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A Study of Inventory Model under Weibull Amelioration and Constant Demand

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ABSTRACT

This paper is concerned with the development of an inventory model for both ameliorating and deteriorating items. Generally it can be seen with the items such as broiler, duck, pig etc. when these items are kept in the farm or in the sales counter, they will increase in value due to their growth and decrease in value due to feeding expenses and /or diseases. Here in this paper an inventory model for above types of items with constant demand rate, without shortage. Also the model is studied for minimization of total average cost if some extra inventory is added into or removed out of the lot. This paper investigates the optimal time for addition or removal of inventories so that the average cost will be minimum. The result is illustrated with numerical example.

Keywords: Amelioration, deterioration, inventory system, optimal control, optimal time.

AMS Classification No: 90B05

1.0. Introduction :

In daily life, the deterioration of goods is a common phenomenon. Deterioration is defined as decay or damage such that the item cannot be used for its original purpose. One of the most important problems faced in inventory management is how to regulate and maintain the inventories of deteriorating items. The effect of deterioration of physical goods can't be neglected in any inventory system because almost all the physical goods deteriorate over time. Food items, photo graphic films, drugs, pharmaceutical, chemicals, electronic components and radioactive substances are some examples of items in which sufficient deterioration may occur during the normal storage period of the units and consequently this loss must be taken into

account while analyzing the inventory system [1]. A number of mathematical models have been presented for these deteriorating items. To get an idea of the trends of recent research in this area, one may refer to the literature [2, 7, 8, and 12].

In the existing literature, practitioners did not give much attention for fast growing animals like broiler, ducks, pigs etc. in the poultry farm, highbred fishes in bhery (pond) which are known as ameliorating items. When these items are in storage, the stock increases (in weight) due to growth of the items and also decrease due to death, various diseases or some other factors.

The first reference to inventory systems with both ameliorating and deteriorating items seems to be that of Hwang [3]. He assumes that items ameliorate while at a breeding yard, such as fish culture facility, and deteriorate when in the distribution centre. Hwang [4] extends his earlier results by developing three models; the economic order quantity (EOQ), the partial selling quantity (PSQ), and the economics production quantity (EPQ). These models are particularly well suited for items that ameliorate and deteriorate at the same time. Fast growing animals such as fish, chickens, ducks, and rabbits are such examples. While in the farm, they will increase in value due to their growth and once grown, they are used to produce food. For this reason, the inventory of such items grows faster in a first period and then starts to decline. Mondal et al.[9] consider the case where the demand rate depends on the price of the item. Hwang [5] studies a set-covering location problem and determines the number of storage facilities. Moon et al.[10, 11] generalize the EOQ model with ameliorating and deteriorating items by allowing shortages and by taking into account the effects of inflation and time value of money. Law and Wee [6] study the EPQ model with ameliorating and deteriorating items by allowing shortages, by taking into account time discounting and by incorporating the manufacturer-retail cooperation. Tadj et al. [13] have considered a production inventory model for both ameliorating and deteriorating items.

In the present paper, an economic order quantity model is developed for both ameliorating and deteriorating items. The model is formed by taking a constant initial amount of inventory. The deterioration rate is assumed to be constant whereas the rate of amelioration is considered to be two parameter weibull distributions. In the process some extra amount of inventories may be added into the existing lot or may be taken out for some reason. The aim is to find the optimal time for adding or removing the inventories so that the total average cost will be minimum. Both the above cases of adding and removing items are illustrated in the formulation of model. The rest of the paper is organized as follows. In the next section, the assumptions and notations related to this study are presented. In Section 3, the formulation of the model is given. In the last two sections, numerical examples are discussed to illustrate the procedure of solving the model and conclusions are provided.

2.0. Assumptions and Notations :

Following assumptions are made for the proposal model:

- i. Demand rate is constant with respect to time.
- ii. Single inventory will be used.
- iii. Lead time is zero.
- iv. Shortages are not allowed.

- v. Replenishment rate is infinite but size is finite.
- vi. Time horizon is finite.
- vii. There is no repair of deteriorated items occurring during the cycle.
- viii. Amelioration and deterioration occur when the item is effectively in stock.
- ix. The time-value of money and inflation are considered.

Following notations are made for the given model:

$I(t)$ = On hand inventory at time t .

$R(t)$ = Constant demand rate.

θ = The constant deterioration rate where $0 \leq \theta \leq 1$

$I(0)$ = Inventory at time $t = 0$

T = Duration of a cycle.

$A(t) = \alpha\beta t^{\beta-1}$ = Amelioration rate. Here $\beta > 0, 0 \leq \alpha < 1$ where α is scale parameter and β is shape parameter

c_a = Cost of amelioration per unit.

c_p = The purchasing cost per unit item.

c_d = The deterioration cost per unit item.

c_h = The holding cost per unit item.

CT = The total average cost of the system.

3.0. Formulation

The model is studied for addition and removal of inventories to optimize the cost in the following cases.

Case – 1:

The objective of the model is to determine the optimal time for addition of extra inventory in order to keep the total relevant cost as low as possible. It is assumed that the extra inventory is added at time $t = t_1$.

If $I(t)$ be the on hand inventory at time $t \geq 0$, then at time $t + \Delta t$, the on hand inventory in the interval $[0, t_1]$ will be

$$I(t + \Delta t) = I(t) + \alpha\beta t^{\beta-1} I(t) \Delta t - \theta I(t) \Delta t - R \Delta t$$

Dividing by Δt and then taking as $\Delta t \rightarrow 0$ we get

$$\frac{dI}{dt} + \theta I = \alpha\beta t^{\beta-1} I - R, 0 \leq t \leq t_1 \quad (3.1)$$

with conditions $I(0) = I_0, I(t_1) = I_1$.

After $t = t_1$, we have following differential equation.

$$\frac{dI}{dt} + \theta I = \alpha \beta t^{\beta-1} I - R, t_1 \leq t \leq T \quad (3.2)$$

with conditions $I(t_1) = I_{t_1} + I_0, I(T) = 0$

Now on solving equation (3.1)

$$\frac{dI}{dt} + (\theta - \alpha \beta t^{\beta-1}) I = -R$$

we have

$$I = -R \left(t - \frac{\theta t^2}{2} \right) + K \left(1 + \alpha t^\beta - \theta t + \frac{\theta^2 t^2}{2} \right) \quad (3.3)$$

Using $I(0) = I_0,$

$$k = I_0$$

From equation (3.3), the solution of equation (3.1) is

$$I(t) = -R \left(t - \frac{\theta t^2}{2} \right) + I_0 \left(1 + \alpha t^\beta - \theta t + \frac{\theta^2 t^2}{2} \right) \quad 0 \leq t \leq t_1 \quad (3.4)$$

But $I(t_1) = -R \left(t_1 - \frac{\theta t_1^2}{2} \right) + I_0 \left(1 + \alpha t_1^\beta - \theta t_1 + \frac{\theta^2 t_1^2}{2} \right)$

i.e. $I_{t_1} = -R \left(t_1 - \frac{\theta t_1^2}{2} \right) + I_0 \left(1 + \alpha t_1^\beta - \theta t_1 + \frac{\theta^2 t_1^2}{2} \right)$ in $0 \leq t \leq t_1$ (3.5)

Again using $I(T) = 0,$ in equation (3.3), we have

$$k = R \left(T + \frac{\theta T^2}{2} \right)$$

The solution in the interval $t_1 \leq t \leq T$ is

$$I(t) = -R \left(t - \frac{\theta t^2}{2} \right) + R \left(T + \frac{\theta T^2}{2} \right) \left(1 + \alpha t^\beta - \theta t + \frac{\theta^2 t^2}{2} \right) \quad (3.6)$$

But $I(t_1) = -R \left(t_1 - \frac{\theta t_1^2}{2} \right) + R \left(T + \frac{\theta T^2}{2} \right) \left(1 + \alpha t_1^\beta - \theta t_1 + \frac{\theta^2 t_1^2}{2} \right)$

We have $I(t_1) = I_{t_1} + I_0$ in $t_1 \leq t \leq T .$

i.e.

$$I_{t_1} + I_0 = -R\left(t_1 - \frac{\theta t_1^2}{2}\right) + R\left(T + \frac{\theta T^2}{2}\right)\left(1 + \alpha t_1^\beta - \theta t_1 + \frac{\theta^2 t_1^2}{2}\right) \quad (3.7)$$

Now from equation (3.5) and (3.7)

$$\begin{aligned} & -R\left(t_1 - \frac{\theta t_1^2}{2}\right) + I_0\left(1 + \alpha t_1^\beta - \theta t_1 + \frac{\theta^2 t_1^2}{2}\right) + I_0 = \\ & -R\left(t_1 - \frac{\theta t_1^2}{2}\right) + R\left(T + \frac{\theta T^2}{2}\right)\left(1 + \alpha t_1^\beta - \theta t_1 + \frac{\theta^2 t_1^2}{2}\right) \end{aligned}$$

i.e.

$$T + \frac{\theta T^2}{2} = \frac{I_0}{R} + \frac{I_0}{R}\left(1 + \alpha t_1^\beta - \theta t_1 + \frac{\theta^2 t_1^2}{2}\right)^{-1}$$

$$T = \frac{-1 \pm \sqrt{1 + \frac{2\theta I_0}{R}\left\{1 + \left(1 + \alpha t_1^\beta - \theta t_1 + \frac{\theta^2 t_1^2}{2}\right)^{-1}\right\}}}{\theta} \quad (3.8)$$

The total inventory holding during the time interval $[0, T]$ is given by

$$\begin{aligned} I_T &= \int_0^{t_1} I dt + \int_{t_1}^T I dt \\ &= \int_0^{t_1} -R\left(t - \frac{\theta t^2}{2}\right) + I_0\left(1 + \alpha t^\beta - \theta t + \frac{\theta^2 t^2}{2}\right) dt \\ &+ \int_{t_1}^T -R\left(t - \frac{\theta t^2}{2}\right) + R\left(T + \frac{\theta T^2}{2}\right)\left(1 + \alpha t^\beta - \theta t + \frac{\theta^2 t^2}{2}\right) dt \\ &= I_0\left(t_1 - \frac{\theta t_1^2}{2}\right) - \frac{RT^2}{2} + R\left(T + \frac{\theta T^2}{2}\right)\left(T - t_1 - \frac{\theta T^2}{2} + \frac{\theta t_1^2}{2}\right) \end{aligned} \quad (3.9)$$

The total number of deteriorated units in $[0, T]$ is given by

$$D_T = \theta \int_0^T I dt = \theta I_T$$

i.e. $D_T = \theta I_T$ (3.10)

The total number of ameliorated units over the cycle is given by

$$A_T = \int_0^T \alpha \beta t^{\beta-1} I dt \quad (3.11)$$

i.e.

The cost function consists of following elements :

(i) Purchasing cost per cycle

$$2C_p I_0 T$$

(ii) Holding cost per cycle

$$C_h \int_0^T I dt = C_h I_T$$

(iii) Deterioration cost per cycle

$$C_d \int_0^T \theta I(t) dt = C_d \theta I_T$$

(iv) Amelioration cost per cycle

$$\begin{aligned} & C_a \int_0^T \alpha \beta t^{\beta-1} I(t) dt \\ &= C_a \alpha \beta \left[-R \left\{ \frac{t_1^{\beta+1}}{\beta+1} - \frac{\theta t_1^{\beta+2}}{2(\beta+2)} \right\} + I_0 \left\{ \frac{t_1^\beta}{\beta} + \frac{\alpha t_1^{2\beta}}{2\beta} - \frac{\theta t_1^{\beta+1}}{\beta+1} + \frac{\theta t_1^{\beta+2}}{2(\beta+2)} \right\} \right. \\ & - R \left\{ \frac{T^{\beta+1}}{\beta+1} - \frac{\theta T^{\beta+2}}{2(\beta+2)} - \frac{t_1^{\beta+1}}{\beta+1} + \frac{\theta t_1^{\beta+2}}{2(\beta+2)} \right\} \\ & \left. + R \left(T + \frac{T^2}{2} \right) \left\{ \frac{T^\beta}{\beta} + \frac{\alpha T^{2\beta}}{2\beta} - \frac{\theta T^{\beta+1}}{\beta+1} + \frac{\theta^2 T^{\beta+2}}{2(\beta+2)} - \frac{t_1^\beta}{\beta} - \frac{\alpha t_1^{2\beta}}{2\beta} + \frac{\theta t_1^{\beta+1}}{\beta+1} - \frac{\theta^2 t_1^{\beta+2}}{2(\beta+2)} \right\} \right] \end{aligned}$$

Taking the relevant costs mentioned above, the average total cost per unit time is given by

Average total cost = CT

$$= \frac{1}{T} [\text{Purchasing cost} + \text{Holding cost} + \text{Deterioration cost} + \text{Amelioration cost}]$$

$$= \frac{1}{T} \left[2C_p I_0 T + (C_h + C_d \theta) I_T + C_a \alpha \beta \int_0^T t^{\beta-1} I(t) dt \right]$$

$$\begin{aligned}
\text{i.e. } CT(T, t_1) &= \frac{1}{T} \left[2 C_p I_0 T + (C_h + C_d \theta) \right. \\
&= \frac{1}{T} \left[2 C_p I_0 T + (C_h + C_d \theta) \left\{ I_0 \left(t_1 - \frac{\theta t_1^2}{2} \right) - \frac{RT^2}{2} + R \left(T + \frac{\theta T^2}{2} \right) \left(T - t_1 - \frac{\theta T^2}{2} + \frac{\theta t_1^2}{2} \right) \right\} \right. \\
&+ C_a \alpha \beta \left[-R \left\{ \frac{t_1^{\beta+1}}{\beta+1} - \frac{\theta t_1^{\beta+2}}{2(\beta+2)} \right\} + I_0 \left\{ \frac{t_1^\beta}{\beta} + \frac{\alpha t_1^{2\beta}}{2\beta} - \frac{\theta t_1^{\beta+1}}{\beta+1} + \frac{\theta t_1^{\beta+2}}{2(\beta+2)} \right\} \right. \\
&- R \left\{ \frac{T^{\beta+1}}{\beta+1} - \frac{\theta T^{\beta+2}}{2(\beta+2)} - \frac{t_1^{\beta+1}}{\beta+1} + \frac{\theta t_1^{\beta+2}}{2(\beta+2)} \right\} \\
&\left. \left. + R \left(T + \frac{T^2}{2} \right) \left\{ \frac{T^\beta}{\beta} + \frac{\alpha T^{2\beta}}{2\beta} - \frac{\theta T^{\beta+1}}{\beta+1} + \frac{\theta^2 T^{\beta+2}}{2(\beta+2)} - \frac{t_1^\beta}{\beta} - \frac{\alpha t_1^{2\beta}}{2\beta} + \frac{\theta t_1^{\beta+1}}{\beta+1} - \frac{\theta^2 t_1^{\beta+2}}{2(\beta+2)} \right\} \right] \right] \quad (3.12)
\end{aligned}$$

After eliminating T using (3.8) from (3.12) as it is difficult to solve the problem by deriving a closed equation of the solution Matlab software has been used to determine optimal t_1^* and hence the optimal $I(t_1)$ with optimal time T .

Case: 2

Suppose some constant amount of inventories is to be taken out from the existing lot. The problem is to calculate the time at which the constant amount will be taken out, so that the average total cost will be minimum.

Suppose $\frac{I_0}{2}$ amount of inventory will be taken out at some time t_1 . Now in the interval

$t_1 \leq t \leq T$ the condition will be $I(t_1) = I_t - \frac{I_0}{2}$.

$$\text{i.e. } I_t - \frac{I_0}{2} = -R \left(t_1 - \frac{\theta t_1^2}{2} \right) + R \left(T + \frac{\theta T^2}{2} \right) \left(1 + \alpha t_1^\beta - \theta t_1 + \frac{\theta^2 t_1^2}{2} \right) \quad (3.13)$$

From equation (3.5) and (3.7)

$$\begin{aligned}
&-R \left(t_1 - \frac{\theta t_1^2}{2} \right) + I_0 \left(1 + \alpha t_1^\beta - \theta t_1 + \frac{\theta^2 t_1^2}{2} \right) - \frac{I_0}{2} = -R \left(t_1 - \frac{\theta t_1^2}{2} \right) \\
&+ R \left(T + \frac{\theta T^2}{2} \right) \left(1 + \alpha t_1^\beta - \theta t_1 + \frac{\theta^2 t_1^2}{2} \right) \\
\text{i.e. } T + \frac{\theta T^2}{2} &= \frac{I_0}{R} - \frac{I_0}{2R} \left(1 + \alpha t_1^\beta - \theta t_1 + \frac{\theta^2 t_1^2}{2} \right)^{-1}
\end{aligned}$$

$$T = \frac{-1 \pm \sqrt{1 + \frac{\theta I_0}{R} \left(2 - \left(1 + \alpha t_1^\beta - \theta t_1 + \frac{\theta^2 t_1^2}{2} \right)^{-1} \right)}}{\theta} \quad (3.14)$$

Again after eliminating T using (3.14) from (3.12) as it is difficult to solve the problem by deriving a closed equation of the solution Matlab software has been used to determine optimal t_1^* and hence the optimal $I(t_1)$ with optimal time T .

4.0. Numerical Examples:

Example 1

Case 1

The values of the parameters are considered as follows:

$$\theta = 0.05, A = 0.075, R = 15, I_0 = 100 \text{ Units}, \alpha = 0.001, \beta = 2,$$

$$c_a = \$5/\text{unit}, c_h = \$3/\text{unit}/\text{year}, c_p = \$5/\text{unit}, c_d = \$8/\text{unit}$$

Using equation (3.8) in (3.12) and on solving we obtain the optimal $t_1^* = 2.4851$ years. The optimal inventory cost, $CT(t_1) = 647.4531$ \$, optimal time $T^* = 13.9025$ years.

Case 2

The values of the parameters are considered as follows:

$$\theta = 0.05, A = 0.075, R = 15, I_0 = 100 \text{ Units}, \alpha = 0.001, \beta = 2,$$

$$c_a = \$5/\text{unit}, c_h = \$3/\text{unit}/\text{year}, c_p = \$5/\text{unit}, c_d = \$8/\text{unit}$$

Using equation (3.14) in (3.12) and on solving we obtain the optimal $t_1^* = .000067$ years. The optimal inventory cost, $CT(t_1) = 542.073$ \$, optimal time $T^* = 5.5605$ years.

5.0. Conclusion:

Here we have derived an inventory model for some special types of items where both amelioration and deterioration rates are assumed to be constant. In this model shortages are not allowed and it is studied for minimization of total average cost if some extra inventory is added or removed out from existing lot where it is required to find the optimal time for addition or removal of inventories. The cost parameters are determined for constant demand rate. Numerical examples are used to illustrate the result.

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