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Effect of Variable Viscosity and Prandtl Number on the Flow and Heat Transfer along a Wedge

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ABSTRACT

In this paper, the steady laminar water boundary layer flow along a porous wedge with temperature-dependent viscosity and Prandtl number is considered. The effect of viscous dissipation and suction (injection) is included in the analysis. The system of partial differential governing equations are reduced to a system of non-linear ordinary differential equations by similarity transformations and then solved by an implicit finite difference scheme along with quasilinearization technique. Numerical computations are carried out for different values of dimensionless parameters for the wedge angle ranging from 0^o to 90^o, to study their effect on the flow and heat transfer in the presence of variable fluid properties, and analyzed graphically.

1. INTRODUCTION

It is well known that fluid viscosity and thermal conductivity are the main governing fluid properties in laminar boundary layer forced flow. Further, as these thermo-physical properties are temperature dependent, their variations are most easily accomplished in the boundary layer by maintaining a temperature difference between the solid wall and the fluid. In fact for technological applications, surface heating is an effective means of controlling the laminar boundary layer since heating promotes stability through the interplay among the thermal boundary layer, the temperature dependent viscosity, and momentum balance in the crucial region near the wall [1].

In their pioneering work, Falkner and Skan [2] have analysed two-dimensional wedge flows, to illustrate the applications of Ludwig Prandtl's boundary layer theory. Thereafter several investigations [3-5] have studied this classical Falkner-Skan problem employing various analytical and numerical methods for different flow as well as heat transfer situations. However, in all these studies, the fluid properties were assumed to be constant which is unrealistic in many engineering and technological applications. As such, it is imperative to consider such problems by assuming temperature-dependent viscosity and Prandtl number, since this property of the fluid varies significantly when large temperature difference exists. Indeed, both momentum and energy equations are coupled, and, in such situations each equation affects the other.

The first attempt to solve the Falkner-Skan problem including the variation of viscosity with temperature was made by Herwig and Wickern[6]. Hossain et al. [7] studied the flow of a fluid with variable viscosity past a permeable wedge with uniform surface heat flux. Pantokratoras [8] presented Falkner-Skan flow with constant wall temperature and variable viscosity. The objective of the present paper is to present the solutions of Falkner-Skan wedge flow with temperature-dependent viscosity and Prandtl number. The fluid considered here is water, as it is one of the most common working fluids found in scientific and engineering applications.

2. GOVERNING EQUATIONS

Consider the steady, two-dimensional laminar forced convection flow (of water) past a sharp porous wedge with temperature-dependent viscosity and Prandtl number. A schematic diagram illustrating the flow domain and the coordinate system is given in Figure 1.

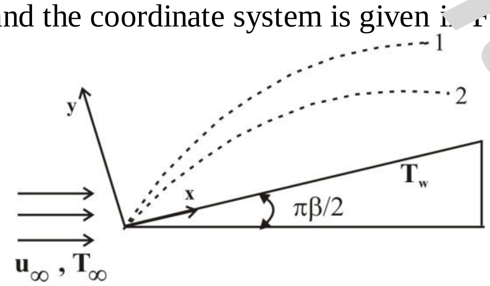

Fig.1. Physical model and co-ordinate system for Falkner- Skan wedge flow, where 1 and 2 represent edge of thermal and momentum boundary layers, respectively.

The fluid is assumed to flow with moderate velocities, and the temperature difference between the surface of the wedge and the free stream is small (< 40^0C). In the range of temperature considered (i.e. 0-40^0C) the variation of both density (ρ) and specific heat (c_p) of water with temperature, is less than 1% (See Table 1) and hence they are taken as constants. However, since the thermal conductivity (k) and viscosity (μ) [and hence Prandtl number (Pr)] variation with temperature is quite significant, the viscosity and Prandtl number are assumed to vary as an inverse linear function of temperature [9]:

$$\mu = 1/(b_1 + b_2 T) \tag{1}$$

$$\Pr = 1/(c_1 + c_2 T) \tag{2}$$

where

$$b_1 = 53.41, \quad b_2 = 2.43, \quad c_1 = 0.068, \quad \text{and} \quad c_2 = 0.004 \tag{3}$$

Table 1. Values of thermo physical properties of water at different temperature [10]

Temperature (T) (^{0}C)	**Density(ρ) (g/cm^3)**	**Specific heat(c_p) (J × 10^7/kg K)**	**Thermal conductivity (k) (erg × 10^5/cms K)**	**Viscosity(μ) (g × 10^{-2}/cm-s)**	**Prandtl number (Pr)**
0	1.00228	4.2176	0.5610	1.7930	13.48
10	0.9997[illegible]	4.1921	0.5800	1.3070	9.45
20	0.99821	4.1818	0.5984	1.0060	7.03
30	0.99565	4.1784	0.6154	0.7977	5.12
40	0.99222	4.178[illegible]	0.6305	0.6532	4.32
50	0.98803	[illegible].180[illegible]	0.6435	0.5470	3.55

The numerical data, used for these correlations, are taken from Ref. [10]. The relation (1) and (2) are reasonably good approximations for liquids such as water, particularly for small wall and ambient temperature differences. As the fluid is incompressible, the contribution of heating due to compression is very small and it has been neglected. The effect of viscous dissipation is included in the analysis. It is assumed that the injected fluids possess the same physical properties as the boundary layer fluid.

Under the aforesaid assumptions, the equations governing the water boundary layer Falkner-Skan flow past a wedge are given by

$$u_x + v_x = 0 \tag{4}$$

$$uu_x + vu_y = UU_x + \rho^{-1}\left(\mu u_y\right)_y \tag{5}$$

$$uT_x + vT_y = \rho^{-1}\left(\frac{\mu}{\Pr}T_y\right)_y + \frac{\mu}{c_p\rho}\left(u_y\right)^2 \tag{6}$$

where u and v are respectively, velocity components in x and y - directions of the flow; U is the reference velocity at the edge of the boundary layer and is a function of x; α is the thermal diffusivity of the fluid; T is the temperature in the vicinity of the wedge;
The boundary conditions are given by

$$\begin{aligned} &at\ \ y = 0 : u = v = 0 \ \ and\ T = T_w \\ &as\ y \to \infty : u \to U(x) = u_\infty (x/L)^m \ \ and \ \ T = T_\infty \\ &at\ x = 0 : u = u_\infty \ \ and\ T = T_\infty \end{aligned} \tag{7}$$

where L the length of the wedge, m is the Falkner-Skan power-law parameter and x is measured from the tip of the wedge. The subscript ∞ denotes conditions at infinity.
Introducing the following transformations to Eqns. (4) - (6)

$$u=\frac{\partial \psi}{\partial y}\ , \qquad v=-\frac{\partial \psi}{\partial x}, \qquad f(\eta)=\sqrt{\left(\frac{1+m}{2}\right)\left(\frac{L^m}{\nu\, u_\infty}\right)}\left(\frac{\psi}{x^{(1+m)/2}}\right),$$
$$\eta=\sqrt{\left(\frac{1+m}{2}\right)\left(\frac{u_\infty}{\nu\, L^m}\right)}\left(\frac{y}{x^{(1-m)/2}}\right),\quad G(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \tag{8}$$

we see that the continuity Eqn.(4) is identically satisfied, and Eqns.(5) and (6) reduce to non – dimensional form, respectively, as:

$$(NF')' + fF' + \beta\left(1-F^2\right)=0 \tag{9}$$
$$(N\,\mathrm{Pr}^{-1}G')' + fG' + NEc(F')^2 = 0 \tag{10}$$

where

$$N=\left(\frac{\mu}{\mu_\infty}\right)=\frac{b_1+b_2T_\infty}{b_1+b_2T}=\frac{1}{1+a_1G},\quad \mathrm{Pr}=\frac{1}{c_1+c_2T}=\frac{1}{a_2+a_3G},\ Ec=\frac{U^2}{c_p(T_w-T_\infty)}$$
$$a_1=\left(\frac{b_2}{b_1+b_2T_\infty}\right)\Delta T_w,\quad a_2=c_1+c_2T_\infty,\quad a_3=c_2\Delta T_w,\quad \Delta T_w=(T_w-T_\infty),\ \mathrm{Re}_L=\frac{u_\infty L}{\nu}$$
$$u=Uf'=UF\ ,\qquad v=-\sqrt{\left(\frac{2}{1+m}\right)\left(\frac{\nu\, u_\infty}{L^m}\right)}\ x^{(m-1)/2}\left[\frac{m+1}{2}f+\ \eta\frac{m-1}{2}F\right] \tag{11}$$
$$f=\int_0^\eta F\,d\eta+f_w,\ f_w=-\int_0^x\left(\sqrt{\frac{2}{m+1}}\right)\left(\sqrt{\frac{L^m}{\nu\, u_\infty}}\right)v_w\frac{d}{dx}\left(x^{\frac{1-m}{2}}\right),\ A=-\left(\frac{v_w}{u_\infty}\right)(\mathrm{Re}_L)^{1/2}$$

The transformed boundary conditions are:

$$\begin{aligned} F&=0; \quad G=1 \quad at \quad \eta=0\\ F&=1; \quad G=0 \quad as \quad \eta\to\infty \end{aligned} \tag{12}$$

Here, f and ψ are dimensional and dimensionless stream functions respectively; Re_L is the local Reynolds number, Pr is the Prandtl number; η is the transformed coordinate, ν is the kinematic viscosity of the fluid, β and m are related through the expression $\beta=\frac{2m}{(1+m)}$. If the normal velocity at the surface v_w is taken as constant, then A is constant since u_∞ and (Re_L) are constants. The mass transfer parameter $A>0$ or $A<0$, according as there is suction or injection.

When N = 1 and Pr = 0.7 (air), Eqns. (9) and (10) reduces to

$$F'' + fF' + \beta\left(1-F^2\right)=0 \tag{13}$$
$$\mathrm{Pr}^{-1}G'' + fG' + NEc(F')^2 = 0 \tag{14}$$

for governing the flow past the wedge with constant fluid properties.

The skin friction and heat transfer coefficients in the form of Nusselt number, can be expressed respectively, as

$$C_f(\mathrm{Re}_L)^{1/2} = 2\sqrt{\left(\frac{1}{2-\beta}\right)}(F')_{\eta=0} \tag{15}$$

$$Nu(\mathrm{Re}_L)^{-1/2} = -\sqrt{\left(\frac{1}{2-\beta}\right)}(G')_{\eta=0} \tag{16}$$

3. RESULTS AND DISCUSSION

The set of ordinary differential equations (9) and (10) along with the boundary conditions (12) using the relations (13)-(14) has been solved numerically using an implicit finite difference scheme along with quasilinearization technique. Since the method is described in great detail in Ref. [11], its description is omitted here for the sake of brevity.

Table 2. Comparison of numerical results for the case of $\beta = 0$, Ec = 0, when N = 1 and Pr = 0.7

η	$f(\eta)$		$F = f'(\eta)$	
	Present	**White[12]**	**Present**	**White[12]**
0.0	0.00000	0.00000	0.00000	0.00000
0.1	0.00236	0.00235	0.04698	0.04696
0.2	0.00940	0.00939	0.09395	0.09391
0.3	0.02111	0.02113	0.14079	0.14081
0.4	0.03760	0.03755	0.18763	0.18761
0.5	0.05866	0.05864	0.23425	0.23423
0.6	0.08450	0.08439	0.28061	0.28058
0.7	0.11469	0.11474	0.32655	0.32653
0.8	0.14969	0.14967	0.37198	0.37196
0.9	0.18915	0.18911	0.41677	0.41672
1.0	0.23300	0.23299	0.46064	0.46063
2.0	0.88700	0.88680	0.81700	0.81669
3.0	1.79558	1.79557	0.96900	0.96905
4.0	2.78390	2.78388	0.99779	0.99777
5.0	3.78325	3.78323	1.00000	0.99994

In order to assess the accuracy of our method we have computed the results of stream function (f) and velocity (F), for constant fluid properties (N = 1 and Pr = 0.7) at the leading edge of the wedge ($\beta = 0$) in the absence of viscous dissipation (Ec = 0), by solving the equations (13) and (14) and compared with those of White [12]. As seen from Table.2, our results are in very good agreement.

The variation of skin friction $[C_f(\mathrm{Re}_L)^{-1/2}]$ and heat transfer coefficients $[Nu(\mathrm{Re}_L)^{-1/2}]$ with wedge angle (β) in the presence of variable fluid properties [T_∞ =18.7°C, ΔT_w= 10.0] and constant fluid properties [N = 1 and Pr = 7.0] is shown in Fig. 2. It is observed from these figures both $[C_f(\mathrm{Re}_L)^{-1/2}]$ and $[Nu(\mathrm{Re}_L)^{-1/2}]$ increase with the increase of wedge angle (β). This behavior is same at all streamwise locations, for an increase of β from 0 to 1.0. In fact, the percentages of increase in $[C_f(Re_L)^{-1/2}]$, at $\beta = 0.5$ is 13.38% [Fig.2(a)], whereas in the case of $[Nu(Re_L)^{-1/2}]$ it is about 2.75 % [Fig.2 (b)].

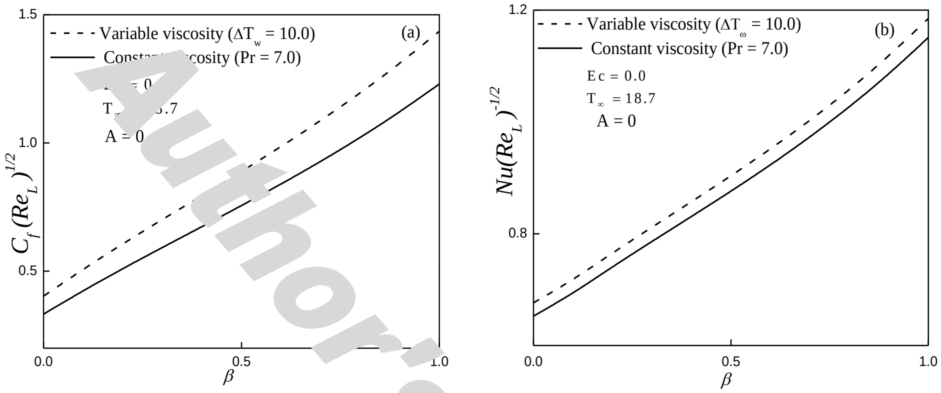

Fig.2. Comparison of variable fluid property results with constant fluid properties results. (a) skin friction coefficient (b) heat transfer coefficient.

The velocity and temperature profiles for constant and variable fluid properties for the values β of ranging from 0.0 and 0.5(i.e., wedge angle ranging from 0^o to 45^o) are displayed in Fig. 3. It is observed that both, momentum boundary layer and thermal boundary layer thicknesses increase when β increases. Indeed, for constants fluids properties, the momentum boundary layer thickness increases about 9.8%. Whereas for variable fluids properties increases about 8.42% [Fig.3 (a)], near η = 2.0. Similarly, at the same location thermal boundary layer thickness increases about 0.64%, for constant fluid properties and 0.39 % in the case of variable fluid properties [Fig.3 (b)].

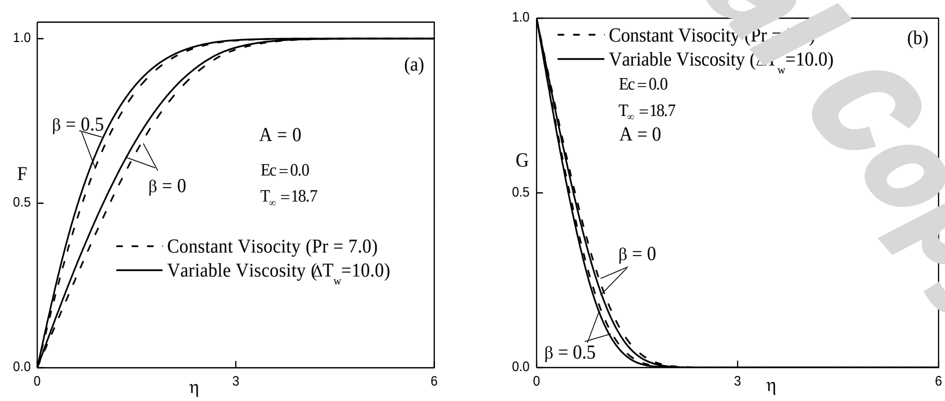

Fig. 3. Comparison of (a) velocity and (b) temperature profiles for constant fluid and variable fluid properties

To see the effect of difference in the temperature (ΔT_w) between the wall and fluid, which actually causes the variation of viscosity and Prandtl number across the boundary layer, the skin friction and heat transfer coefficients have been plotted against ΔT_w [See in Fig.4]. Since $T_\infty = 18.7^oC$, the maximum value of ΔT_w taken is 10^oc so as to keep the temperature within the allowed value (< 40^oC). It is observed from this figure that both $[C_f(Re_L)^{1/2}]$ and $[Nu(Re_L)^{-1/2}]$

monotonically increase with the increase of ΔT_w. Further, these coefficients are found to show their increasing trend along the wedge angle ranging from 0^o to 90^o.

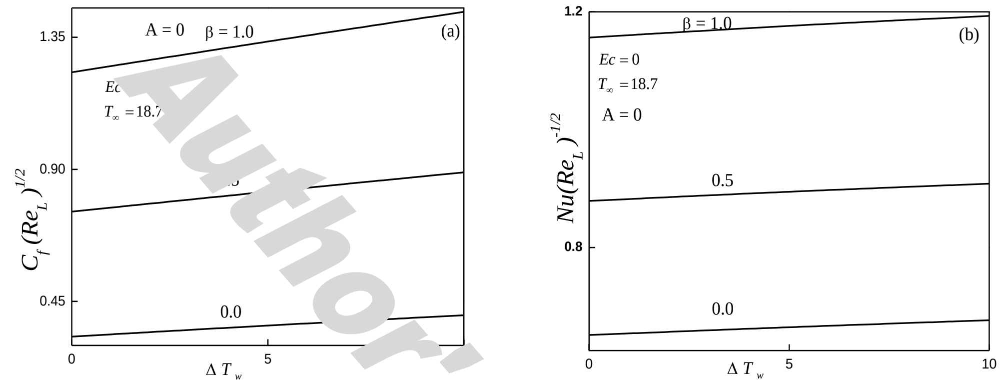

Fig. 4. Variation of (a) skin friction and (b) heat transfer coefficients with ΔT_w

The effect of viscous dissipation parameter (Ec) on heat transfer coefficient $[Nu(Re_L)^{-1/2}]$ and temperature profile (G) is depicted in Fig. 4. It is observed from Fig.4 (a), $[Nu(Re_L)^{-1/2}]$ decreases with increase of Ec. Due to viscous dissipation, the fluid near the wall heats up and its temperature becomes more than the wedge surface, although originally the surface was at higher temperature [T_∞ =18.7°C , T_w= 28.7°C, ΔT_w= 10.0°C]. Thus, the cooler free steam unable to cool the hot surface due to the heat cushion provided by frictional heating. However, $[Nu(Re_L)^{-1/2}]$ found to increase as wedge angle increases. In addition, it is noteworthy to observe the undershoot in the temperature profiles for Ec = -0.5, accounting for the change in the direction of heat transfer [See Fig.5 (b)]. The effect of viscous dissipation on $[C_f\,(Re_L)^{1/2}]$ is negligible because the term Ec is appear only in energy equation

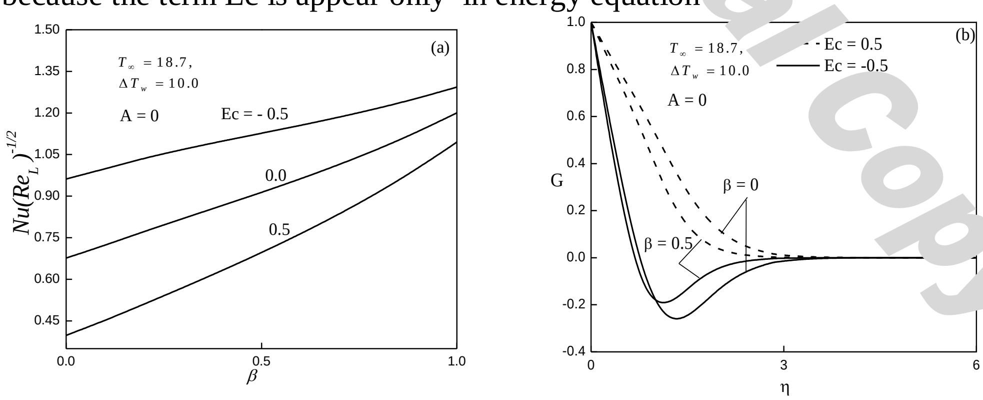

Fig. 5. Effect of mass transfer (Ec) on (a) heat transfer coefficient and (b) temperature profile

Figure 6 shows the effect of mass transfer parameter (A) with wedge angle (β) on $[C_f\,(Re_L)^{1/2}]$ and $[Nu(Re_L)^{-1/2}]$. It is observed that during suction (A> 0), skin friction decreases from the tip of the wedge (i.e., at β = 0) and starts increasing monotonically from the wedge angle 18^o (approximately) up to the angle 90^o (i.e., at β = 1.0). On the other hand, skin friction increases sharply from the tip of wedge, during injection (A< 0), and coincides with the same value around the wedge angle 18^o, as in the case of suction, and becomes monotonic in the remaining regime of the wedge angle (i.e.,in $18^o < \beta < 90^o$)[See Fig. 6(a)]. Similar trend is observed in the case of

$[Nu(Re_L)^{-1/2}]$ [See Fig. 6(b)]. However, heat transfer rate found to decrease penetratingly in $0^o < \beta < 18^o$ in case of suction.

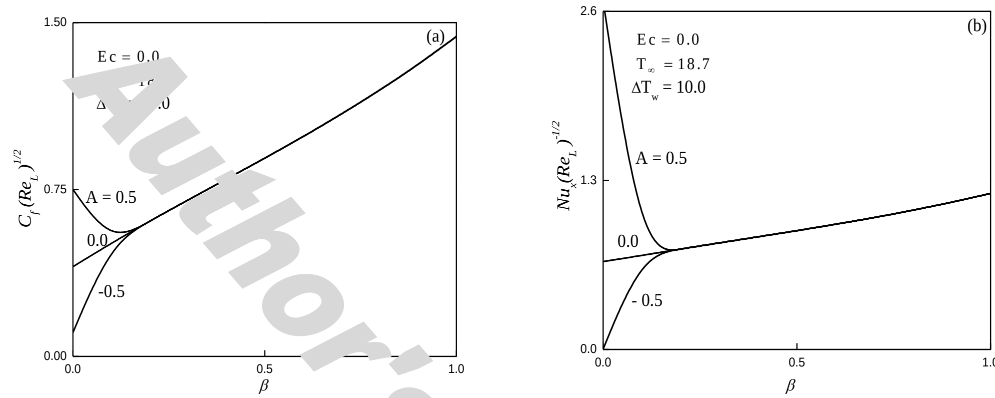

Fig. 6. Effect of mass transfer (A) on (a) skin friction and (b) heat transfer coefficients

4. CONCLUSIONS

Under the assumption of variable viscosity and Prandtl number, the steady water boundary layer flow past a wedge is numerically investigated. The effects of suction (injection) and viscous dissipation on skin friction coefficient and on the rate of heat transfer are discussed. The obtained results show that the flow field and thermal characteristics are significantly influenced by suction (injection) and viscous dissipation parameters, in the presence of variable viscosity and Prandtl number. From the present study it is concluded that when the viscosity and Prandtl number of a fluid is sensitive to temperature variations it is important to consider the effect of temperature-dependent viscosity, otherwise substantial errors may occur in the characteristics of flow and heat transfer processes.

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