2$; b , c $\in$ T; A_2 is a two sided ideal of T

$\Rightarrow$ bca , $abc \in A_2 \subseteq A_1 \cup A_2$ = A $\Rightarrow$bca , $abc \in$ A.

Therefore a $\in$ A, b, c $\in$ T $\Rightarrow$ bca, $abc \in$ A and hence A is a two sided ideal of T.

THEOREM 3.24 : The union of any family of two sided ideals of a ternary semigroup T is a two sided ideal of T.

Proof : Let $A_\alpha \ _{\alpha \in \Delta}$ be a family of two sided ideal of a ternary semigroup T.

Let A = $\bigcup_{\alpha \in \Delta} A_\alpha$. Clearly A is a non-empty subset of T. Let $a \in$ A ; b , $c \in$ T.

$a \in$ A $\Rightarrow a \in \bigcup_{\alpha \in \Delta} A_\alpha \Rightarrow a \in A_\alpha$ for some $\alpha \in \Delta$.

$a \in A_\alpha$, b , $c \in$ T, A_α is a two sided ideal of T $\Rightarrow$ bca, $abc \in A_\alpha \subseteq \bigcup_{\alpha \in \Delta} A_\alpha$ = A

Therefore bca, $abc \in$ A and hence A is a two sided ideal of T.

DEFINITION 3.25 : A nonempty subset A of a ternary semigroup T is said to be ***ternary ideal*** or simply an ***ideal*** of T if $b, c \in$ T , $a \in$ A implies $bca \in$ A, $bac \in$ A, $abc \in$ A.

NOTE 3.26 : A nonempty subset A of a ternary semigroup T is an ideal of T if and only if it is left ideal, lateral ideal and right ideal of T .

EXAMPLE 3.27 : Let N be the set of all natural numbers. Define the ternary operation from $N \times N \times N \rightarrow$ N as (a, b, c) = a.b.c where '.' is a multiplication. Then N is a ternary semigroup and A = 3 N is an ideal of the ternary semigroup N.

THEOREM 3.28 : The nonempty intersection of any two ideals of a ternary semigroup T is an ideal of T.

Proof: Let A , B be two ideals of T. Let $a \in A \cap B$ and $b, c \in T$

$a \in A \cap B \Rightarrow a \in A$ and $a \in B$.

$a \in A$; $b, c \in T$, A is an ideal of T $\Rightarrow$ bca, *bac*, *abc* $\in$ A.

$a \in B$; $b, c \in T$, B is an ideal of T $\Rightarrow$ bca, *bac*, *abc* $\in$ B.

$\therefore$ bca, *bac*, *abc* $\in$ A; bca, *bac*, *abc* $\in$ B $\Rightarrow$ bca, *bac*, *abc* $\in A \cap B$ and hence $A \cap B$ is an ideal of T.

THEOREM 3.29 : The nonempty intersection of any family of ideals of a ternary semigroup T is an ideal of T.

Proof: Let $A_{\alpha\ \alpha \in \Delta}$ be a family of ideals of T and let A = $\bigcap_{\alpha \in \Delta} A_\alpha$

Let $a \in A$; $b, c \in T$.

$a \in A \Rightarrow a \in \bigcap_{\alpha \in \Delta} A_\alpha \Rightarrow a \in A_\alpha$ for each $\alpha \in \Delta$.

$a \in A_\alpha$, b , $c \in T$, A_α is a ideal of T $\Rightarrow$bca, *bac*, *abc* $\in A_\alpha$

bca, *bac*, *abc* $\in A_\alpha$ for all $\alpha \in \Delta \Rightarrow$bca, *bac*, *abc* $\in \bigcap_{\alpha \in \Delta} A_\alpha$

$\Rightarrow$ bca, *bac*, *abc* $\in$ A. Therefore A is an ideal of T.

THEOREM 3.30 : The union of any two ideals of a ternary semigroup T is an ideal of T.

Proof: Let A_1, A_2 be two ideals of a ternary semigroup T.

Let A = $A_1 \cup A_2$. Clearly A is a non-empty subset of T. Let $a \in A$; $b, c \in T$.

$a \in A \Rightarrow a \in A_1 \cup A_2 \Rightarrow a \in A_1$ (or) $a \in A_2$

Suppose $a \in A_1$. So $a \in A_1$; b , $c \in T$; A_1 is an ideal of T

$\Rightarrow$ bca, *bac*, *abc* $\in A_1 \subseteq A_1 \cup A_2$ = A $\Rightarrow$ bca, *bac*, *abc* $\in$ A.

Suppose $a \in A_2$. So $a \in A_2$; b , $c \in T$; A_2 is an ideal of T

$\Rightarrow$ bca, *bac*, *abc* $\in A_2 \subseteq A_1 \cup A_2$ = A $\Rightarrow$ bca, *bac*, *abc* $\in$ A.

Therefore $a \in A$, $b, c \in T \Rightarrow$ bca, *bac*, *abc* $\in$ A and hence A is an ideal of T.

THEOREM 3.31 : The union of any family of ideals of a ternary semigroup T is an ideal of T.

Proof : Let $A_{\alpha}\;_{\alpha\in\Delta}$ be a family of ideals of a ternary semigroup T.

Let A = $\bigcup_{\alpha\in\Delta} A_{\alpha}$. Clearly A is a non-empty subset of T. Let $a \in$ A ; b , $c \in$ T.

$a \in$ A $\Rightarrow a \in \bigcup_{\alpha\in\Delta} A_{\alpha} \Rightarrow a \in A_{\alpha}$ for some $\alpha \in \Delta$.

$a \in A_{\alpha}$, b , $c \in$ T, A_{α} is an ideal of T

$\Rightarrow$bca, *bac*, *abc*$\in A_{\alpha} \subseteq \bigcup_{\alpha\in\Delta} A_{\alpha}$ = A $\Rightarrow$bca, *bac*, *abc* $\in$ A. Therefore A is an ideal of T.

DEFINITION 3.32 : An ideal A of a ternary semigroup T is said to be a ***proper ideal*** of T if A is different from T.

DEFINITION 3.33 : An ideal A of a ternary semigroup T is said to be a ***trivial ideal*** provided T\ A is singleton.

DEFINITION 3.34 : An ideal A of a ternary semigroup T is said to be a ***maximal left ideal*** provided A is a proper left ideal of T and is not properly contained in any proper left ideal of T.

DEFINITION 3.35 : An ideal A of a ternary semigroup T is said to be a ***maximal lateral ideal*** provided A is a proper lateral ideal of T and is not properly contained in any proper lateral ideal of T.

DEFINITION 3.36 : An ideal A of a ternary semigroup T is said to be a ***maximal right ideal*** provided A is a proper right ideal of T and is not properly contained in any proper right ideal of T.

DEFINITION 3.37 : An ideal A of a ternary semigroup T is said to be a ***maximal two sided ideal*** provided A is a proper two sided ideal of T and is not properly contained in any proper two sided ideal of T.

DEFINITION 3.38 : An ideal A of a ternary semigroup T is said to be a ***maximal ideal*** provided A is a proper ideal of T and is not properly contained in any proper ideal of T.

DEFINITION 3.39 : An ideal A of a ternary semigroup T is said to be ***globally idempotent*** if A^3 = A.

DEFINITION 3.40 :A ternary semigroup T is said to be ***globally idempotent*** if T^3 = T.

THEOREM 3.41 : **If A is an ideal of a ternary semigroup T with unity 1 and 1$\in$ A then A = T.**

Proof : Clearly A $\subseteq$ T. Let s,t $\in$T. 1$\in$ A ; *st* $\in$T, A is an ideal of T

$\Rightarrow$ *st* (1) $\in$ A $\Rightarrow$ *st* $\in$A $\Rightarrow$T $\subseteq$ A. A $\subseteq$ T, T $\subseteq$ A $\Rightarrow$ T = A.

THEOREM 3.42 : If T is a ternary semigroup with unity 1 then the union of all proper ideals of T is the unique maximal ideal of T.

Proof : Let M be the union of all proper ideals of T. Since 1 is not an element of any proper ideal of T, 1$\notin$M. Therefore M is a proper subset of T. By theorem 3.31, M is an ideal of T. Thus M is a proper ideal of T. Since M contains all proper ideals of T , M is a maximal ideal of T. If M_1 is any maximal ideal of T, then $M_1 \subseteq$ M $\subset$ T and hence M_1 = M. Therefore M is the unique maximal ideal of T.

DEFINITION 3.43 : Let T be a ternary semigroup and A be a non-empty subset of T. The smallest left ideal of T containing A is called ***left ideal of T generated by A***.

THEOREM 3.44 : The left ideal of a ternary semigroup T generated by a non-empty subset A is the intersection of all left ideals of T containing A.

Proof : Let Δ be the set of all left ideals of T containing A . Since T itself is a left ideal of T containing A, T$\in\Delta$. So $\Delta\neq\varnothing$. Let $S^* = \bigcap_{S\in\Delta} S$ .Since A $\subseteq$S for all S $\in\Delta$, A $\subseteq$ S^*.

By theorem 3.4, S^* is a left ideal of S. Let K be a left ideal of T containing A, K is a left ideal of T. Clearly A ⊆ K. Therefore K $\in\Delta\Rightarrow S^* \subseteq$ K and hence S^* is the left ideal of T generated by A.

DEFINITION 3.45 :Let T be a ternary semigroup and A be a non-empty subset of T. The smallest lateral ideal of T containing A is called ***lateral ideal of T generated by A***.

THEOREM 3.46 : The lateral ideal of a ternary semigroup T generated by a non-empty subset A is the intersection of all lateral ideals of T containing A.

Proof : Let Δ be the set of all lateral ideals of T containing A . Since T itself is a lateral ideal of T containing A, T $\in \Delta$. So $\Delta \neq \emptyset$. Let $S^* = \bigcap_{S \in \Delta} S$. Since A $\subseteq$S for all S$\in \Delta$, A$\subseteq S^*$. By theorem 3.10, S^* is a lateral ideal of S.

Let K be a lateral ideal of T containing A. Clearly A $\subseteq$ K and K is a lateral ideal of T. Therefore K$\in \Delta \Rightarrow S^* \subseteq$ K and hence S^* is the lateral ideal of T generated by A.

DEFINITION 3.47 : Let T be a ternary semigroup and A be a non-empty subset of T. The smallest right ideal of T containing A is called ***right ideal of T generated by A***.

THEOREM 3.48 : The right ideal of a ternary semigroup T generated by a nonempty subset A is the intersection of all right ideals of T containing A.

Proof : Let Δ be the set of all right ideals of T containing A . Since T itself is a right ideal of T containing A, T$\in \Delta$. So $\Delta \neq \emptyset$. Let $S^* = \bigcap_{S \in \Delta} S$.

Since A $\subseteq$S for all S $\in \Delta$, A $\subseteq$ S^*. By theorem 3.16, S^* is a right ideal of S.

Let K be a right ideal of T containing A. Clearly A $\subseteq$ K and K is a right ideal of T .

Therefore K$\in \Delta \Rightarrow$ $S^* \subseteq$ K. Therefore S^* is the right ideal of T generated by A.

DEFINITION 3.49 : Let T be a ternary semigroup and A be a non-empty subset of T. The smallest two sided ideal of T containing A is called ***two sided ideal of T generated by A***.

THEOREM 3.50 : The two sided ideal of a ternary semigroup T generated by a non-empty subset A is the intersection of all two sided ideals of T containing A.

Proof : Let Δ be the set of all two sided ideals of T containing A . Since T itself is a two sided ideal of T containing A, T$\in \Delta$. So $\Delta \neq \theta$. Let $S^* = \bigcap_{S \in \Delta} S$.

Since A $\subseteq$S for all S $\in \Delta$, A $\subseteq$ S^*. By theorem 3.22, S^* is a two sided ideal of S.

Let K be a two sided ideal of T containing A.

Clearly A $\subseteq$ K and K is a two sided ideal of T. Therefore K$\in \Delta \Rightarrow S^* \subseteq$ K.

Therefore S^* is the two sided ideal of T generated by A.

DEFINITION 3.51 : Let T be a ternary semigroup and A be a non-empty subset of T. The smallest ideal of T containing A is called ***ideal of T generated by A***.

THEOREM 3.52 : The ideal of a ternary semigroup T generated by a non-empty subset A is the intersection of all ideals of T containing A.

Proof : Let Δ be the set of all ideals of T containing A . Since T itself is an ideal of T containing A, T $\in \Delta$. So $\Delta \neq \theta$. Let $S^* = \bigcap_{S\in\Delta} S$.

Since A $\subseteq$ S for all S $\in \Delta$, A $\subseteq$ S^*.By theorem 3.29 , S^* is an ideal of S.

Let K be an ideal of T containing A. Clearly A $\subseteq$ K and K is an ideal of T .

Therefore K $\in \Delta \Rightarrow S^* \subseteq$ K. Therefore S^* is the ideal of T generated by A.

DEFINITION 3.53 : A left ideal A of a ternary semigroup T is said to be the ***principal left ideal generated by a*** if A is a left ideal generated by a for some a $\in$ T. It is denoted by L (a) or $< a >_l$.

THEOREM 3.54 : If T is a ternary semigroup and a $\in$ T then L (a) = $a \cup$ TTa.

Proof : Let $s, t \in$ T, $r \in a \cup$ TTa.

r $\in a \cup$ TT$a \Rightarrow r = a$ (or) $r = uva$ for some $u, v \in$ T

If $r = a$ then $str = sta \in$ TT$a \subseteq$ a$\cup$TTa.

If $r = uva$ then str = $s\,t\,(uva) = (stu)\,v\,a \in$ TT$a \subseteq a\cup$TTa

Therefore $str \in a\cup$TTa and hence $a\cup$TTa is a left ideal of T.

Let L be a left ideal of T containing a.

Let $r \in a\cup$TTa. Then $r = a$ (or) $r = uva$ for some $u,v \in$ T

If $r = a$ then $r = a \in$ L. If $r = uva$ then $r = uva \in$ L

Therefore $a\cup$TT$a \subseteq$ L and hence $a\cup$TTa is the smallest left ideal containing a.

Therefore L(a) = a$\cup$TTa.

NOTE 3.55 : if T is ternary semigroup and $a \in$ T then L(a) = T^1T^1a .

DEFINITION 3.56 : A lateral ideal A of a ternary semigroup T is said to be the ***principal lateral ideal generated by a*** if A is a lateral ideal generated by a for some $a \in$ T. It is denoted by M (a) (or) $< a >_m$.

THEOREM 3.57 : If T is a ternary semigroup and a $\in$ T then
M (a) = $a\cup$TaT$\cup$TTaTT.

Proof : ***Proof*** : Let $s, t \in$ T , $r \in a\cup$TaT$\cup$TTaTT

$r \in a\cup$TaT$\cup$TTaTT$\Rightarrow r = a$ (or) $r = uav$ (or) $r = uvapq$ for some $u,v,p,q \in$ T.

If $r = a$ then $srt = sat \in TaT \subseteq a \cup TaT \cup TTaTT$

If $r = uav$ then $srt = s(uav)t = suavt \in TaT \subseteq a \cup TaT \cup TTaTT$.

If $r = uvapq$ then $srt = s\ (uvapq)\ t = (suv)\ a\ (pqt) \in TaT \subseteq a \cup TaT \cup TTaTT$

Therefore $srt \in a \cup TaT \cup TTaTT$ and hence $a \cup TaT \cup TTaTT$is a lateral ideal of T.

Let M be a lateral ideal of T containing a.

Let $r \in a \cup TaT \cup TTaTT$ then $r = a$ (or) $r = uav$ (or) $r = uvapq$ for some $u,v,p,q \in T$

If $r = a$ then $r = a \in M$

If $r = uav$ then $r = uav \in M$

If $r = uvapq$ then $r = uvapq \in M$

Therefore $a \cup TaT \cup TTaTT \subseteq M$ and hence $a \cup TaT \cup TTaTT$ is the smallest lateral ideal containing a. Therefore $M(a) = a \cup TaT \cup TTaTT$.

DEFINITION 3.58 : A right ideal A of a ternary semigroup T is said to be a ***principal right ideal generated by a*** if A is a right ideal generated by a for some $a \in T$. It is denoted by R (a) (or) $< a >_r$.

THEOREM 3.59 : If T is a ternary semigroup and a $\in$ T then R (a) = a$\cup$ aTT.

Proof : ***Proof*** : Let $s, t \in T$, $r \in a \cup aTT$.

$r \in a \cup aTT \Rightarrow r = a$ (or) $r = auv$ for some $u,v \in T$.

If $r = a$ then $rst = ast \in aTT \subseteq a \cup aTT$.

If $r = auv$ then $rst = (auv)st = a(uvs)t \in aTT \subseteq a \cup aTTT$.

Therefore $rst \in a \cup aTT$ and hence $a \cup aTT$is a right ideal of T.

Let R be a right ideal of T containing a

Let $r \in a \cup aTT$, then $r = a$ (or) $r = auv$ for some $u,v \in T$

If $r = a$ then $r = a \in R$. If $r = auv$ then $r = auv \in R$

Therefore $a \cup aTT \subseteq R$ and hence $a \cup aTT$ is the smallest right ideal containing a.

Therefore $R(a) = a \cup aTT$.

NOTE 3.60 : If T is a ternary semigroup and $a \in T$ then R (a) = a T^1T^1

DEFINITION 3.61 : A two sided ideal A of a ternary semigroup T is said to be the ***principal two sided ideal*** provided A is a two sided ideal generated by a for some $a \in T$. It is denoted by T (a) (or) $< a >_t$.

THEOREM 3.62 : If T is a ternary semigroup and $a \in$ T then
T (a) = $a \cup$ TT$a \cup a$TT $\cup$ TTaTT.

Proof : ***Proof*** : Let **s,t** $\in$ T , $r \in a \cup$ TT$a \cup a$TT$\cup$ TTaTT

$r \in a \cup$ TT$a \cup a$TT$\cup$ TTaTT

$\Rightarrow r = a$ (or) $r = uva$ (or) $r = auv$ (or) $r = uvapq$ for some $u,v,p,q \in$ T.

If $r = a$ then $str = sta \in$ TT$a \subseteq a \cup$ TT$a \cup a$TT$\cup$ TTaTT.

If $r = uva$ then $rst = (uva)st = uvast \in$ TTaTT $\subseteq a \cup$ TT$a \cup a$TT$\cup$ TTaTT.

If $r = auv$ then $rst = (auv)st = a\,(uvs)\,t \in a$TT $\subseteq a \cup$ TT$a \cup a$TT$\cup$ TTaTT.

If $r = uvapq$ then $rst = (uvapq)\,st = uva\,(pqs)t \in$ TTaTT $\subseteq a \cup$ TT$a \cup a$TT$\cup$ TTaTT.

Therefore $rst \in a \cup$ TT$a \cup a$TT$\cup$ TTaTT and hence $a \cup$ TT$a \cup a$TT$\cup$ TTaTT is a two sided ideal of T. Let T_1 be a two sided ideal of T containing a.

Let $r \in a \cup$ TT$a \cup a$TT$\cup$ TTaTT then r = a (or) $r = uva$ (or) r = auv (or) $r = uvast$ for some $u,v,s,t \in$ T. If $r = a$ then $r = a \in T_1$. If $r = uva$ then $r = uva \in T_1$.

If $r = auv$ then $r = auv \in T_1$. If $r = uvast$ then $r = uvast \in T_1$.

Therefore $a \cup$ TT$a \cup a$TT$\cup$ TTaTT $\subseteq T_1$ and hence a$\cup$ TTa$\cup$ aTT$\cup$ TTaTTis the smallest two sided ideal containing 'a'. Therefore T(a) = $a \cup$ TT$a \cup a$TT$\cup$ TTaTT.

DEFINITION 3.63 : An ideal A of a ternary semigroup T is said to be ***a principal ideal*** provided A is an ideal generated by a for some $a \in$ T. It is denoted by J (a) (or) < a >.

THEOREM 3.64 : If T is a ternary semigroup and a $\in$ T then
J (a) = $a \cup a$TT$\cup$ TT$a \cup$ TaT$\cup$ TTaTT.

Proof : ***Proof*** : Let $s,t \in$ T , $r \in a \cup a$TT$\cup$ TT$a \cup$ TaT$\cup$ TTaTT.

$r \in a \cup a$TT$\cup$ TT$a \cup$ TaT$\cup$ TTaTT

$\Rightarrow r = a$ (or) $r = auv$ (or) $r = uva$ (or) $r = uav$ (or) $r = uvapq$ for some $u,v,p,q \in$ T.

If $r = a$ then $rst = ast \in a$TT $\subseteq a \cup a$TT$\cup$ TT$a \cup$ TaT$\cup$ TTaTT.

If $r = auv$ then $rst = (auv)$st = auvst $\in a$TT $\subseteq a \cup a$TT$\cup$ TT$a \cup$ TaT$\cup$ TTaTT.

If $r = uva$ then $rst = (uva)$st = uvast $\in$ TaT $\subseteq a \cup a$TT$\cup$ TT$a \cup$ TaT$\cup$ TTaTT.

If $r = uav$ then $rst = (uav)$ st = ua (ust) $\in$ TaT $\subseteq a \cup a$TT$\cup$ TT$a \cup$ TaT$\cup$ TTaTT.

If $r = uvapq$ then $rst = (uvapq)$ st $\in$ TTaTT $\subseteq a \cup a$TT$\cup$ TT$a \cup$ TaT$\cup$ TTaTT.

$\therefore\ rst \in a \cup a\mathrm{TT} \cup \mathrm{TT}a \cup \mathrm{T}a\mathrm{T} \cup \mathrm{TT}a\mathrm{TT} \Rightarrow a \cup a\mathrm{TT} \cup \mathrm{TT}a \cup \mathrm{T}a\mathrm{T} \cup \mathrm{TT}a\mathrm{TT}$ is an ideal of T. Let J be an ideal of T containing a. Let $r \in a \cup a\mathrm{TT} \cup \mathrm{TT}a \cup \mathrm{T}a\mathrm{T} \cup \mathrm{TT}a\mathrm{TT}$ then

$r = a$ (or) $r = auv$ (or) $r = uva$ (or) $r = uav$ (or) $uvapq$ for some $u,v,p,q \in$ T.

If $r = a$ then $r = a \in$ J. If $r = auv$ then $r = auv \in$ J. If $r = uva$ then $r = uva \in$ J.

If $r = uav$ then $r = uav \in$ J. If $r = uvapq$ then $r = uvapq \in$ J.

$\therefore\ a \cup a\mathrm{TT} \cup \mathrm{TT}a \cup \mathrm{T}a\mathrm{T} \cup \mathrm{TT}a\mathrm{TT} \subseteq$ J and hence $a \cup a\mathrm{TT} \cup \mathrm{TT}a \cup \mathrm{T}a\mathrm{T} \cup \mathrm{TT}a\mathrm{TT}$ is the smallest ideal containing 'a'. Therefore $\mathrm{J}(a) = a \cup a\mathrm{TT} \cup \mathrm{TT}a \cup \mathrm{T}a\mathrm{T} \cup \mathrm{TT}a\mathrm{TT}$.

NOTE 3.65 : If T is a ternary semigroup and a $\in$ T then

$\mathrm{J}(a) = a \cup a\mathrm{TT} \cup \mathrm{TT}a \cup \mathrm{T}a\mathrm{T} \cup \mathrm{TT}a\mathrm{TT} = T^1 T^1 a T^1 T^1$.

THEOREM 3.66 : In any ternary semigroup T, the following are equivalent.

1) **Principal ideals of T form a chain.**
2) **Ideals of T form a chain.**

Proof : (1) $\Rightarrow$ (2): Suppose that principal ideals of T form a chain.

Let A, B be two ideals of T. Suppose if possible A $\not\subseteq$ B , B $\not\subseteq$ A.

Then there exist a $\in$ A\B and b $\in$ B\A

$a \in$ A $\Rightarrow < a > \subseteq$ A and $b \in$ B $\Rightarrow < b > \subseteq$ B.

Since principal ideals form a chain, either $< a > \subseteq < b >$ or $< b > \subseteq < a >$.

If $< a > \subseteq < b >$, then $a \in$ <b> $\subseteq$ B. It is a contradiction.

If $< b > \subseteq < a >$, then $b \in < a > \subseteq$ A. It is also a contradiction.

Therefore A $\subseteq$ B or B $\subseteq$ A and hence ideals form a chain.

(2) $\Rightarrow$ (1) : Suppose that ideals of T form a chain.

Then clearly principal ideals of T form a chain.

DEFINITION 3.67 : A ternary semigroup T is said to be ***left simple ternary semigroup*** if T is its only left ideal.

THEOREM 3.68 : A ternary semigroup T is a left simple ternary semigroup if and only if TT*a* = T for all *a* $\in$ T.

Proof : Suppose that T is a simple ternary semigroup and $a \in$ T. Let $s \in \mathrm{TT}a$; $t, u \in$ T

$s \in$ TTA $\Rightarrow s = vwa$ where $v,w \in$ T. Now $uts = ut(vwa) = (utv)\, wa \in \mathrm{TT}a$

$\Rightarrow$ TTa is a left ideal of T. Since T is a left simple ternary semigroup , TTa = T

Therefore TTa = T for all $a \in$ T.

Conversely suppose that T*Ta* = T for all $a \in$ T.

Let L be a left ideal of T. Let $l \in$ L. Then $l \in$ T . By assumption TTl = T.

Let $t \in$ T then $t \in$ TT$l \Rightarrow t = uvl$ for some $u,v \in$ T.

$l \in$ L; $u,v \in$ T and L is a left ideal $\Rightarrow uvl \in$ L $\Rightarrow$ $t \in$ L.

Therefore T $\subseteq$ L. Clearly L $\subseteq$ T and hence L = T.

Therefore T is the only left ideal of T. Hence T is left simple ternary semigroup.

DEFINITION 3.69: A ternary semigroup T is said to be ***lateral simple ternary semigroup*** if T is its only lateral ideal.

THEOREM 3.70 : A ternary semigroup T is a lateral simple ternary semigroup if and only if T*a*T = TT*a*TT = T for all *a*$\in$T.

Proof : ***Proof*** : Suppose that T is a lateral simple ternary semigroup and $a \in$ T.

Let $s \in$ TaT ; $t, u \in$ T

$s \in$ TaT $\Rightarrow s = vaw$ where $v,w \in$ T.

Now $ust = u(vaw)t = uvawt \in$ TTaTT = TaT $\Rightarrow$ TaT is a lateral ideal of T.

Since T is a lateral simple ternary semigroup, TaT = T

Therefore TaT = T for all $a \in$ T.

Conversely suppose that TaT = T for all a $\in$ T.

Let M be a lateral ideal of T. Let $m \in$ M. Then $m \in$ T. By assumption TmT = T.

Let $t \in$ T . Then $t \in$ TmT $\Rightarrow t = umv$ for some $u,v \in$ T.

$m \in$ M; $u,v \in$ T and M is a lateral ideal $\Rightarrow umv \in$ M $\Rightarrow$ $t \in$ M.

Therefore T $\subseteq$ M. Clearly M $\subseteq$ T and hence M = T.

Therefore T is the only lateral ideal of T. Hence T is lateral simple ternary semigroup.

DEFINITION 3.71 : A ternary semigroup T is said to be ***right simple ternary semigroup*** if T is its only right ideal.

THEOREM 3.72 : A ternary semigroup T is a right simple ternary semigroup if and only if *a*TT = T for all *a*$\in$T.

Proof : ***Proof*** : Suppose that T is a right simple ternary semigroup and $a \in$ T.

Let $s \in a$TT ; $t, u \in$ T. $s \in a$TT $\Rightarrow s = avw$ where $v,w \in$ T.

Now $sut = (avw)ut = a(vwu)$t $\in a$TT $\Rightarrow$ aTT is a right ideal of T.

Since T is a right simple ternary semigroup , aTT = T. Therefore aTT = T for all $a \in$ T.

Conversely suppose that aTT = T for all $a \in$ T.

Let R be a right ideal of T. Let $r \in$ R. Then $r \in$ T. By assumption rTT = T.

Let $t \in$ T . Then $t \in$ rTT $\Rightarrow t = ruv$ for some $u,v \in$ T.

$r \in$ R; $u,v \in$ T and R is a right ideal $\Rightarrow ruv \in$ R $\Rightarrow t \in$ R.

Therefore T $\subseteq$ R. Clearly R $\subseteq$ T and hence R = T.

Therefore T is the only right ideal of T. Hence T is right simple ternary semigroup.

DEFINITION 3.73 : A ternary semigroup T is said to be ***simple ternary semigroup*** if T is its only ideal of T.

THEOREM 3.74 : If T is a left simple ternary semigroup (or) a lateral simple ternary semigroup (or) a right simple ternary semigroup then T is a simple ternary semigroup.

Proof : Suppose that T is a left simple ternary semigroup. Then T is the only left ideal of T. If A is an ideal of T, then A is a left ideal of T and hence A = T.

Therefore T itself is the only ideal of T and hence T is a simple ternary semigroup.

Suppose that T is a lateral simple ternary semigroup. Then T is the only lateral ideal of T. If A is an ideal of T, then A is a lateral ideal of T and hence A = T.

Therefore T itself is the only ideal of T and hence T is a simple ternary semigroup. Similarly if T is right simple ternary group then T is simple ternary semigroup.

THEOREM 3.75: A ternary semigroup T is simple ternary semigroup if and only if TT*a*TT = T for all *a* $\in$ T.

Proof : Suppose that T is a simple ternary semigroup and $a \in$ T.

Let $t \in$ TTaTT ; $u,v \in$ T. $t \in$ TTaTT $\Rightarrow t = s_1 s_2 a s_3 s_4$ where $s_1, s_2, s_3, s_4 \in$ T.

Now $uvt = uv\,(s_1 s_2 a s_3 s_4) = (uv s_1)\, s_2 a s_3 s_4 \in$ TTaTT

$utv =$ u $s_1 s_2 a s_3 s_4$ v $\in$ TTaTT and $tuv = s_1 s_2 a s_3 s_4 uv = s_1 s_2 a s_3\,(s_4 uv) \in$ TTaTT

Therefore TTaTT is an ideal of T. Since T is a simple ternary semigroup, T itself is the only ideal of T and hence TTaTT = T.

Conversely suppose that TTaTT = T for all $a \in$ T. Let I be an ideal of T.

Let $a \in$ I. Then $a \in$ T. So TTaTT = T.

Let $t \in$ T. Then $t \in$ TTaTT $\Rightarrow t = t_1 t_2 a t_3 t_4$ for some $t_1 t_2 a t_3 t_4 \in$ I $\Rightarrow t \in$ I.

Therefore T $\subseteq$ I. Clearly I $\subseteq$ T and hence T = I.

Therefore T is the only ideal of T. Hence T is simple ternary semigroup.

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