ISSN 0976 – 1411

Available online at www.internationaleJournals.com

International eJournals

INTERNATIONAL eJOURNAL OF MATHEMATICS AND ENGINEERING

www.InternationaleJournals.com

International eJournal of Mathematics and Engineering 19 (2010) 191 - 202

DIFFUSION OF INTERACTING MARINE SPECIES IN OCEAN

V.P. Saxena[1], Neeta Mazumdar[2]

[1]V. P. Saxena, Director, Sagar Institute of Research, Technology and Science, Bhopal-462041, India. E-mail: vinodpsaxena@gmail.com

[2]Ms.Neeta Mazumdar, Rosary College of Commerce and Arts, Navelim, Salcette, Goa, India. E-mail: neetam53@rediffmail.com

---

Abstract:
In this paper, the model deals with competition in populations which diffuse in a circular bounded area. Migration of interacting marine animal species in ocean is considered. Pseudo Analytic Finite Partition Method (PAFPM) has been employed to find out the approximate solution of the dispersion problem in the non-homogeneous region. A two dimensional circular region is considered. For angular direction the Fourier series has been used assuming angular uniformity in each part. After numerical verification graphs are plotted between the angle θ and population density of species for constant time.

Key words: *Discretisation, diffusion, Euler -Lagrange's equation, intrinsic growth, Fourier series, parabolic variation, perturbation, Pseudo Analytic Finite Partition Method, Ritz Finite Element Method, variational form*

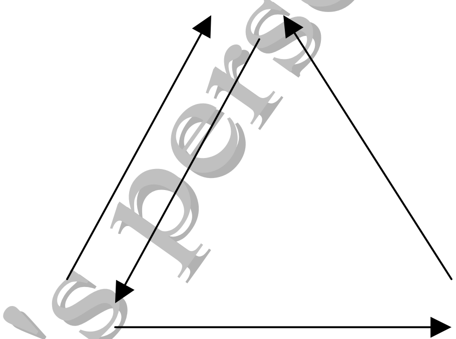
AMS Mathematics subject classification: 92D25,35K55

Introduction: Migration of interacting marine animal species in ocean is considered in this paper. This situation may occur in rivers or oceans areas of arbitrary geometrical shapes. Pseudo Analytic Finite Partition Method (PAFPM) has been employed to find out the approximate solution of the dispersion problem in the non-homogeneous region. A two dimensional circular region is considered. In this region, the environmental properties vary in radial direction and therefore, it is partitioned into annular sub regions to apply Ritz Finite Element Method. For angular direction the Fourier series has been used assuming angular uniformity in each part.

In its normal use, the word migration or diffusion means to move from one place to another. When applied to animals it has a special meaning: a migration is a coming and going with the seasons, with a 'once-a-year' implication, and the most obvious examples are provided by birds. Here there are many types of migratory movement and Landsborough Thompson (1942) groups those under three headings:

1. Local and seasonal movements
2. Dispersals
3. True migrations

The local seasonal movements are merely change of ground at a particular time of the year. Some of these movements maybe very small, others larger while still confines within one geographical area. An example of the British Isles is the autumn migration of the Starlings to the south and west, and their return to the north and east in the following spring. The second group of movements is more extensive and classed as dispersals or wanderings. Only the breeding area is well defined and the movement is, ideally, an even and outward spread from this



centre.

In particular, the pattern of fish migration is thought to confirm to that shown in figure 1. The young stages leave the spawning grounds at A for the nursery grounds at B; from the nursery grounds the juveniles recruit to the adult stock on their feeding grounds at C; and the mature and ripening fish move from the feeding grounds back to the spawning grounds at A. Then as spents, they return to the feeding grounds. The migration pattern is believed to be related to that of currents. The young stages drift with the current to the nursery ground; the spawning migrate from B to C against the current, and the spents return to the feeding ground with the current.

Meek (1915) introduced the terms denatant and contranatant movements to describe movements of fish in relation to water current. Denatant means swimming or drifting or migration with the current, contranatant means swimming against the current.

There are several different types of currents in the oceans, leaving for the moment tidal currents, the remainder maybe divided into drift and gradient currents. Drift currents are produced by the winds. Gradient currents are produced by gradients of pressure on the surrounding masses of water. The surface circulation in the main ocean basins is wind-driven. Munk (1955) has given a clear and simple account of the relation between wind and water movement. Below the surface currents which may persist down to 400m or more, there are deeper currents. These are slower than those at the surface and may move as counter-currents, in the opposite direction. Such counter currents are present below the Baltic outflow, below the North Atlantic current along the line of Norwegian shelf, below the axis of the Florida current on the western boundaries of the atlantic; and, as the equatorial currents in the pacific and Atlantic oceans.

The really spectacular migrations of mature adult fish are for spawning and for feeding. They breeding one area, but grow up and feed in another .examples are provided by the cod of the Arcto-Norwegian stock and the European eel. In summer months the Arcto-Norwegian cod feed in area roughly bounded by spitsbergen, Bear Island,Novaya Zemlya, and the Murman coast. In October and November the older fish start to move southwards after the arctic night has set in and four months later, in February and March, most of the mature and ripe fish arrive on their main spawning ground at the Lofoten Islands, within the West Fjord,700 miles or more distant. In the spring and early summer the spent and larval fish return to the feeding ground in the north.

The European eel is believed to spawn somewhere between the Bermudas and the Bahamas in an area corresponding more or less with the Sargasso sea, where the water is warm and of relatively high salinity. The larval eels, known as leptocephali, are carried towards the European coastline where as elvers, now about two and half years old, they enter rivers. Ten or more years later, as fully grown eels, they leave freshwater and are thought to make the return journey of over 3,000 miles back to their birthplace, to spawn and to die.

Spectacular feeding and spawning migrations are made by place and herring. Then there is the example of the salmon which spawns in freshwater but spends most of its adult life in the sea. Salmon are believed to return to freshwater to spawn in the stream in which they grew up.

The model deals with competition in populations which diffuse in a circular bounded area.
Here we take a competition model, in which two competitive population species diffuse between circular patches in a given area. The system of non-linear partial differential equations for the above case is

$$\frac{\partial N_{1i}}{\partial t} = N_{1i}\left(a_{1i} - b_{1i}N_{1i} - c_{1i}N_{2i}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(rD_{r1i}\frac{\partial N_{1i}}{\partial r}\right)$$

$$+\frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{D_{\theta 1i}}{r}\frac{\partial N_{1i}}{\partial \theta}\right)$$

$$\frac{\partial N_{2i}}{\partial t} = N_{2i}\left(a_{2i} - b_{2i}N_{2i} - c_{2i}N_{1i}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(rD_{r2i}\frac{\partial N_{2i}}{\partial r}\right)$$

$$+\frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{D_{\theta 2i}}{r}\frac{\partial N_{2i}}{\partial \theta}\right) \qquad (1)$$

Here

N_{1i} =Density of first species population

N_{2i} = Density of second species population

a_{1i} =Intrinsic birth rate of first species population

b_{1i}, b_{2i} =Intraspecific interaction coefficients

a_{2i} = Intrinsic birth rate of second species population

c_{1i}, c_{2i} =Interspecific interaction coefficients

$D_{1ri}, D_{1\theta i}, D_{2ri}, D_{2\theta i},$ =Diffusion coefficients

Each of the above quantity is pertaining to the i^{th} patch, (i=1,2)

Equilibrium points of the equations (1) are

E_1 (0, 0) and $E_2\left(\frac{a_{1i}c_{2i} - a_{2i}b_{1i}}{c_{1i}c_{2i} - b_{1i}b_{2i}}, \frac{a_{1i}b_{2i} - a_{2i}c_{1i}}{b_{1i}b_{2i} - c_{1i}c_{2i}}\right)$

We take small perturbations u_{1i} and u_{2i} near the non zero equilibrium point i.e.

$$N_{1i} = N_{1i}^* + u_{1i}$$
$$N_{2i} = N_{2i}^* + u_{2i} \qquad (2)$$

where

$$|u_{1i}| << 1, |u_{2i}| << 1, \text{ i=1,2}$$

Then the system of equations (1) becomes

$$\frac{\partial u_{1i}}{\partial t} = u_{1i}\left(a_{1i} - 2b_{1i}N_{1i}^* - c_{1i}N_{2i}^*\right) - c_{1i}N_{1i}^*u_{2i} + \frac{1}{r}\frac{\partial}{\partial r}\left(rD_{r1i}\frac{\partial N_{1i}}{\partial r}\right)$$

$$+\frac{1}{r}\frac{\partial}{\partial\theta}\left(\frac{D_{\theta 1i}}{r}\frac{\partial N_{1i}}{\partial\theta}\right)$$

$$\frac{\partial u_{2i}}{\partial t}=u_{2i}\left(a_{2i}-2b_{2i}N_{2i}^{*}-c_{2i}N_{1i}^{*}\right)-c_{2i}N_{2i}^{*}u_{1i}+\frac{1}{r}\frac{\partial}{\partial r}\left(rD_{r2i}\frac{\partial N_{2i}}{\partial r}\right)+\frac{1}{r}\frac{\partial}{\partial\theta}\left(\frac{D_{\theta 2i}}{r}\frac{\partial N_{2i}}{\partial\theta}\right) \quad (3)$$

The initial conditions are taken as

$$u_{1i}(r_0,\theta)=G_{1i}(\theta),$$
$$u_{2i}(r_0,\theta)=G_{2i}(\theta) \quad (4)$$

where G_{1i} and G_{2i} are suitable functions. This gives initial population distribution close to the innermost boundary.

Interface conditions at $r=r_1$ are assumed to be

$$D_{11}\left.\frac{\partial u_{11}}{\partial r}\right|_{r_1}=-D_{12}\left.\frac{\partial u_{12}}{\partial r}\right|_{r_2}$$
$$D_{21}\left.\frac{\partial u_{21}}{\partial r}\right|_{r_1}=-D_{22}\left.\frac{\partial u_{22}}{\partial r}\right|_{r_2} \quad (5)$$

where r_1 and r_2 denote region-I and region-II respectively. These conditions ensure no disappearance, no return and no stop of the animal species at the barriers.

Boundary conditions associated with the system of equations are,

$$\left.\frac{\partial u_{11}}{\partial r}\right|_{r=r_0}=\left.\frac{\partial u_{12}}{\partial r}\right|_{r=r_2}=0$$

$$\left.\frac{\partial u_{21}}{\partial r}\right|_{r=r_0}=\left.\frac{\partial u_{22}}{\partial r}\right|_{r=r_2}=0 \quad (6)$$

This implies that the innermost and outermost boundaries are sealed or prohibited for the animals to cross over.

Solution:

To solve this model, we apply the Finite Element Method in radial direction.
Comparing system of equations (3) with Euler -Lagrange's equation, we get

$$I_i = \int_{r_{i-1}}^{r_i} F_i dr \qquad (7)$$

where

$$F_i = A_i u_{1i}^2 r + B_i u_{2i}^2 r + C_i u_{1i} u_{2i} r + D_i r\left(u'_{r1i}\right)^2 + E_i r\left(u'_{r2i}\right)^2$$
$$-\frac{1}{2}\frac{\partial}{\partial\theta}\left(\frac{D_{\theta 1i}}{r}\frac{\partial}{\partial\theta}\right)u_{1i}^2 - \frac{1}{2}\frac{\partial}{\partial\theta}\left(\frac{D_{\theta 2i}}{r}\frac{\partial}{\partial\theta}\right)u_{2i}^2 \qquad (8)$$

where

$$u_{1ir} = \frac{\partial u_{1i}}{\partial r}, u_{2ir} = \frac{\partial u_{2i}}{\partial r}, \quad \text{i=1,2}$$

Here

$$A_i = \frac{1}{2}\left(\frac{\partial}{\partial t} - \left(a_{1i} - 2b_{1i}N_{1i}^* - c_{1i}N_{2i}^*\right)\right)$$
$$B_i = \frac{1}{2}\left(\frac{\partial}{\partial t} - \left(a_{2i} - 2b_{2i}N_{2i}^* - c_{2i}N_{1i}^*\right)\right)$$
$$C_i = c_{1i}N_{1i}^* + c_{2i}N_{2i}^* \qquad (9)$$
$$D_i = \frac{D_{r1i}}{2}, \quad E_i = \frac{D_{r2i}}{2}$$

We take linear shape function as

$$u_{1i} = A_{1i} + B_{1i}r, \quad r_0 \le r \le r_1$$
$$u_{2i} = A_{2i} + B_{2i}r, \quad r_1 \le r \le r_2, \text{ i=1,2} \qquad (10)$$
$$u_{1i} = u_{11(i-1)} \text{ at } r = r_{i-1}$$
$$u_{1i} = u_{11i} \text{ at } r = r_i \qquad (11)$$

$$u_{2i} = u_{22(i-1)} \text{ at } r = r_{i-1}$$
$$u_{2i} = u_{22i} \text{ at } r = r_i \qquad (12)$$

Using these values, we get

$$A_{1i} = \frac{u_{11(i-1)}r_i - u_{11i}r_{i-1}}{r_i - r_{i-1}}$$
$$B_{1i} = \frac{u_{11i} - u_{11(i-1)}}{r_i - r_{i-1}} \qquad (13)$$
$$A_{2i} = \frac{u_{22(i-1)}r_i - u_{22i}r_{i-1}}{r_i - r_{i-1}}$$

$$B_{2i} = \frac{u_{22i} - u_{22(i-1)}}{r_i - r_{i-1}} \qquad (14)$$

Substituting above values in equations (8), we get

$$I_i = A_i\left[A_{1i}^2\alpha_{1i} + B_{1i}^2\alpha_{2i} + 2A_{1i}B_{1i}\alpha_{3i}\right] + B_i\left[A_{2i}^2\alpha_{1i} + B_{2i}^2\alpha_{2i} + 2A_{2i}B_{2i}\alpha_{3i}\right]$$

$$+ C_i\left[A_{1i}A_{2i}\alpha_{1i} + \left(A_{1i}B_{2i} + A_{2i}B_{1i}\right)\alpha_{3i} + B_{1i}B_{2i}\alpha_{2i}\right] + D_iB_{1i}^2\alpha_{1i} + E_iB_{2i}^2\alpha_{1i}$$

$$-\frac{1}{2}\frac{\partial}{\partial\theta}\left(D_{\theta 1i}\frac{d}{d\theta}\right)\left(A_{1i}^2\alpha_{4i} + B_{1i}^2\alpha_{1i} + 2A_{1i}B_{1i}\alpha_{5i}\right)$$

$$-\frac{1}{2}\frac{\partial}{\partial\theta}\left(D_{\theta 2i}\frac{d}{d\theta}\right)\left(A_{2i}^2\alpha_{4i} + B_{2i}^2\alpha_{1i} + 2A_{2i}B_{2i}\alpha_{5i}\right)$$

(15)

where

$$I = I_1 + I_2 \qquad (16)$$

Now differentiating I with respect to u_{111} and u_{221} and putting

$$\frac{\partial I}{\partial u_{111}} = \frac{\partial I}{\partial u_{221}} = 0,$$

we get

$$\left(A_1\beta_{11} + D_1\beta_{12} + D_{1\theta 1}\frac{\partial^2}{\partial\theta^2}\beta_{13}\right)u_{110}$$

$$+ u_{111}\left(A_1\beta_{14} + D_1\beta_{15} + A_2\beta_{21} + D_2\beta_{22} + D_{1\theta 1}\frac{d^2}{d\theta^2}\beta_{16} + D_{1\theta 2}\frac{d^2}{d\theta^2}\beta_{23}\right)$$

$$+\left(A_2\beta_{24} + D_2\beta_{25} + D_{1\theta 2}\beta_{26}\frac{d^2}{d\theta^2}\right)u_{112} + u_{220}\left(C_1\beta_{17}\right)$$

$$+ u_{221}\left(C_1\beta_{18} + C_2\beta_{27}\right) + u_{222}\left(C_2\beta_{28}\right) = 0 \qquad (17)$$

$$u_{110}\left(C_1\beta_{17}\right) + u_{111}\left(C_1\beta_{18} + C_2\beta_{27}\right) + u_{112}\left(C_2\beta_{28}\right)$$

$$+\left(B_1\beta_{11} + E_1\beta_{12} + D_{2\theta 1}\frac{\partial^2}{\partial\theta^2}\beta_{13}\right)u_{220}$$

$$+\left(B_1\beta_{14}+E_1\beta_{15}+B_2\beta_{21}+E_2\beta_{22}+D_{2\theta 1}\frac{d^2}{d\theta^2}\beta_{16}+D_{2\theta 2}\frac{d^2}{d\theta^2}\beta_{23}\right)u_{221}$$

$$+\left(B_2\beta_{24}+E_2\beta_{25}+D_{2\theta 2}\beta_{26}\frac{d^2}{d\theta^2}\right)u_{222}=0 \qquad (18)$$

We can write these equations in the form

$$\left(P_{11}+\frac{d^2}{d\theta^2}Q_{11}\right)u_{110}+\left(P_{12}+\frac{d^2}{d\theta^2}Q_{12}\right)u_{111}+\left(P_{13}+\frac{d^2}{d\theta^2}Q_{13}\right)u_{112}$$

$$\left(P_{14}+\frac{d^2}{d\theta^2}Q_{14}\right)u_{220}+\left(P_{15}+\frac{d^2}{d\theta^2}Q_{16}\right)u_{221}+\left(P_{16}+\frac{d^2}{d\theta^2}Q_{16}\right)u_{222}=0$$

(19)

$$\left(P_{21}+\frac{d^2}{d\theta^2}Q_{21}\right)u_{110}+\left(P_{22}+\frac{d^2}{d\theta^2}Q_{22}\right)u_{111}+\left(P_{23}+\frac{d^2}{d\theta^2}Q_{23}\right)u_{112}$$

$$+\left(P_{24}+\frac{d^2}{d\theta^2}Q_{24}\right)u_{220}+\left(P_{25}+\frac{d^2}{d\theta^2}Q_{25}\right)u_{221}+\left(P_{26}+\frac{d^2}{d\theta^2}Q_{26}\right)u_{222}=0$$

(20)

To solve the above system of differential equations we use Fourier series

$$u_{110}=a_{10}+\sum\left(a_{1n}\cos n\theta+b_{1n}\sin n\theta\right)$$
$$u_{111}=a_{20}+\sum\left(a_{2n}\cos n\theta+b_{2n}\sin n\theta\right)$$
$$u_{112}=a_{30}+\sum\left(a_{3n}\cos n\theta+b_{3n}\sin n\theta\right)$$
$$u_{220}=a'_{10}+\sum\left(a'_{1n}\cos n\theta+b'_{1n}\sin n\theta\right)$$
$$u_{221}=a'_{20}+\sum\left(a'_{2n}\cos n\theta+b'_{2n}\sin n\theta\right)$$
$$u_{222}=a'_{30}+\sum\left(a'_{3n}\cos n\theta+b'_{3n}\sin n\theta\right)$$

(21)

where all the summations are taken from $n=1$ to $n=\infty$.
However, assuming fast convergence of the series on the right we terminate the series after three terms.
Substituting these values in equations (17) and (18) and comparing constant terms and coefficients of $\cos\theta$ and $\sin\theta$,we arrive at

$$P_{12}a_{20} + P_{15}a'_{20} = -\left[P_{11}a_{10} + P_{13}a_{30} + P_{14}a'_{10} + P_{16}a'_{30}\right] \quad (22)$$

$$P_{22}a_{20} + P_{25}a'_{20} = -\left[P_{21}a_{10} + P_{23}a_{30} + P_{24}a'_{10} + P_{26}a'_{30}\right] \quad (23)$$

$$\begin{aligned}&\left(P_{12} - n^2Q_{12}\right)a_{2n} + \left(P_{15} - n^2Q_{15}\right)a'_{2n}\\ &= -\left(P_{11} - n^2Q_{11}\right)a_{1n} - \left(P_{13} - n^2Q_{13}\right)a_{3n} - \left(P_{14} - n^2Q_{14}\right)a'_{1n}\\ &\quad -\left(P_{16} - n^2Q_{16}\right)a'_{3n}\end{aligned} \quad (24)$$

$$\begin{aligned}&\left(P_{22} - n^2Q_{22}\right)a_{2n} + \left(P_{25} - n^2Q_{25}\right)a'_{2n}\\ &= -\left(P_{21} - n^2Q_{21}\right)a_{1n} - \left(P_{23} - n^2Q_{23}\right)a_{3n} - \left(P_{24} - n^2Q_{24}\right)a'_{1n}\\ &\quad -\left(P_{26} - n^2Q_{26}\right)a'_{3n}\end{aligned} \quad (25)$$

$$\begin{aligned}&\left(P_{12} - n^2Q_{12}\right)b_{2n} + \left(P_{15} - n^2Q_{15}\right)b'_{2n}\\ &= -\left(P_{11} - n^2Q_{11}\right)b_{1n} - \left(P_{13} - n^2Q_{13}\right)b_{3n} - \left(P_{14} - n^2Q_{14}\right)b'_{1n}\\ &\quad -\left(P_{16} - n^2Q_{16}\right)b'_{3n}\end{aligned} \quad (26)$$

$$\begin{aligned}&\left(P_{22} - n^2Q_{22}\right)b_{2n} + \left(P_{25} - n^2Q_{25}\right)b'_{2n}\\ &= -\left(P_{21} - n^2Q_{21}\right)b_{1n} - \left(P_{23} - n^2Q_{23}\right)b_{3n} - \left(P_{24} - n^2Q_{24}\right)b'_{1n}\\ &\quad -\left(P_{26} - n^2Q_{26}\right)b'_{3n}\end{aligned} \quad (27)$$

Since on inner boundary

$$u_{11}(r,\theta) = G_{11}(\theta),$$

$$u_{21}(r,\theta) = G_{21}(\theta),$$

Parabolic variation of perturbation along the circular boundary has been taken in the form

$$G_{j1}(\theta) = A_j + B_j\theta + C_j\theta^2 \quad ,$$

$u_{j\alpha}$ and $u_{j\beta}$ are perturbation at $\theta = 0$ and $\theta = \pi$, j=1,2.

Then, we get

$$A_j = u_{j\alpha},\ B_j = 2\left(u_{j\beta} - u_{j\alpha}\right)/\pi,\ C_j = \left(u_{j\alpha} - u_{j\beta}\right)/\pi^2 \quad , \text{j=1,2}$$

By Fourier series, we get

$$a_{10} = \left(2u_{1\alpha} + 4u_{1\beta}\right)/3$$

$$a_{1n} = 4\left(u_{1\alpha} - u_{1\beta}\right)/n^2\pi^2$$

$$b_{1n} = 0,$$

and

$$a'_{10} = (2u_{2\alpha} + 4u_{2\beta})/3$$
$$a'_{1n} = 4(u_{2\alpha} - u_{2\beta})/n^2\pi^2$$
$$b'_{1n} = 0,$$

Now substituting these values and taking Laplace transform of equations (22) to (27), we get

$$(\gamma_{11}p + \gamma_{12})\overline{a}_{20} + \gamma_{13}\overline{a}'_{20} = \frac{1}{p}\gamma_{14} + \gamma_{11}a_{20}(0) \quad (28)$$

$$\gamma_{21}\overline{a}_{20} + (\gamma_{22}p + \gamma_{23})\overline{a}'_{20} = \frac{1}{p}\gamma_{24} + \gamma_{22}a'_{20}(0) \quad (29)$$

$$(\gamma_{31}p + \gamma_{32})\overline{a}_{2n} + \gamma_{33}\overline{a}'_{2n} = \frac{1}{p}\gamma_{34} + \gamma_{31}a_{2n}(0) \quad (30)$$

$$\gamma_{41}\overline{a}_{2n} + (\gamma_{42}p + \gamma_{43})\overline{a}'_{2n} = \frac{1}{p}\gamma_{43} + \gamma_{42}a'_{2n}(0) \quad (31)$$

$$(\gamma_{51}p + \gamma_{52})\overline{b}_{2n} + \gamma_{53}\overline{b}'_{2n} = \frac{1}{p}\gamma_{54} + \gamma_{51}b_{2n}(0) \quad (32)$$

$$\gamma_{61}\overline{b}_{2n} + (\gamma_{62}p + \gamma_{63})\overline{b}'_{2n} = \frac{1}{p}\gamma_{44} + \gamma_{62}b'_{2n}(0) \quad (33)$$

where all $\overline{a}'s$ and $\overline{b}'s$ are Laplace transforms of the corresponding $a's$ and $b's$ and $a_{20}(0), a'_{20}(0), a_{2n}(0), a'_{2n}(0)$, $b_{20}(0)$, $b'_{20}(0), b_{2n}(0), b'_{2n}(0)$ are initial values.

Solving these algebraic equation, we get the values of $\overline{a}_{20}, \overline{a}'_{20}, \overline{a}_{2n}$, $\overline{a}'_{2n}, \overline{b}_{2n}$ and $\overline{b}'_{2n}$, and taking inverse Laplace transform of these we can get the values of $a_{20}, a'_{20}, a_{2n}, a'_{2n}$, b_{2n} and b'_{2n}.

Substituting these values in equation (21), we can get the u_{111} and u_{221}.

Numerical Computation:

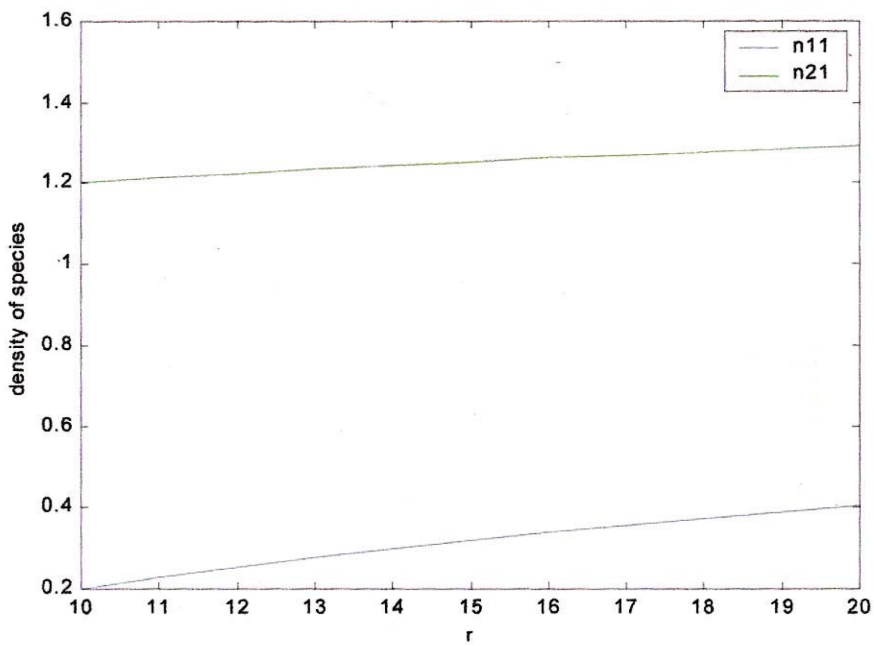
We make use of the following values of parameters and constants

$a_{11} = 5, a_{12} = 4, a_{21} = 3, a_{22} = 4, c_{11} = 5, c_{12} = 6,$
$b_{11} = 5, b_{12} = 3, b_{21} = 5, b_{22} = 6, c_{21} = 6, c_{22} = 5, d_{r11} = 2,$
$d_{r12} = 3, d_{r21} = .8, d_{r22} = .9, d_{t11} = 2,$
$d_{t12} = 1, d_{t21} = .8, d_{t22} = .9$ $r_0 = 10, r_1 = 20, r_2 = 30, a_{2200} = .2,$
$a_{2200d} = .3, a_{22n0} = .2, a_{22nd0} = .3,$ $b_{22n0} = .2, b_{22nd0} = .3$

$u_{11\alpha} = .1$, $u_{11\beta} = 1$,

Discussion:

Graphs are plotted between θ and population density of species for constant time. Graphs show that density of both species decreases with θ at any time in the first half part after that density of both species increases with θ at any time in the second half part in the first and second patch. Graphs also show that the density of both increases with time in the first and second patch.



Graph for competition model between radius and density in the first patch at time t=1.

References:

1. Adhikari, J.N., Sinha, B.H.[1980]: Fundamentals of biology of animals, New Central Book Agency, Calcutta, India, 117-153.
2. Carthy, J.D.[1979]: The study of behaviour of animals, The Camelot Press Ltd., Southampton Great Britain, 39-43.
3. Cushing, J. M. [1980]: Two species competition in a periodic environment. J. Math. Bio., **10**, 385-400.

4. Ebenman, B. [1987]: Niche differences between age classes and intraspecific competition in age- structure populations. J. Ther. Bio., 1**24**,25-33
5. Ei, U.and Yanagida, E. [1994]: Dynamics of interfaces in competition-diffusion systems. SIAM J. on App. Math. , **54,**1355-1373.
6. Murray, J.D. [1989]: Mathematical Biology. Biomathematics Texts, Vol19, Springer Verlay, Berlin.
7. Lou, Y. [2004]: On a limiting system in the Lotka-Volterra competition with cross-diffusion. Discrete and Cont.Dynamical system, **10,**435-458.
8. Misra, O.P. and Jadon, B.P. [1999]: Stability of system of two competing species with convective and dispersive migration in a heterogeneous habitat. Proc. Nat. acad. Sci., India, **69,**56-66.
9. Orr, R.T.; Animals in Migration, New York, Macmillan Co.,1970