ISSN 0976 – 1411

Available online at www.internationaleJournals.com

International eJournals

INTERNATIONAL eJOURNAL OF MATHEMATICS AND ENGINEERING

International eJournal of Mathematics and Engineering 195 (2012) 1864 - 1874

www.InternationaleJournals.com

A MATHEMATICAL MODEL OF FLUID FLOW BETWEEN POROUS PARALLEL OSILATING PLATES

Odelu Ojjela
Dept of Applied Mathematics , DIAT (DU), Pune , India – 411 025
e-mail: **odelu3@yahoo.co.in**

Abstract:

In this paper , the flow of a viscous fluid between two parallel porous plates generated due to the periodic oscillations of the plates is considered. The stream function and pressure are taken as power series in oscillation Reynolds number parameters. The equations for zeroth order, first order and second order flow are derived. The flow pattern is obtained for different situations like constant motion, square wave motion and sinusoidal motion of the plates. The flow pattern is represented in the form of graphs.

Keywords: porous plates, oscillation,suction, injection, viscous fluid

1.Introduction:

The problems involving disturbances of known frequency are encountered very often in natural and biological flows; for example motion of water waves over a shallow beach, the flow of blood in the arteries and the mechanics of cochlea in the human ear. These effects of the periodic disturbances at the boundary on the flow can be easily analyzed theoretically and experimentally in the simple geometry of parallel plates. The flow of a fluid between two parallel plates is treated by several researchers, not only in view of the Mathematical simplicity, but also due to its important applications in many devices such as aerodynamic heating, electrostatic precipitation, polymer technology, petroleum industry etc.

Many researchers considered this problem of viscous flow between parallel plates. Abraham S.Berman(1953) considered the problem where in both channel plates have equal permeability and the flow at the center line of the channel attains maximum. John.R.Sellors (1955) considered the same problem whose solution is valid for high Reynold's number. Latter F.M.White et al(1958) extended this problem in detail to get a solution for a wide range of suction Reynold's number. R.M.Terrill and G.Shrestha(1965) examined the problem when the suction normal velocities at the walls are different(i.e the asymmetric suction problem). Stephen M.Cox(1991) studied the more general problem of symmetric and asymmetric suction driven flow between two parallel walls. Several authors considered different varieties of problems having parallel plate geometry. Nabil T.M El-Dabe and Salwa M.G.Mohandis(1995) considered the flow of couple stress fluid between two parallel plates due to pulsating pressure gradient under constant magnetic field. D.Srinivasacharya, J.V.Ramana Murthy and D.Venugopalam(2001) considered the stokes flow of micropolar fluid between two parallel plates when the plates are subjected to periodic suction or injection, by neglecting the nonlinear terms. N.M.Bujurke et al.(2004) have examined a flow between two parallel rectangular/ circular plates due to motion of the upper plate. The solution is obtained by using similarity technique. H.A.Attaia(2005) examined the effect of suction and injection on the flow of viscous fluid between two plates with variable viscosity. In a recent paper , N.Ch.Pattabhiramacharyulu et al (2006) have examined the flow of a thermo viscous fluid between two parallel plates due to the motion of the upper plate. The temperature distribution is obtained by finite difference methods.

In this paper we consider the flow of a viscous fluid between two parallel plates subjected to periodic oscillation without the neglect of nonlinear terms. The flow variables are expanded in a series of powers of suction Reynolds number and we obtain the flow variables up to the second order.

2.Statement and Formulation of the Problem:

We consider an incompressible viscous fluid present in between two infinite parallel plates. The flow of the fluid is generated due to periodic oscillation at the plates. We assume that the oscillation is an even function and can be represented as Real{Σ a_n exp(iωnt)} at the lower plate and Real{Σ b_n exp(iωnt)} at the upper plate.

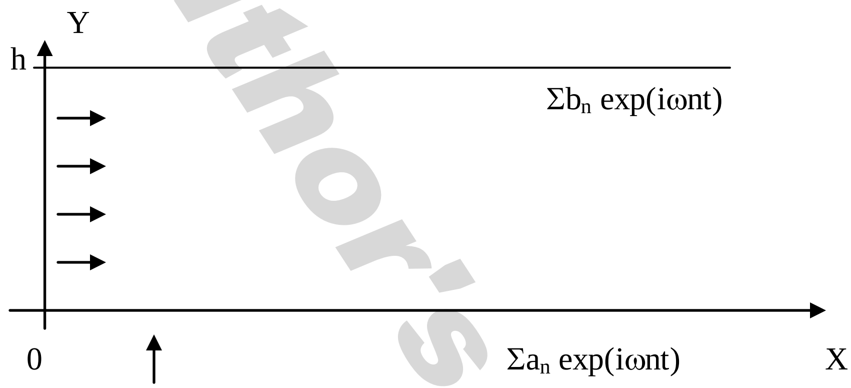


Fig1. Flow due to periodic oscillation of the plates.

We take Cartesian system (X,Y) on the lower plate with X axis along the plate and Y axis perpendicular to the plate. The plates are given by Y = 0 and Y = h planes. The flow is two dimensional in X-Y plane.

The incompressibility condition and equations of motion are given by

$$\text{Div } \overline{Q} = 0 \tag{1}$$

$$\rho \left(\frac{\partial \overline{Q}}{\partial t} + \overline{Q} \cdot grad\, \overline{Q} \right) = -grad\, P - \mu \,\text{rot}\,\text{rot}\, \overline{Q} \tag{2}$$

where $\overline{Q}$ is fluid velocity, P is pressure, ρ is density, μ is coefficient of viscosity and t is time. Let ω be the frequency of oscillation of at the plates, V_0 and V_1 be amplitudes of oscillation of velocities at the lower plate and upper plate and U_0 be average velocity along the plates. Now we introduce the following non-dimensional scheme.

$\overline{Q} = V_0\ \overline{q}$, $P = \rho V_0^2\, p$, $X = hx$, $Y = hy$, $t = h\tau / V_0$, $N = U_0/V_0$ and $\varepsilon = V_1/V_0$

and

$\sigma = \omega h / V_0$ = frequency parameter, $R = \rho V_0 h/ \mu$ suction Reynolds number (3)

By the non-dimensional scheme (3), the first term in the complex form of Fourier series expansion of periodic suction we have $a_0 = 1$ and the equations (1) and (2) take the following form:

$$\nabla \cdot \bar{q} = 0 \tag{4}$$

$$R\left(\frac{\partial \bar{q}}{\partial t} + \bar{q} \cdot \nabla \bar{q}\right) = -R\,\nabla \mathrm{p} + \nabla^2 \bar{q} \tag{5}$$

Since the flow is two dimensional, we introduce the non dimensional stream function ψ through:

$$\bar{q} = ui + v\,j = \frac{\partial \psi}{\partial y} i - \frac{\partial \psi}{\partial x} j \tag{6}$$

We notice that

$$\nabla \times \bar{q} = -\nabla^2 \psi \;\; k \tag{7}$$

To solve the non linear equation (5), we introduce a regular perturbation series for the non dimensional quantities as follows:

$$\left.\begin{aligned} \bar{q} &= \bar{q}_0 + R\,\bar{q}_1 + R^2\,\bar{q}_2 + \ldots, \\ p &= p_0 + R\,p_1 + R^2 p_2 + \ldots, \\ \psi &= \psi_0 + R\,\psi_1 + R^2\,\psi_2 + \ldots \end{aligned}\right\} \tag{8}$$

Substituting (8) in (5) we get the following linear equations.

$$\nabla^2 \bar{q}_0 = 0$$

$$\nabla^2 \bar{q}_n = \nabla p_{n-1} + \frac{\partial \bar{q}_{n-1}}{\partial \tau} + \sum_{k=0}^{n-1} \bar{q}_k \cdot \nabla \bar{q}_{n-1-k} \qquad n = 1,2\ 3 \tag{9}$$

Eliminating the non dimensional pressure p_{n-1} from (9) we get,

$$\nabla^4 \psi_0 = 0 \tag{10}$$

$$\nabla^4 \psi_n = \frac{\partial \nabla^2 \psi_{n-1}}{\partial \tau} - \sum_{k=0}^{n-1}\left(\frac{\partial \psi_k}{\partial x}\frac{\partial}{\partial y}\nabla^2 \psi_{n-1-k} - \frac{\partial \psi_k}{\partial y}\frac{\partial}{\partial x}\nabla^2 \psi_{n-1-k} + \frac{\partial^2 \psi_{n-1-k}}{\partial x \partial y} L^2 \psi_k - \frac{\partial^2 \psi_k}{\partial x \partial y} L^2 \psi_{n-1-k}\right)$$

where $L^2 = \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}$, n = 1, 2, 3 … (11)

We now introduce a function $f_n(y,\tau)$ as follows

$$\psi_n = (N - x)\, f_n(y, \tau) \tag{12}$$

Introducing the equation(12) in the equations (10) and (11) we get

International eJournal of Mathematics and Engineering 195 (2012) 1864 – 1874
Odelu Ojjela

$$\frac{\partial^4 f_0}{\partial y^4} = 0 \tag{13}$$

$$\frac{\partial^4 f_n}{\partial y^4} = \frac{\partial^3 f_{n-1}}{\partial y^2 \partial \tau} + \sum_{k=0}^{n-1}\left(f_k \frac{\partial^3 f_{n-1-k}}{\partial y^3} - \frac{\partial^2 f_k}{\partial y^2}\frac{\partial f_{n-1-k}}{\partial y}\right) \quad n = 1, 2, 3, \ldots \tag{14}$$

To match with the periodic oscillation of the boundary we take the function f_n as ;

$$f_n(y,\tau) = \mathrm{Real}\sum_{k=0}^{\infty} g_{nk}(y) e^{i\sigma k \tau} \tag{15}$$

By the use of the equation (15) , the equation (13) and (14) are reduced to ordinary differential equations as follows:

$$g_{ok}^{iv}(y) = 0 \tag{16}$$

$$g_{nk}^{iv}(y) = i\sigma k\, g''_{n-1,k} + \sum_{i=0}^{n-1}\sum_{p=0}^{k}\left(g_{ip}\, g'''_{n-1-i,k-p} - g''_{ip}\, g'_{n-1-i,k-p}\right), \quad n = 1, 2, 3, \ldots \tag{17}$$

These set of equations (16) and (17) are to be solved under the conditions

(i) horizontal velocity is zero at the lower and upper plates and

(ii) the vertical velocity at the lower and upper plates is equal to oscillation velocities.

i.e. (i) $f_0(0,\tau) = 0$ at $y = 0$ and $y=1$, (18)

(ii) $$\frac{\partial f_0(0,\tau)}{\partial y} = \mathrm{Real}\{\Sigma\, a_n \exp(i\sigma n\tau)\} \text{ and } \frac{\partial f_0(1,\tau)}{\partial y} = \mathrm{Real}\{\Sigma\, b_n \exp(i\sigma n\tau)\}$$

and $f_n(0, \tau) = f_n(1, \tau) = 0$ for $n = 1, 2, 3, \ldots$ (19)

These conditions in turn can be expressed in terms of $g_{nk}(y)$ as follows:

(i) $g_{0k}(0) = g_{0k}(1) = 0$, $g'_{ok}(0) = a_k$, $g'_{0k}(1) = b_k$, and (20)

(ii) $g'_{nk}(0) = g'_{nk}(1) = g_{nk}(0) = g_{gk}(1) = 0$ for $n = 1, 2, 3, \ldots$ (21)

Solving the set of equations (16) and (17) for zeroth order, first order and second order flow under the conditions (20) and (21) we get the solutions as:

$g_{0k}(y) = A_{1k}(y^3 - y^2) - A_{2k}(y^2 - y)$ (22)

$g_{1k}(y) = i\sigma k\,[A_{1k}(3y^5 - 5y^4 + y^3 + y^2)/60 + A_{2k}(-y^4 + 2y^3 - y^2)/12] + A^2_{1k}(-3y^7 + 7y^6 - 7y^5 + 8y^3 - 5y^2)/210 + A_{1k}A_{2k}(2y^6 - 4y^5 + 5y^4 - 6y^3 + 3y^2)/60 + A^2_{2k}(-2y^5 + 5y^4 + 4y^3 - 7y^2)/60$ (23)

$g_{2k}(y) = i\sigma k[B_{1k}(60y^7 - 140y\ 8y5 + 210y^4 - 154y^3 + 45y^2) + C_{2k}(448y^{11} - 2464y^{10} + 3080y^9 + 12705y^8 - 50424y^7 + 83776y^6 - 99792y^5 + 99330y^4 - 64378y^3 + 17719y^2) + C_{3k}(2y^8 - 8y^7 + 14y^6 -$

$28y^5+43y^4-30y^3+7y^2) + C_{4k}(2y^5-5y^4+4y^3-y^2) + C_{5k}(20y^7-70y^6+84y^5-35y^4-2y^3+3y^2) +$
$i\{C_{6k}(10y^9-45y^8+90y^7-210y^6+387y^5-285y^4-y^3+54y^2) + C_{7k}(6y^6-18y^5+15y^4-3y^2) +$
$C_{8k}(10y^9-45y^8+117y^7-252y^6+378y^5-315y^4+119y^3-12y^2) + C_{9k}(y^6-3y^5+3y^4-y^3)\}$ (24)

where

$A_{1k} = (a_k - b_k)$, $A_{2k} = b_k$, $B_{1k} = -\sum_{p=0}^{k} A_{1p}A_{1,k-p}/70$, $B_{2k} = \sum_{p=0}^{k} A_{2p}A_{1,k-p}/2$ and $B_{3k} = \sigma k A_{1k}/20$,

$C_{1k} = -\sum_{p=0}^{k} A_{1p}B_{2,k-p}/420$, $C_{2k} = \sum_{p=0}^{k} A_{1p}B_{1,k-p}/9240$, $C_{3k} = \sum_{p=o}^{k} A_{2p}B_{1,k-p}/4$, $C_{4k} = \sum_{p=0}^{k} A_{2p}B_{2,k-p}/10$,

$C_{5k} = -\sigma k B_{3k}/420$, $C_{6k} = \sigma k B_{1k}/180$, $C_{7k} = \sigma k B_{2k}/180$, $C_{8k} = -\sum_{p=0}^{k} A_{1p}B_{3,k-p}/315$ and

$C_{9k} = \sum_{p=0}^{k} A_{2p}B_{3,k-p}/3$

Pressure: Pressure can be found from equation (9)

$$\nabla^2 \bar{q}_n = \nabla p_{n-1} + \frac{\partial \bar{q}_{n-1}}{\partial \tau} + \sum_{k=0}^{n-1} \bar{q}_k \cdot \nabla \bar{q}_{n-1-k} \qquad n = 1,2\ 3$$

Substituting the expression for ψ from (12) in these equations and using the integral of equation (14) we get

$$\frac{\partial p_{n-1}}{\partial x} = (N-x)\left\{\frac{\partial^3 f_n}{\partial y^3} - \frac{\partial^2 f_{n-1}}{\partial \tau \partial y} + \sum_{k=0}^{n-1}\left(\frac{\partial f_k}{\partial y}\frac{\partial f_{n-k-1}}{\partial y} - f_k \frac{\partial^2 f_{n-k-1}}{\partial y^2}\right)\right\} = (N-x)\lambda_{1,n-1} \qquad (25)$$

and

$$\frac{\partial p_{n-1}}{\partial y} = \frac{\partial^2 f_n}{\partial y^2} - \frac{\partial f_{n-1}}{\partial \tau} + \sum_{k=0}^{n-1} f_k \frac{\partial f_{n-k-1}}{\partial y} \qquad (26)$$

Now integrating these equations (25) and (26), we get pressure as

$$p_{n-1} = \lambda_{1,n-1}(Nx - x^2/2) + \frac{\partial f_n}{\partial y} - \int \frac{\partial f_{n-1}}{\partial \tau} dy + \sum_{k=0}^{n-1} \int f_k \frac{\partial f_{n-k-1}}{\partial y} dy + \lambda_{2,n-1} \qquad (27)$$

The constants of integration $\lambda_{1,n-1}$ can be obtained by integrating equation (14) and evaluating it at y=0 and the constant of integration $\lambda_{2,n-1}$ can be known when pressure at any point is known.

For n=1,

$$p_0 = \lambda_{20} + \lambda_{10}(Nx - x^2/2) + \mathrm{Real}\left\{\sum_{k=0}^{\infty}\left\{g'_{1k} - \frac{1}{2}\sum_{r=0}^{k} g_{0r}\, g_{0,k-r} - i\sigma k \int g_{0k}\, dy\right\} e^{i\sigma k \tau}\right\} \qquad (28)$$

The first order pressure can be given similarly. When the constants a_k and b_k are given, it will be a big expression.

Special Cases: Now we consider various special cases of the flow by taking the Fourier series expansion of oscillation of the plates to illustrate the problem clearly.

Case (i): We consider the case of constant motion at the lower plate and same type of motion at the upper plate . In this case the periodic functions at the lower plate and upper plate are given by $V(\tau) = 1$ and hence we have

$$a_0 = b_0 = 1 \text{ and } a_k = b_k = 0 \text{ for } k \geq 1$$

In this case the flow is along the direction of X-axis and stream lines are given by y = constant.

Case (ii): We consider the case of constant motion at the lower plate and upper plate moves at the same speed in opposite. In this case the periodic functions at the lower plate and upper plate are given by $V_0(\tau) = 1$ and $V_1(\tau) = -1$ and hence we have

$$a_0 = -b_0 = 1 \text{ and } a_k = b_k = 0 \text{ for } k \geq 1$$

positive values and the stream lines with the same amount of negative values are placed symmetrically about the horizontal line y = 0.5.

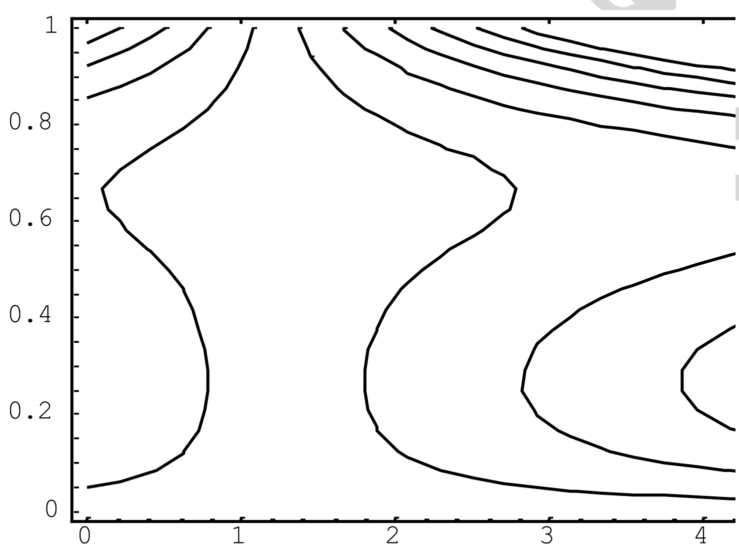
Case (iii) : We consider now Square wave oscillation at the plates as follows.

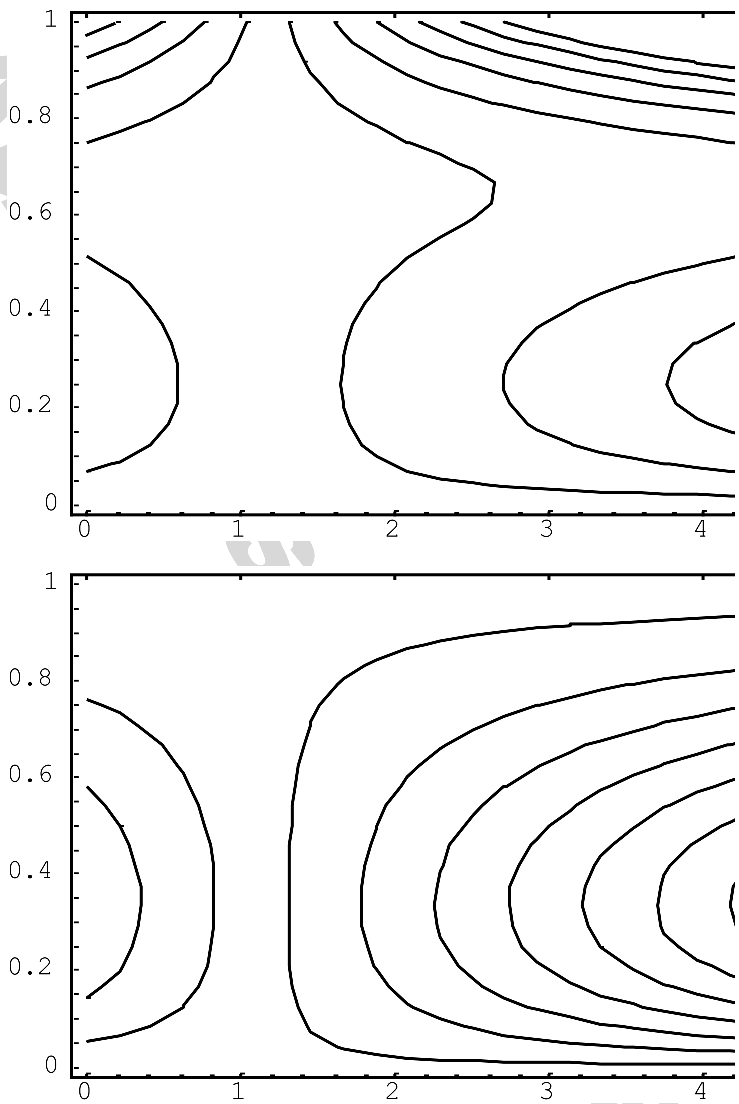
$$V_0(\tau) = 1 \text{ for } 0 < |\tau| < \pi/2$$
$$= 0 \text{ for } \pi/2 < |\tau| < \pi \quad \text{with period } 2\pi$$

Similarly at the upper plate, but $V_1(\tau)$ with negative sign. Now expanding these functions in Fourier series, we have

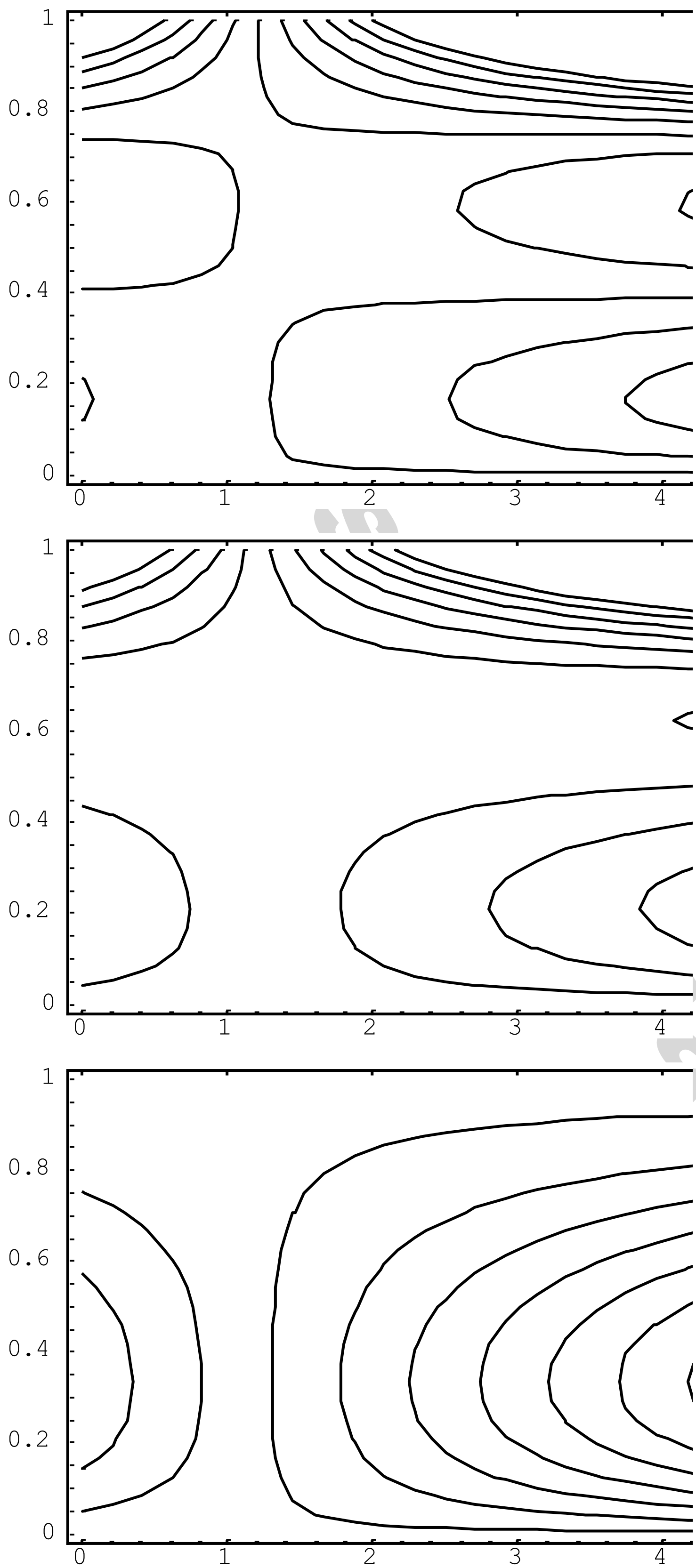
$$a_0 = -b_0 = ½ \text{ and } a_k = b_k = 2(-1)^m/(\pi k) \text{ if } k = (2m+1) \text{ and } a_k = b_k = 0 \text{ if } k = 2m.$$

In this case the stream functions are shown below.

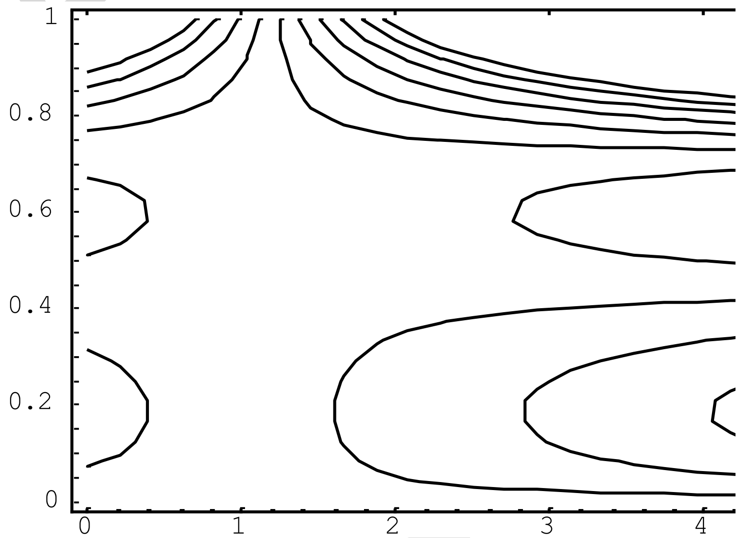


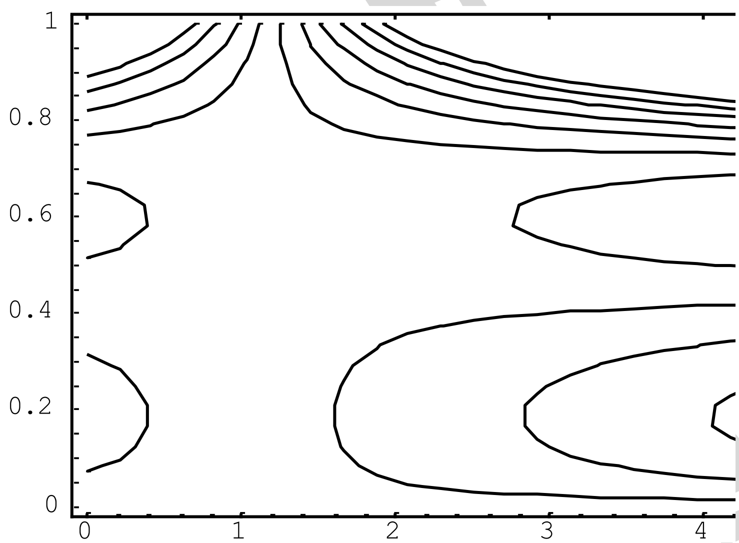


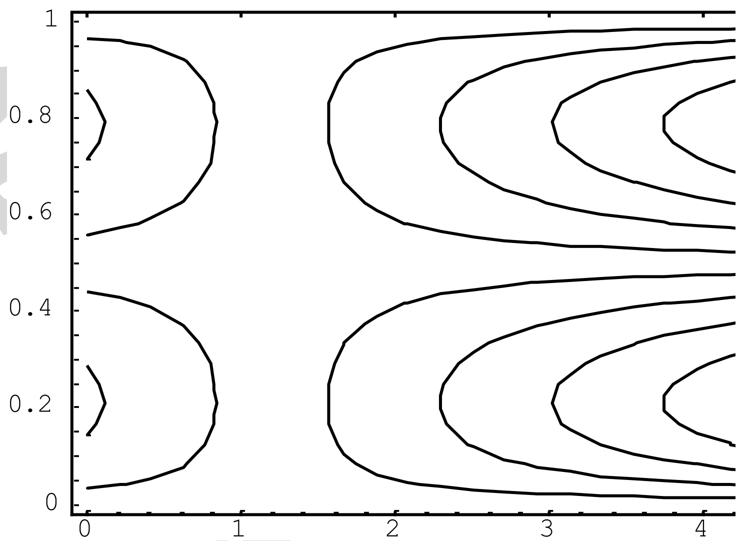
when the plates are oscillating with square wave nature and both are in opposite phase.


1
0.8
0.6
0.4
0.2
0
0
1
2
3
4

when the lower plate is kept constant and upper plate oscillates with square wave nature.







when the plates are oscillating with square wave nature and both are in same phase.

Conclusions:

The problem of viscous fluid flow between two porous parallel walls with periodic suction and injection is analyzed. The flow pattern is obtained for different situations like when the plates are oscillating with square wave nature and both are in opposite phase , when the lower plate is kept constant and upper plate oscillates with square wave nature and when the plates are oscillating with square wave nature and both are in same phase.

References :

1. Abraham S. Berman, "*Laminar flow in channels with porous walls",* J. Appl. Physics, Vol. 24, pp 1232 – 1235, 1953.
2. John R. Sellors, "*Laminar flow in channels with porous walls at high suction Reynolds number*", J.Appl. Physics, Vol 26, pp 489-490, 1955.
3. F. M. White Jr., B. F. Barfield, M. J. Goglia, " *Laminar flow in a uniformly porous channel",* J. Appl. Mechanics , pp 613-617, Dec.1958.
4. R.M. Terrill, G. Shrestha, " *Laminar Flow Through Parallel and Uniformly porous walls of Different permeability",* Z A M P, Vol 16, pp 470-482, 1965.
5. Stephen M. Cox, " *Two dimensional flow of a viscous fluid in a channel with porous walls",* J. Fluid Mech., Vol 227, pp1-33, 1991.
6. Nabil T.M. El-Dabe, Salwa M.G.Mohandis, "*Effect of couple stresses on pulsatile hydromagnetic poiseuille flow",* Fluid Dynamics Research, Vol 15, pp 313-324, 1995.

7. D.Srinivasacharya, J.V.Ramana.Murthy and D.Venugopalam, "*Unsteady Stokes flow of Micro polar fluid between two parallel porous plates",* Int J. Engg. Sci., Vol 39, pp1557-563, 2001.
8. N. M. Bujurke, N. P. Pai, N.N. Katagi and V.B. Awati, *"Analytic continuation of series solution representing flow between plates",* Indian J. Pure & Appl. Math., Vol.35, pp587-589, 2004.
9. H. A. Attaia, " *The effect of suction and injection on the unsteady flow between two parallel plates with variable properties",* Tamkang J.Sci. Engg., Vol 8, pp 17-22, 2005.
10. N. Ch. Pattabhiramacharyulu, B. Krishna Gandhi and K. Anuradha*, " Steady flow of a thermo-viscous fluid between two parallel plates in relative motion",* Int. J. of Math. Sci, Vol. 5, pp 147- 164, 2006.
11. J.V.Ramana Murthy, N.Srinivasacharyulu, O.Odelu (2007): *"viscous fluid flow between two parallel plates with periodic suction and injection"*, AMSE Journal on Modeling, Series B: Mechanics and Thermics Vol. 52, no.2, pp 19-27, 2007.