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RADIAL VIBRATIONS IN MICROPOLAR THIN SPHERICAL SHELL

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Abstract : *The frequency equations are derived for the radial vibrations in micropolar elastic thin spherical shell. It is interesting to observe that a new type of wave is propagated which is not found in classical theory of elasticity. The frequency equation of the classical case is obtained as a particular case of this paper.*

Key Words: Radial vibrations, Micro polar elastic spherical shell

INTRODUCTION

In the micro polar theory, a volume element V is assumed to be a collection of micro elements $\Delta V^{(\alpha)}$ ($\alpha = 1,2,\ldots.N$). In addition to the classical deformation it considers the rotation of micro elements about the centre of mass of V.

The mechanical behavior of elastic materials with micro-structure has been the subject of intensive studies in recent years. The theory of micro polar elasticity is formulated by Eringen [1] and stems from the non-linear theory of micro-elastic solids which was formulated by Eringen and Suhubi [2].

The problem of radial vibrations of isotropic elastic sphere and hollow sphere are discussed by Ghosh [3], Love's [5] treatise contains the forced vibrations of a sphere due to body forces derivable from a potential. Love [6] considered the sphere problem in connection with the problems of geodynamics. Grey and Eringen [4] obtained the complete solution of sphere subject to dynamic surface tractions and computed the natural frequencies of the free oscillations. T. Sree Lakshmi and K.Sambaiah [7] obtained the frequency equations for radial vibrations in micro polar elastic hollow sphere.

In this paper, we discussed the radial vibrations in micro polar thin spherical shell and obtained the frequency equations. It is interesting to observe that an additional frequency equation is obtained which is not encountered in the classical elasticity. Further the result of classical case is obtained as a particular case of it. The equations are reduced to non-dimensional forms and graphs are drawn by assuming certain values of non-dimensional quantities.

BASIC EQUATIONS

The fundamental equations for the motion of a micropolar elastic solid are given by the following.

(i) The balance of momentum equation

$$(\lambda + \mu)u_{l,lk} + (\mu + k)u_{k,ll} + k \in_{klm} \phi_{m,l} + \rho(f_K - \ddot{u}_k) = 0 \qquad (1)$$

(ii) The balance of the stress moment equation

$$(\alpha + \beta)\phi_{l,lk} + \gamma\phi_{k,ll} + k\in_{klm} u_{m,l} - 2k\,\phi_k + \rho(l_k - j\ddot{\phi}_k) = 0 \qquad (2)$$

In the above equations, $\bar{u}$ is displacement vector, $\bar{f}$ is the body force, $\bar{l}$ is the body couple vector, ρ is the density, j is the micro – inertia, an index (say k) following a comma indicates differentiation with respect to the coordinate (X_K), dot superposed on a symbol denotes differentiation with respect to the time t and λ, μ, k, α, β, γ are the material coefficients which satisfy the following inequalities.

$$3\lambda + 2\mu + k \geq 0\,,\quad 2\mu + k \geq 0\,,\quad k \geq 0$$

$$3\alpha + \beta + \gamma \geq 0\,,\quad -\gamma \leq \beta \leq \gamma\,,\quad \gamma \geq 0 \qquad (3)$$

The stress tensor t_{kl} and couple stress tensor m_{kl} are given by

$$t_{kl} = \lambda\, u_{r,r}\,\delta_{kl} + \mu\,(u_{k,l} + u_{l,k}) + k(u_{l,k} - \in_{klr}\phi_r) \qquad (4)$$

$$m_{kl} = \lambda\phi_{r,r}\,\delta_{kl} + \beta\phi_{k,l} + \gamma\phi_{l,k} \qquad (5)$$

where δ_{kl} is the kronecker delta and $\in_{klm}$ is the permutation symbol.

FORMULATION AND SOLUTION OF THE PROBLEM

The frequency equations of radial vibrations in micropolar elastic hollow sphere are given by [7]

$$\frac{(h^2a^2 - s)\tanh a + sha}{(h^2a^2 - s) - has\tanh a} = \frac{(h^2b^2 - s)\tanh b + shb}{(h^2b^2 - s) - hbs\tanh b} \qquad (6)$$

and

$$\frac{(h_1{}^2a^2 - s_1)\tanh_1 a + s_1h_1a}{(h_1{}^2a^2 - s_1) - h_1as_1\tanh_1 a} = \frac{(h_1{}^2b^2 - s_1)\tanh_1 b + s_1h_1b}{(h_1{}^2b^2 - s_1) - h_1bs_1\tanh_1 b} \qquad (7)$$

where $h^2 = \dfrac{p^2\rho}{\lambda + 2\mu + k}$, $h_1{}^2 = \dfrac{p^2\rho j}{\alpha+\beta+\gamma}$,

$s = \dfrac{4\mu + 2k}{\lambda + 2\mu + k}$, $s_1 = \dfrac{2(\beta+\alpha)}{\alpha+\beta+\gamma}$,

p is frequency, a and b are the radii of inner and outer sphere of hollow sphere.

Now, we suppose the shell is bounded by sphere of radius a and a + da, where da is small, then the required frequency equation is

$$f(a) = f(a + da) \tag{8}$$

where $f(a) = \dfrac{(h^2a^2 - s)\tanh a + has}{(h^2a^2 - s) - has\tanh a}$

Using the Taylor's series expansion and neglecting the second and higher order terms of da then the equation (8) reduces to $\dfrac{\partial}{\partial a}(f(a)) = 0$ (9)

Introducing ha = l we have

$$\frac{\partial}{\partial l}\left[\frac{(l^2 - s^2)\tan l + ls}{(l^2 - s) - ls\tan l}\right] = 0 \tag{10}$$

Simplifying the equation (10) we get

$$l^2 = 3s - s^2 \tag{11}$$

This is the frequency equation in thin spherical shell. Substituting the values of l, s and h in the equation (11) we get the frequency equation as

$$p^2 = \frac{4\mu + 2k}{\rho a^2}\left(\frac{3\lambda + 2\mu + k}{\lambda + 2\mu + k}\right) \tag{12}$$

Allowing k $\to 0$, the classical result [3] can be obtained. Similarly, corresponds to micro – rotation we have another frequency equation.

$$l_1{}^2 = 3s_1 - s_1{}^2 \tag{13}$$

where $l_1 = h_1 a$

Now substituting the values of l_1, h_1 and s_1 in the equation (13) we get the frequency equation as

$$p^2 = \frac{2(\beta+\alpha)}{\rho j a^2(\alpha+\beta+\gamma)} \tag{14}$$

The additional frequency (14) is not encountered in classical elasticity and it corresponds to micro rotation.

The equation (12) reduced to non dimensional form

$$p^2 = \frac{(4 + 2m_1)m_3(3m_2 + 2 + m_1)}{a^2(m_2 + 2 + m_1)} \tag{15}$$

where $m_1 = \frac{k}{\mu}$, $m_2 = \frac{\lambda}{\mu}$ and $m_3 = \frac{\mu}{\rho}$

and the equation (14) reduced to

$$p^2 = \frac{2(m_4+1)(1+m4+3m5)}{a^2 \rho j (1+m_4+m_5)} \tag{16}$$

where $m_4 = \frac{\beta}{\alpha}$, $m_5 = \frac{\gamma}{\alpha}$

NUMERICAL CALCULATIONS

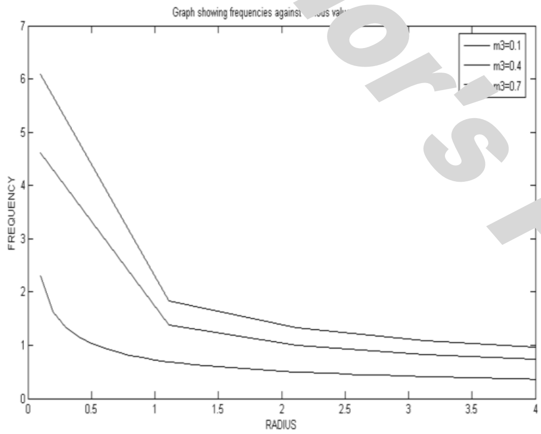

Fig. 1

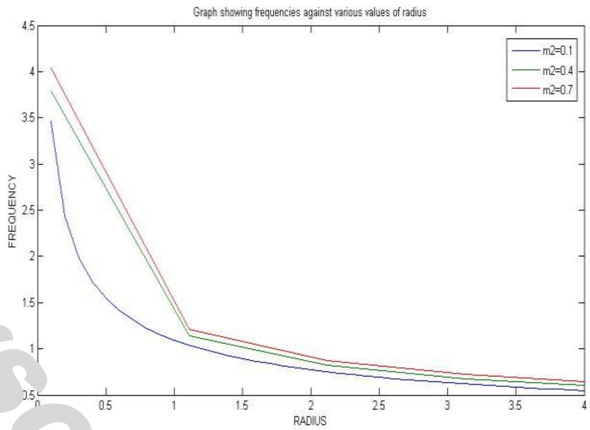

Fig. 2

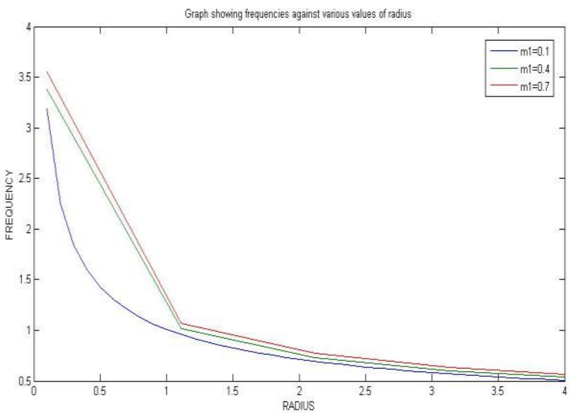

Fig. 3

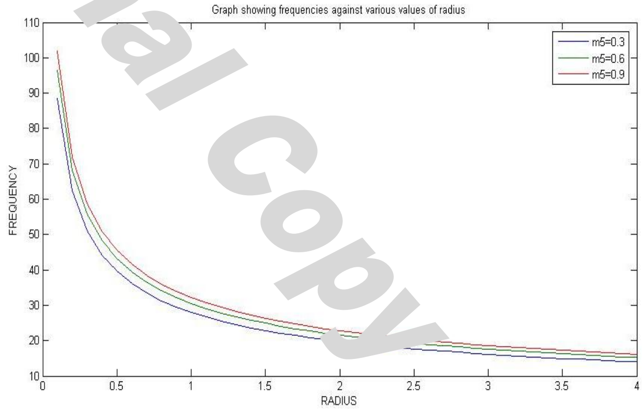

Fig. 4

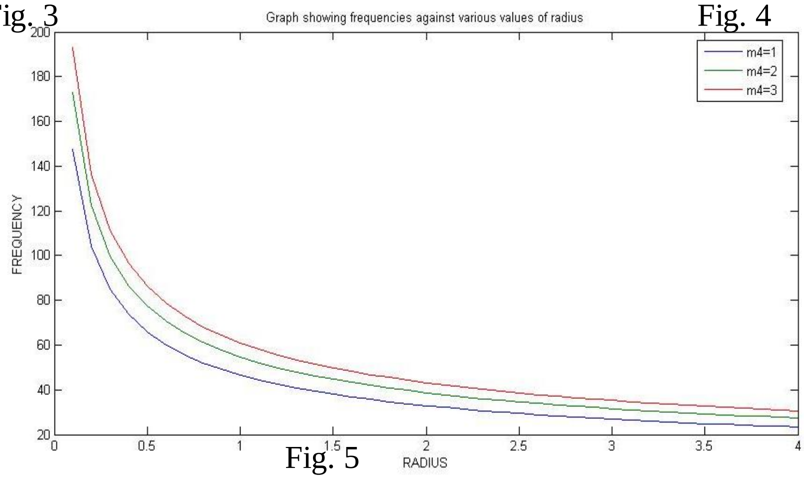

Fig. 5

The figures 1, 2 and 3 are the graphs drawn for the frequency (15) for various values of radii for the following cases respectively.

(i) $m_1 = 0.2$, $m_2 = 0.25$ and $m_3 = 0.1, 0.4, 0.7$

(ii) $m_1 = 0.2$, $m_3 = 0.25$ and $m_2 = 0.1, 0.4, 0.7$

(iii) $m_2 = 0.25$, $m_3 = 0.2$ and $m_1 = 0.1, 0.4, 0.7$

It is observed that the curves corresponding to (i) $m_1 = 0.2$, $m_2 = 0.25$, $m_3 = 0.1$ (ii) $m_1 = 0.2$, $m_2 = 0.1$, $m_3 = 0.25$ (iii) $m_1 = 0.1$, $m_2 = 0.25$, $m_3 = 0.2$ are parabolic shape. In other cases, the curves are parabolic when radius is greater than 1.2 and almost straight line for radius less than 1.2.

The figures 4 and 5 are the graphs drawn for the frequency (16) for various values of radii for the following cases.

(i) $m_4 = 0.2$ and $m_5 = 0.3, 0.6, 0.9$

(ii) $m_5 = 4$ and $m_4 = 1, 2, 3$

It is observed that the curves are parabolic in nature and the frequency decreases when radius increases.

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