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VISCOELASTIC EFFECTS TO MHD OSCILLATORY VISCOELASTIC FLOW IN A CHANNEL FILLED WITH POROUS MEDIUM AND CHEMICAL REACTION

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Abstract: The effect of viscoelastic parameter on an unsteady hydromagnetic flow of a conducting optically thin viscoelastic fluid through a channel filled with saturated porous medium and non-uniform walls temperature are investigated in the presence of first-order chemical reaction and radiative heat transfer. It is assumed that the fluid has small electrical conductivity and the electromagnetic force produced is very small. Closed-form analytical solutions are constructed for the problem. The effects of the radiation and the magnetic field parameters on velocity profile and shear stress for different values of the viscoelastic parameter with the combination of the other flow parameters are illustrated graphically and physical aspects of the problem are discussed.

Keywords: Oscillatory flow, porous medium, heat transfer, magnetic field, viscoelastic, first-order chemical reaction.

1. Introduction

The study of flow of an electrically conducting fluid has many applications in engineering problems such as magnetohydrodynamics (MHD) generators, plasma studies, nuclear reactors, geothermal energy extraction, and the boundary layer control in the field of aerodynamics. In the light of these applications, MHD flow in a channel has been studied by many authors; some of them are Nigam and Singh [1], Soundalgekar and Bhat [2], Vajravelu [3], Attia and Kotb [4]. A survey of MHD studies in the technological fields can be found in Moreau [5]. The flow of fluids through porous media is an important topic because of the recovery of crude oil from the pores of the reservoir rocks; in this case, Darcy's law represents the gross effect. Raptis et al. [6] have analysed the hydromagnetic free convection flow through a porous medium between two parallel plates. Aldoss et al. [7] have studied mixed convection flow from a vertical plate embedded in a porous medium in the presence of a magnetic field. Makinde and Mhone [8] have considered heat transfer to MHD oscillatory flow in a channel filled with porous medium. Choudhury and Das [9] have extended the problem studied by Makinde and Mhone [8] to the case of viscoelastic fluid. Singh and Kumar [10] investigated the effects of chemical reaction and heat generation/absorption on unsteady MHD free convection heat and mass transfer flow of an electrically conducting, viscous, incompressible fluid

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past an infinite hot vertical porous plate through porous medium when the plate temperature is span wise co- sinusoidally fluctuating with time.

In this study an attempt has been made to study the effect of viscoelastic parameter on an unsteady hydromagnetic flow of a conducting optically thin viscoelastic fluid through a channel filled with saturated porous medium and non-uniform walls temperature in the presence of first-order chemical reaction and radiative heat transfer characterised by second-order fluid.

The constitutive equation for the incompressible second-order fluid is of the form

$$\sigma = -pI + \mu_1 A_1 + \mu_2 A_2 + \mu_3 (A_1)^2 \tag{1}$$

where σ is the stress tensor, p is the hydrostatic pressure, I is the unit tensor, $A_n\,(n = 1,2)$ are the kinematic Rivlin-Ericksen tensors, μ_1, μ_2, μ_3 are the material co-efficients describing viscosity, elasticity and cross-viscosity respectively. The material co-efficients μ_1, μ_2, μ_3 are taken constants with μ_1 and μ_3 as positive and μ_2 as negative (Coleman and Markovitz [11]). The equation (1) was derived by Coleman and Noll [12] from that of the simple fluids by assuming that stress is more sensitive to the recent deformation than to the deformation that occurred in the distant past.

2. Mathematical formulation of the problem

Consider the flow of a conducting optically thin fluid in a channel filled with saturated porous medium under the influence of an externally applied homogeneous magnetic field and radiative heat transfer as shown in Fig1. It is assumed that the fluid

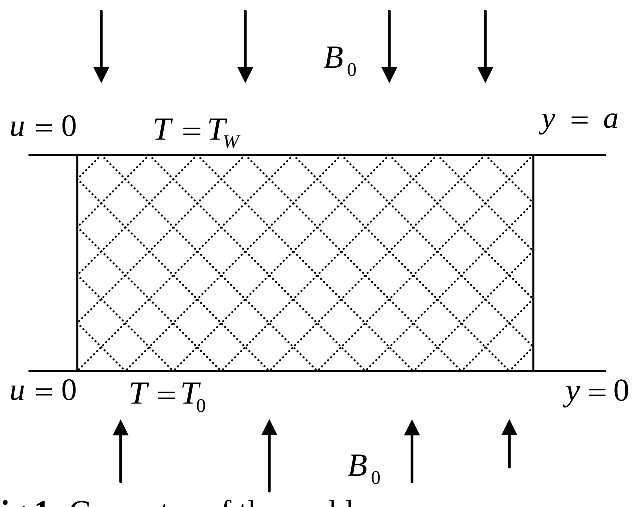

Fig.1: Geometry of the problem

has small electrical conductivity and the electromagnetic force produced is very small. Also, it is assumed that there exists a homogeneous first-order chemical reaction with rate constant K_R between the diffusing species and the fluid. The x-axis is taken along the centre of the channel and the y-axis is taken normal to it. Then, assuming a Boussinesq incompressible fluid model, the equations governing the motion are given as

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \upsilon_1 \frac{\partial^2 u}{\partial y^2} + \upsilon_2 \frac{\partial^3 u}{\partial y^2 \partial t} + g\beta_T (T - T_0) + g\beta_C (C - C_0) - \frac{\upsilon_1 u}{K} - \frac{\sigma_e {B_0}^2 u}{\rho} \qquad (2)$$

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p}\frac{\partial q}{\partial y} \qquad (3)$$

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial y^2} - K_R (C - C_0) \qquad (4)$$

subject to boundary conditions

$$u = 0,\ T = T_w,\ C = C_W \quad \text{on} \quad y = a;$$
$$u = 0,\ T = T_0,\ C = C_0 \quad \text{on} \quad y = 0. \qquad (5)$$

where u is the axial velocity, t the time, T the fluid temperature, C the species concentration, P the pressure, g the gravitational force, q the radiative heat flux, β_T the co-efficient of volume expansion due to temperature, β_C the co-efficient of volume expansion due to mass transfer, C_p the specific heat at constant pressure, k the thermal conductivity, K the porous medium permeability co-efficient, B_0 $(= \mu_e H_0)$ the electromagnetic induction, μ_e the magnetic permeability, H_0 the intensity of the magnetic field, σ_e the conductivity of the fluid, ρ the fluid density and $\upsilon_i = \frac{\mu_i}{\rho}(i = 1,2)$. It is assumed that both walls temperature T_0, T_w are high enough to induce radiative heat transfer. Following Cogley et al. [13], it is assumed that the fluid is optically thin with a relatively low density and the radiative heat flux is given by

$$\frac{\partial q}{\partial y} = 4\alpha^2 (T_0 - T) \qquad (6)$$

where α is the mean radiation absorption co-efficient.

The following non-dimensional quantities are introduced:

$$\text{Re} = \frac{Ua}{\upsilon_1}, \bar{x} = \frac{x}{a}, \bar{y} = \frac{y}{a}, \bar{u} = \frac{u}{U},\ \theta = \frac{T - T_0}{T_w - T_0},\ H^2 = \frac{a^2 \sigma_e {B_0}^2}{\rho \upsilon_1},\ \bar{t} = \frac{tU}{a},\ \bar{P} = \frac{aP}{\rho \upsilon_1 U},$$
$$Da = \frac{K}{a^2}, Gr = \frac{g\beta_T (T_w - T_0) a^2}{\upsilon_1 U},\quad Pe = \frac{Ua\rho C_p}{k},\ N^2 = \frac{4\alpha^2 a^2}{k},\ \phi = \frac{C - C_0}{C_W - C_0},$$
$$Gc = \frac{g\beta_C (C_w - C_0) a^2}{\upsilon_1 U},\ Sc = \frac{D}{Ua},\ K_r = \frac{K_R a}{U}.$$

where U is the flow mean velocity.

The dimensionless governing equations together with the appropriate boundary conditions (neglecting the bars for clarity) can be written as

$$\text{Re}\frac{\partial u}{\partial t} = -\frac{\partial P}{\partial x} + \frac{\partial^2 u}{\partial y^2} - (s^2 + H^2)u + Gr\theta + Gc\phi + \gamma \frac{\partial^3 u}{\partial y^2 \partial t} \qquad (7)$$

$$Pe\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + N^2 \theta \qquad (8)$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{Sc}\frac{\partial^2 \theta}{\partial y^2} - K_r \phi \tag{9}$$

with

$$u = 0,\ \theta = 1, \phi = 1 \text{ on } y = 1;$$
$$u = 0,\ \theta = 0, \phi = 0 \text{ on } y = 0. \tag{10}$$

where $Gr,\ Gc,\ H,\ N,\ Pe,\ \text{Re},\ Da,\ s\left(=\frac{1}{Da}\right), Sc, \gamma = \frac{\upsilon_2 \text{Re}}{a^2}$ are Grashof number for heat transfer, Grashof number for mass transfer, Hartmann number, Radiation parameter, Pe$^{/}$clet number, Reynolds number, Darcy number, porous medium shape factor parameter, Schmidt number, and viscoelastic parameter respectively.

3. Method of solution

In order to solve equations (6) and (7) for purely oscillatory flow, let

$$-\frac{\partial P}{\partial x} = \lambda e^{i\omega t},\ u(y,t) = u_0(y)e^{i\omega t},\ \theta(y,t) = \theta_0(y)e^{i\omega t}, \phi(y,t) = \phi_0(y)e^{i\omega t} \tag{11}$$

where λ is a constant and ω is the frequency of oscillation.

Substituting the above expressions into the equations (7), (8) and (9) and using equation (10), we get

$$(1 + i\gamma\omega)\frac{d^2 u_0}{dy^2} - m_3{}^2 u_0 = -\lambda - Gr\theta_0 - -Gc\phi_0 \tag{12}$$

$$\frac{d^2\theta_0}{dy^2} + m_1{}^2\theta_0 = 0 \tag{13}$$

$$\frac{d^2\phi_0}{dy^2} - m_2{}^2\phi_0 = 0 \tag{14}$$

subject to boundary conditions

$$u_0 = 0,\ \theta_0 = 1, \phi_0 = 1 \quad \text{on} \quad y = 1;$$
$$u_0 = 0,\ \theta_0 = 0, \phi_0 = 0 \text{ on } y = 0. \tag{15}$$

where $m_1 = \sqrt{N^2 - i\omega Pe}$, $m_2 = \sqrt{\frac{K_r + i\omega}{Sc}}$ and $m_3 = \sqrt{S^2 + H^2 + i\omega \text{Re}}$

The equations (12), (13) and (14) are solved under the boundary condition (15) and the solution for the fluid velocity, temperature and concentration profiles are given as follows:

$$u(y,t) = \left\{ C_1 e^{\frac{m_3 y}{L}} + C_2 e^{-\frac{m_3 y}{L}} + \frac{\lambda}{m_3^2} + \frac{Gr}{\sin m_1}\frac{\sin m_1 y}{(m_1^2 L + m_3^2)} - \frac{Gr(e^{m_2 y} - e^{-m_2 y})}{(e^{m_2} - e^{-m_2})(Lm_2^2 - m_3^2)} \right\} e^{i\omega t} \tag{16}$$

$$\theta(y,t) = \frac{\sin(m_1 y)}{\sin m_1} e^{i\omega t} \tag{17}$$

$$\phi(y,t) = \frac{e^{m_2 y} - e^{-m_2 y}}{e^{m_2} - e^{-m_2}} \tag{18}$$

where $C_1 = \frac{\lambda}{m_3^2}\left(\frac{e^{-\frac{m_3}{L}}}{e^{\frac{m_3}{L}} + e^{-\frac{m_3}{L}}} - 1 \right) - \frac{Gr}{m_1^2 L + m_3^2} + \frac{Gr}{m_2^2 L - m_3^2}$,

$$C_2 = -\frac{\lambda}{m_3^2}.\frac{e^{-\frac{m_3}{L}}}{e^{\frac{m_3}{L}}+e^{-\frac{m_3}{L}}} + \frac{Gr}{m_1^2 L + m_3^2} - \frac{Gr}{m_2^2 L - m_3^2}$$

and $L = 1 + i\omega\gamma$

The non-dimensional shear stress σ at the wall $y = 0$ is given by

$$\sigma = \frac{\bar{\sigma}}{\frac{\mu_1 U}{a}} = \left[\frac{\partial u}{\partial y} + \gamma\frac{\partial^2 u}{\partial y \partial t}\right]_{y=0} \qquad (19)$$

The rate of heat transfer across the channel's wall is given as

$$Nu = -\frac{\partial\theta}{\partial y} = -\frac{m_1\cos(m_1)}{\sin(m_1)}e^{i\omega t} \qquad (20)$$

Sherwood number is given by

$$Sh_x = \frac{\partial\phi}{\partial y} \qquad (21)$$

4. Discussions and Conclusion

The purpose of this study is to bring out the effects of the viscoelastic parameter γ on the governing flow with the combination of the other flow parameters. The corresponding results for Newtonian fluid can be deduced from the above results by setting $\gamma = 0$. We have considered the real parts of the results throughout for numerical validation. The velocity profile u against y is plotted in Figs. 2-5 to observe the viscoelastic effects for various sets of values of Hartmann number H and radiation parameter $N(H = 0, N = 1.5;\ H = 0.5,\ N = 1.5;\ H = 0.5,\quad N = 2.5;\ H = 1.5, N = 2.5)$ with fixed values of other flow parameters viz., $Pe = 1$, $\mathrm{Re} = 3$, $s = 1$, $t = 0$, $Gr = 2$, $\lambda = 1$ and $\omega = 1$. It is evident from the Figs. 2-5 that the velocity profile is parabolic in nature and the values of velocity u decrease with the increasing values of the viscoelastic parameter $|\gamma|$ $(\gamma = 0, -0.05, -0.10)$ in comparison with the Newtonian fluid. Again it is found that the velocity decreases with the increasing magnetic parameter for both Newtonian and non-Newtonian fluid. It is because that the application of transverse

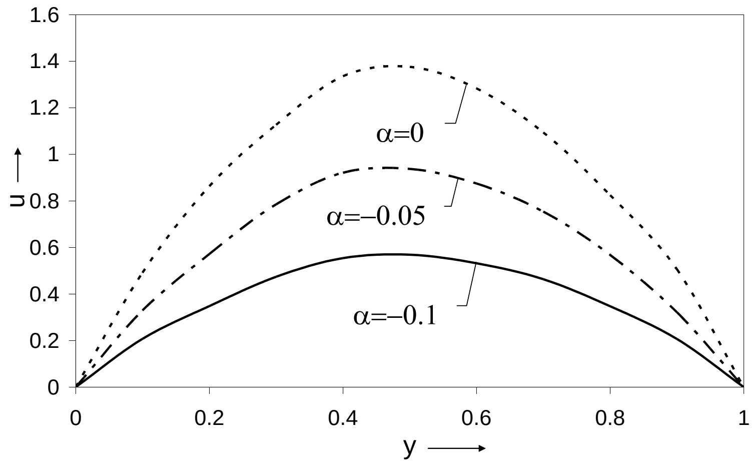

Fig. 2: Variation of u against y when $H = 0, N = 1.5$.

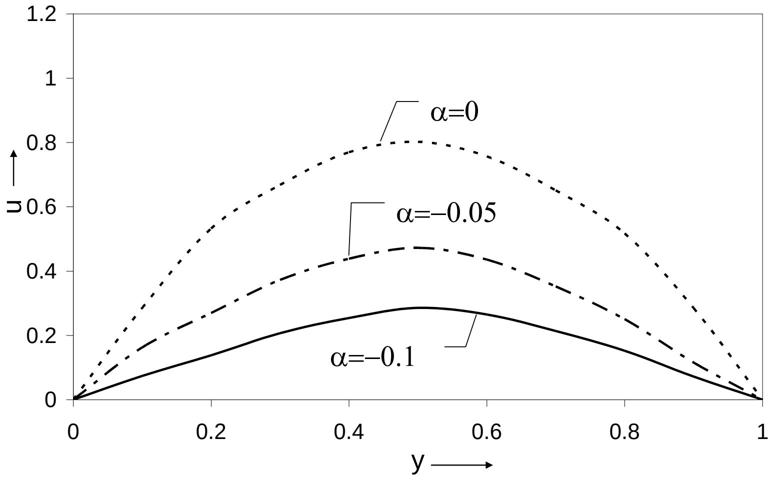

Fig. 3: Variation of u against y when $H = 0.5$, $N = 1.5$.

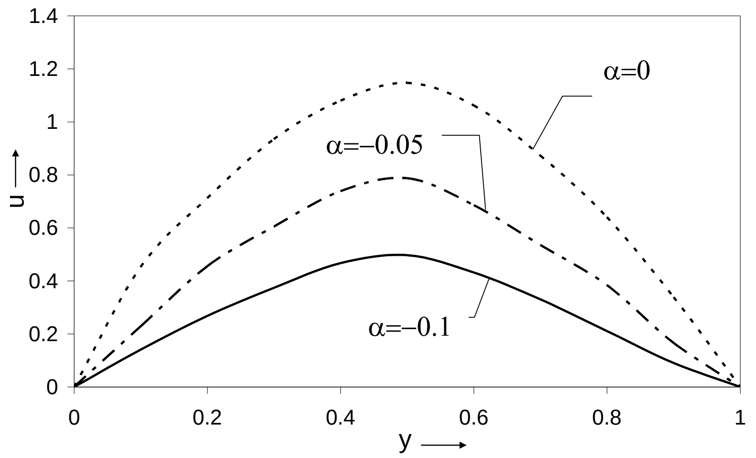

Fig. 4: Variation of u against y when $H = 0.5$, $N = 2.5$.

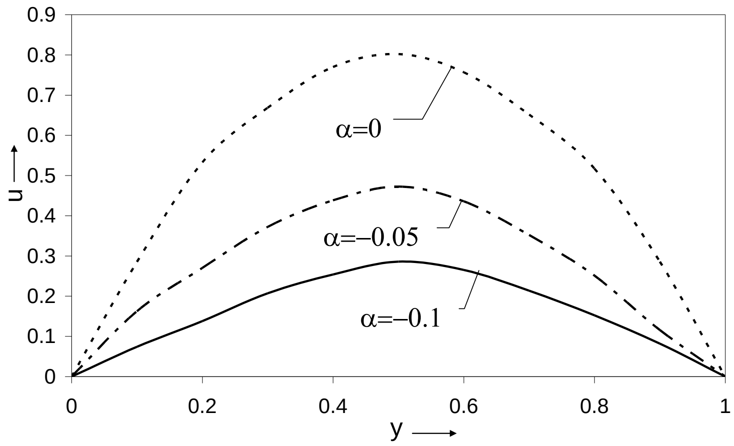

Fig.5: Variation of u against y when $H = 1.5,\ N = 2.5$

magnetic field will result a resistive type force which tends to resist the fluid flow and thus reducing its velocity. It is also noted from the figures that the behaviours of the velocity profiles remain the same with the increasing values of the viscoelastic parameter $|\gamma|$ when (i) the values of the magnetic field parameter H increase with fixed values of the radiation parameter N (Figs. 2, 3 or 4, 5) (ii) the values of the radiation parameter N increase with fixed values of the magnetic field parameter H (Figs. 3, 4) and (iii) both the values of H and N increase (Figs. 2, 4; 2, 5 or 3, 5).

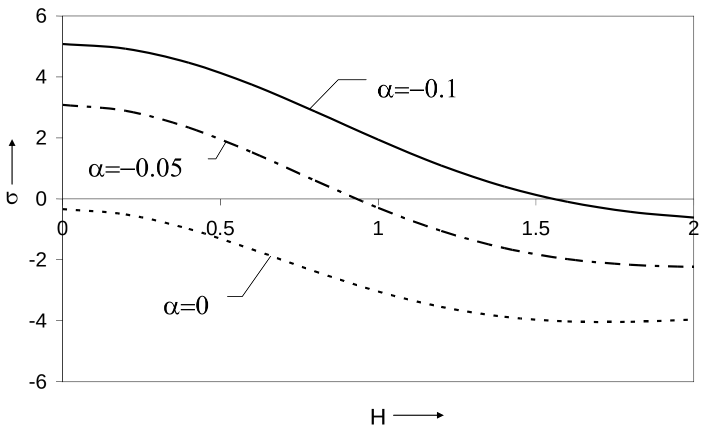

Fig.6: Variation of σ against H when $N = 1.5$.

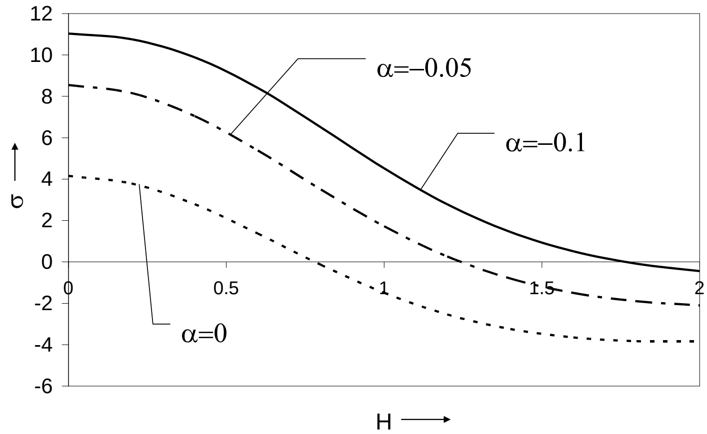

Fig.7: Variation of σ against H when $N = 2.5$.

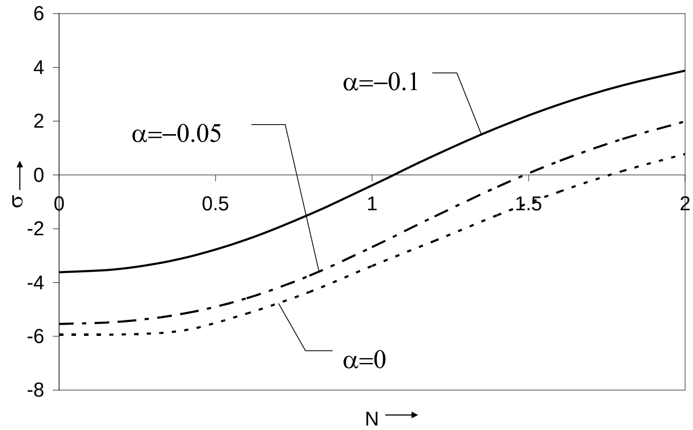

Fig.8: Variation of σ against N when $H = 0.5$.

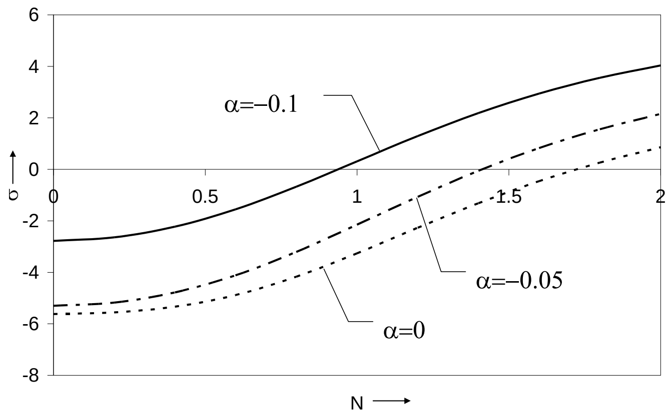

Fig.9: Variation of σ against N when $H = 1.5$.

The Figs. 6, 7 and 8, 9 exhibit the effects of the viscoelastic parameter $|\gamma|$ on skin friction σ against magnetic field parameter H and radiation parameter N respectively with $Pe = 1, \mathrm{Re} = 3,\ s = 1, t = 0,\ Gr = 2,\ \lambda = 1,\ \omega = 1$. Figures 6 and 7 show that for radiation parameter $N = 1.5$ and $N = 2.5$ the shear stress decrease with the increasing values of H for both Newtonian and non-Newtonian fluids while shear stress increase for increasing values of $|\gamma|$ $(\gamma = 0, -0.05, -0.1)$ in comparison with Newtonian fluid. From the Figures 8 and 9, it is evident that the shear stress σ increase with the increasing values of N for both Newtonian and non-Newtonian cases. Also, the Figs. 8, 9 depict that the shear stress increase with the increasing values of the viscoelastic parameter $|\gamma|$ in comparison with Newtonian fluid.

It has also been observed that the temperature field, concentration field are not significantly affected by the viscoelastic parameter.

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