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MHD Flow of Newtonian Fluid through Porous Medium in a Rotating System

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ABSTRACT

The flow of an incompressible viscous fluid through porous medium under the influence of magnetic field is studied. The porous medium contained between infinite parallel plates rotating about an axis perpendicular to the plates. The effect of rotation parameter, permeability parameter and magnetic parameter is examined and the results are presented graphically.

Keywords : Rotation, permeability, porous medium, magnetic parameter.

1. INTRODUCTION

The study of flow through porous medium assumed importance because of the interesting applications in the diverse field of science, Engineering and Technology. The practical applications are in the percolation of water through soil, extraction and filteration of oils from wells, the drainage of water, irrigation and sanitary engineering and also in the inter disciplinary fields such as biomedical engineering, the lung alveolar is an example that finds applications in an animal body. The classical Darcy's law Musakat (1937, 1949), states that the pressure gradient pushes the fluid against the body forces exerted by the medium which can be expressed as $\vec{V} = -\left(\frac{k}{\mu}\right)\nabla P$ with usual notation.

The classical Darcys law gives good result in the situations where the flow is uni directional or at low speed. In general specific discharge in the medium need not be always low. As the specific discharge increases the convective forces get developed and the internal stress generates in the fluid due to viscous nature and produces distortions in the velocity field. Modifications for the classical Darcy's law are considered by Beavers and Jospeh (1967), Saffman (1970) and others. A generalized Darcy's law is proposed by Brinkman (1947)..

$\rho\frac{d\overline{v}}{dt} = \text{Div } S_{ij} - \left(\frac{\mu}{k}\right)\vec{v}$ where S_{ij} is the Stress Tensar of the fluid, ρ is the density, $\overline{v}$ is velocity of the fluid, k is permeability coefficient.

Narasimhacharyulu (1997, 2007), Narasimhacharyulu and Pattabhi Ramacharyulu (1978) and Narasimhacharyulu and Sunder Ram (2010). And several investigators adopted the generalized law proposed by Brinkman.

The geophysical importance of the flows in the rotating frame of reference has attracted the attention of a number of scholars. These appeared a number of studies in the literature viz. Vidyanidhy and Nigam (1967), Gupta (1972) and Janna and Datta (1977). The effect of uniform transverse magnetic field with (or) without suction was studied by Gupta (1972) Soundalekar and Pop (1973) and Mazumder *et al*., (1976). Recently Mazunder (1991) studied an oscillatory Ekman boundary layer flow bounded by two horizontal flat plates, one of which is oscillating about a non zero coefficient mean velocity in it's own plane and the other at rest. Ganaparthy (1994) corrected the method adapted by Mazunder to the problem, by considering the form of solution as the linear combination (Telinois, 1981) Ganapathy presented an alternative solution to the problem.

In the present problem the unstead flow of Newtonian fluid through porous medium between two parallel plates under magnetic field in a rotating system is studied. The impact of all the physical parameters i.e., permeability coefficient, angular velocity, magnetic parameter is studied and represented graphically.

2. Formulation of the Problem:

Let us consider the flow of viscous incompressible fluid through porous medium under magnetic field bounded by two horizontal parallel flat plates separated by a distance 'd'. The lower plate is kept at rest, and upper plate is oscillating in it's own plane with velocity U(t) about a non zero constant mean velocity U_0. The coordinate system *0(x, y, z)* is taken such that z-axis is perpendicular to the plates. x-axis taken parallel to the direction of motion of the upper plate, origin is chosen on the lower plate. The entire system is rotating about the z-axis with angular velocity Ω. All the physical quantities except pressure, dependent on only z and t. The velocity of the fluid is given by $\vec{V}(u,v,w)$.

The induced magnetic effect is considered to be negligible in comparision with the transverse magnetic field. Due to low magnetic Reynold's number, as a result of slightly conducting fluid (Sparrow, 1962). Further the electronic force given by Oms law $J=(E+V\times B)$ where $B=(H_O,H_D,O)$ electrical conductivity is assumed to be a null vector for simplicity of the problem.

The equation of motion of flow of the fluid is

$$\frac{\rho d\vec{v}}{dt} = Div\,S - \sigma\mu_e^2 H_0^2 \bar{v} - \left(\frac{\mu}{k}\right)\bar{v} \qquad \ldots \qquad 2.1$$

where S is stress vector

And equation of continuity is

$$\nabla.\vec{v} = 0 \qquad \ldots \qquad 2.2$$

The equation of continuity gives ω = 0

The equation of motion of fluid in rotating system under magnetic field given by

$$\frac{\partial u}{\partial t}=\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u}{\partial z^2}+2\Omega v-\frac{\nu}{k}u-\frac{\sigma\mu_e^2 H_0^2}{\rho}u \qquad \ldots \qquad 2.3.$$

$$\frac{\partial v}{\partial t}=\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\frac{\partial^2 v}{\partial z^2}-2\Omega u-\frac{\nu}{k}v-\frac{\sigma\mu_e^2 H_0^2}{\rho}v \qquad \ldots \qquad 2.4$$

Where ρ is density of the fluid, σ is electric conductivity, μ_e is magnetic permeability, H_O is the intensity of the magnetic field, ν is kinematic viscosity, t is time, k is permeability of the porous medium, p is modified pressure.

The boundary conditions of the problem considered as

$$u=v=0 \text{ at } z=0$$

$$\ldots \qquad 2.5$$

$$u=U(t)=U_0(1+\in\cos\omega t), v=0 \text{ at } z=d$$

ω is frequency of the oscillations, and ∈ is small +ve constant

Applying usual boundary layer approximations and eliminating pressure term in (2.3) and (2.4) we get.

$$\frac{\partial u}{\partial t}=\nu\frac{\partial^2 u}{\partial z^2}+\frac{\partial U}{\partial t}+2\Omega v-\alpha^2(u-U) \qquad \ldots \qquad 2.6$$

$$\frac{\partial v}{\partial t}=\nu\frac{\partial^2 v}{\partial z^2}-2\Omega(u-U)-\alpha^2 v \qquad \ldots \qquad 2.7$$

where $\alpha^2=\frac{1}{K}+M^2$ and $M=\mu H_0\sqrt{\frac{\sigma}{\rho}}$

Adding equations 2.6 and 2.7

$$\frac{\partial q}{\partial t}=\nu\frac{\partial^2 q}{\partial z^2}+\frac{\partial U}{\partial t}-2i\Omega(q-U)-\alpha^2(q-U) \qquad \ldots \qquad 2.8$$

and corresponding boundary conditions

$$q=0 \text{ at } Z=0$$

$$q=U(t)=U_0\left[1+\frac{\in}{2}\left(e^{i\omega t}+e^{-i\omega t}\right)\right] \text{ at } z=d$$

where $q = u + iv$ … 2.9

To solve the equation 2.8 following Ganapathy we choose

$$q(\eta,t) = U_o\left[q_o(\eta) + \frac{\in}{2}\left\{q_1(\eta)e^{i\omega t} + q_2(\eta)e^{-i\omega t}\right\}\right] \quad \ldots \quad 2.10$$

where $\eta = \frac{z}{d}$, $q_0(\eta) = u_0(\eta) + iv_o(\eta)$

and $\left\{q_1(\eta)e^{i\omega t} + q_2(\eta)e^{-i\omega t}\right\} = u_1(\eta,t) + iv_1(\eta,t)$ … 2.11

Substituting (2.10) in the equation (2.8) and using (2.9)

q_0, q_1, q_2 are solved from the equations.

$$q_0^{\parallel} - \left(2iR + \alpha_1^2\right)q_0 = -(2iR + \alpha_1^2) \quad \ldots \quad 2.12$$

$$q_1^{\parallel} - \left(2iR + i\lambda + \alpha_1^2\right)q_1 = -(2iR + i\lambda + \alpha_1^2) \quad \ldots \quad 2.13$$

$$q_2^{\parallel} - \left(2iR + i\lambda + \alpha_1^2\right)q_2 = -(2iR - i\lambda + \alpha_1^2) \quad \ldots \quad 2.14$$

with boundary conditions

$$\left.\begin{array}{l} q_0 = q_1 = q_2 = 0 \quad at \quad \eta = 0 \\ q_0 = q_1 = q_2 = 1 \quad at \quad \eta = 1 \end{array}\right\} \quad \ldots \quad 2.15$$

Where $R = \frac{\Omega d^2}{\nu}$ is the rotating parameter

$\lambda = \frac{\omega d^2}{\nu}$ is frequency parameter

$$\alpha_1^2 = \alpha^2 d^2 = \frac{d^2}{K} + \frac{\sigma \mu_e^2 H_0^2 d^2}{\mu}$$

Solving 2.12 and 2.14 with boundary conditions 2.15

$$q_0(\eta) = 1 - \frac{\sin h(l(1-\eta))}{\sin\ hl} \quad \ldots \quad 2.16$$

$$q_1(\eta) = 1 - \frac{\sin h(m(1-\eta))}{\sin\ hm} \quad \ldots \quad 2.17$$

$$q_2(\eta) = 1 - \frac{\sin h(n(1-\eta))}{\sin\ hn} \quad \ldots \quad 2.18$$

where $l = \left[2iR + \alpha_1^2\right]^{\frac{1}{2}}, \quad m = \left[2iR + i\lambda + \alpha_1^2\right]^{\frac{1}{2}}$

$$n = \left[2iR - iR + \alpha_1^2\right]^{\frac{1}{2}} \quad \ldots \quad 2.19$$

Substituting (2.16), (2.17), (2.18) in (2.10) we get

$$q(\eta, t) = U + iV \quad \ldots \quad 2.20$$

$$U = U_0 \left\{ \left(2 - \frac{\sin h(m(1-\eta)}{\sin hm} - \frac{\sin h(n(1-\eta)}{\sin hn} \right) \cos \omega t + 1 - \frac{\sin h(l(1-\eta)}{\sin hl} \right\}$$

$$V = U_0 \left\{ \left(2 - \frac{\sin h(m(1-\eta)}{\sin hm} - \frac{\sin h(n(1-\eta)}{\sin hn} \right) \sin \omega t \right\} \quad \ldots \quad 2.21$$

Case 1 :

As $k \to \infty$, the problem reduces to the flow of Newtonian fluid in clear medium under magnetic field.

$$q(\eta, t) = U_o \left[q_o(\eta) + \frac{\in}{2} \left\{ q_1(\eta) e^{i\omega t} + q_2(\eta) e^{-i\omega t} \right\} \right] \quad \ldots \quad 2.22$$

where

$$q_0(\eta) = 1 - \frac{\sin h(l(1-\eta))}{\sin\ hl}$$

$$q_1(\eta) = 1 - \frac{\sin h(m(1-\eta))}{\sin\ hm} \quad \ldots \quad 2.23$$

$$q_2(\eta) = 1 - \frac{\sin h(n(1-\eta))}{\sin\ hn}$$

where

$$l = \left(2iR + M_1^2\right)^{\frac{1}{2}}$$

$$m = \left(2iR + i\lambda + M_1^2\right)^{\frac{1}{2}}$$

$$n = \left(2iR - i\lambda + M_1^2\right)^{\frac{1}{2}}$$

where $M_1 = \dfrac{\sigma\mu_e^2 H_0^2 d^2}{\mu}$ … 2.24

Case 2 :

Flow of Newtonian fluid in the rotating system in the absence of magnetic field.

$$q(\eta,t) = U_o\left[q_o(\eta) + \frac{\in}{2}\left\{q_1(\eta)e^{i\omega t} + q_2(\eta)e^{-i\omega t}\right\}\right] \quad \ldots \quad 2.25$$

where

$$q_0(\eta) = 1 - \frac{\sin h\left(l(1-\eta)\right)}{\sin\ hl}$$

$$q_1(\eta) = 1 - \frac{\sin h\left(m(1-\eta)\right)}{\sin\ hm} \quad \ldots \quad 2.26$$

$$q_2(\eta) = 1 - \frac{\sin h\left(n(1-\eta)\right)}{\sin\ hn}$$

where

$$l = \left(2iR + \beta^2\right)^{\frac{1}{2}}$$

$$m = \left(2iR + i\lambda + \beta^2\right)^{\frac{1}{2}}$$

$$n = \left(2iR - i\lambda + \beta^2\right)^{\frac{1}{2}}$$

where $\beta^2 = \dfrac{d^2}{k}$ … 2.27

Case 3 :

Flow of Newtonian fluid through clear medium in the absence of magnetic field. i.e., $k \to \infty$, M = 0

$$q(\eta,t) = U_o\left[q_o(\eta) + \frac{\in}{2}\left\{q_1(\eta)e^{i\omega t} + q_2(\eta)e^{-i\omega t}\right\}\right] \quad \ldots \quad 2.28$$

where

$$q_0(\eta) = 1 - \frac{\sin h\left(l(1-\eta)\right)}{\sin\ hl}$$
$$q_1(\eta) = 1 - \frac{\sin h\left(m(1-\eta)\right)}{\sin\ hm} \qquad \ldots \qquad 2.29$$
$$q_2(\eta) = 1 - \frac{\sin h\left(n(1-\eta)\right)}{\sin\ hn}$$

where

$$l = (2iR)^{\frac{1}{2}}, \quad m = (2iR + i\lambda)^{\frac{1}{2}}, \quad n = (2iR - i\lambda)^{\frac{1}{2}} \qquad \ldots \qquad 2.30$$

The values of q_0, q_1, q_2 in this case are coincide similar to the solution obtained by Ganapathy.

RESULTS AND DISCUSSION

In the case of steady state the resultant velocity is given by

$$|q_0| = \sqrt{u_0^2 + v_0^2} \qquad \ldots \qquad 2.31$$

where u_0, v_0 are primary and secondary velocity components

The amplitude and phase angle in the steady state are given by

$$|A_0| = u_0^2 + v_0^2, \ \theta_0 = \tan^{-1}\left(\frac{u_0}{v_0}\right) \qquad \ldots \qquad 2.32$$

For large value of R, u_0 and v_0 become

$$u_0(\eta) = 1 - \exp(-l_2\eta)\cos(l_1\eta)$$
$$v_0(\eta) = \exp(-l_2\eta)\cos(l_2\eta) \qquad \ldots \qquad 2.33$$
$$l_1 = \left[\frac{(\alpha_1^2 + 4R^2)^{\frac{1}{2}} + \alpha_1^2}{2}\right]^{\frac{1}{2}}$$
$$l_2 = \left[\frac{(\alpha_1^2 + 4R^2)^{\frac{1}{2}} - \alpha_1^2}{2}\right]^{\frac{1}{2}}$$

There exist a boundary layer of order $O(L_1^{-1})$ in the neighbourhood of the plane which is known as Ekman layer. This layer decreases with increase of rotation parameter R, but increases permeability parameter. Fig. 1, 2, 3 and 4 illustrate the effects of porous medium and magnetic parameter on the velocities q_0, q_1. The values increase with the permeability and magnetic parameter. Rotation parameter R influence the an increase in the values of q_0, q_1.

The Fig. 5, 6, 7, 8 shows the variation of amplitude and phase angle with magnetic parameter M, permeability parameter β.

Fig. 1 : Variation of q_0 with the permeability parameter β

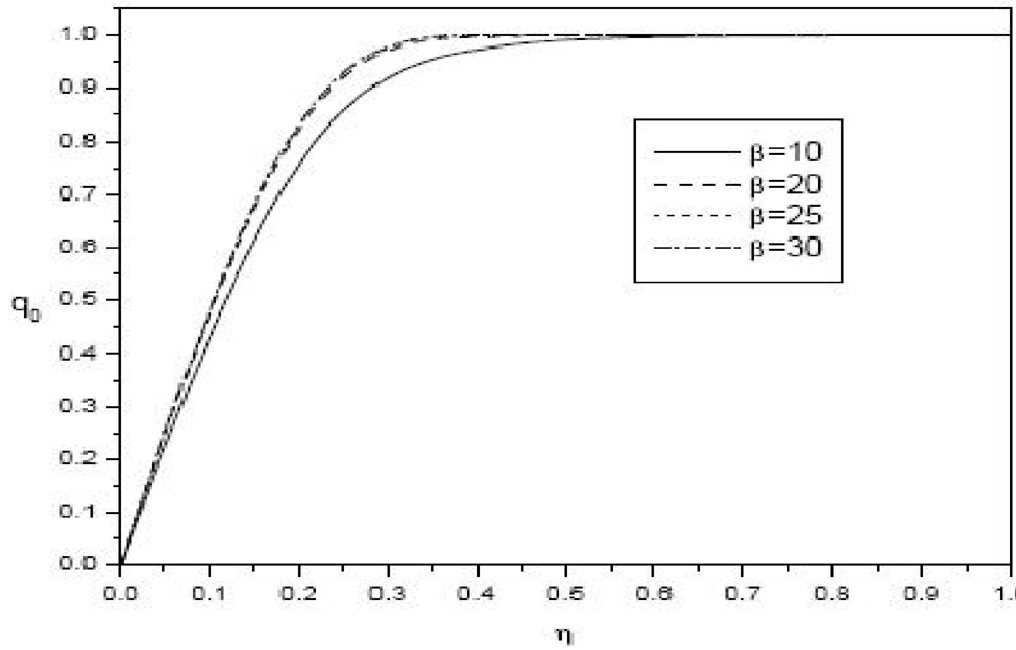


Fig. 2 : Variation of q_1 with the permeability parameter β

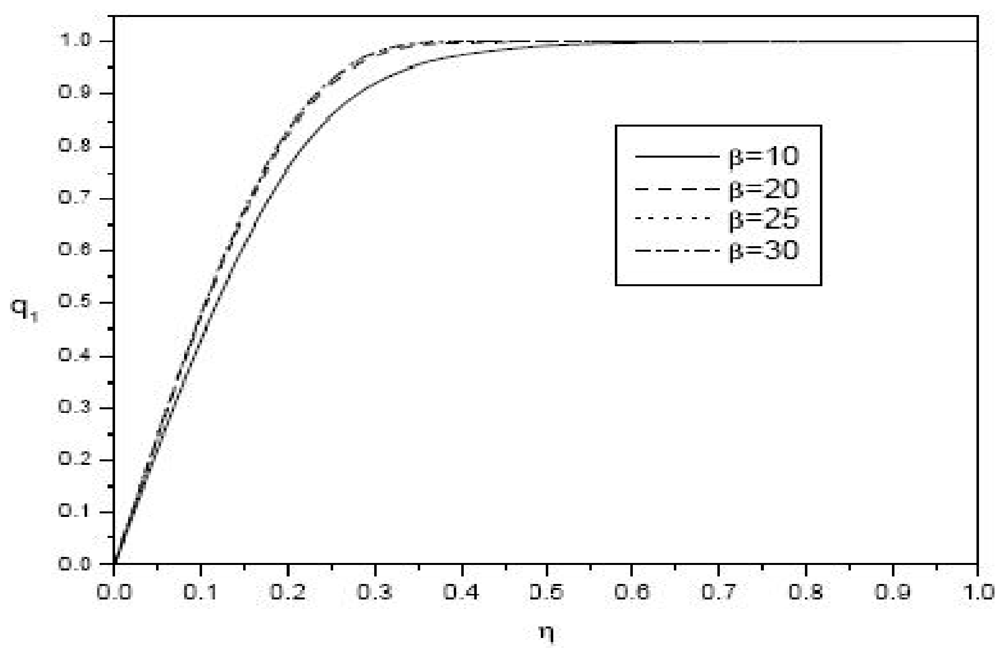


Fig. 3 : Variation of q_0 with the magnetic parameter (M)

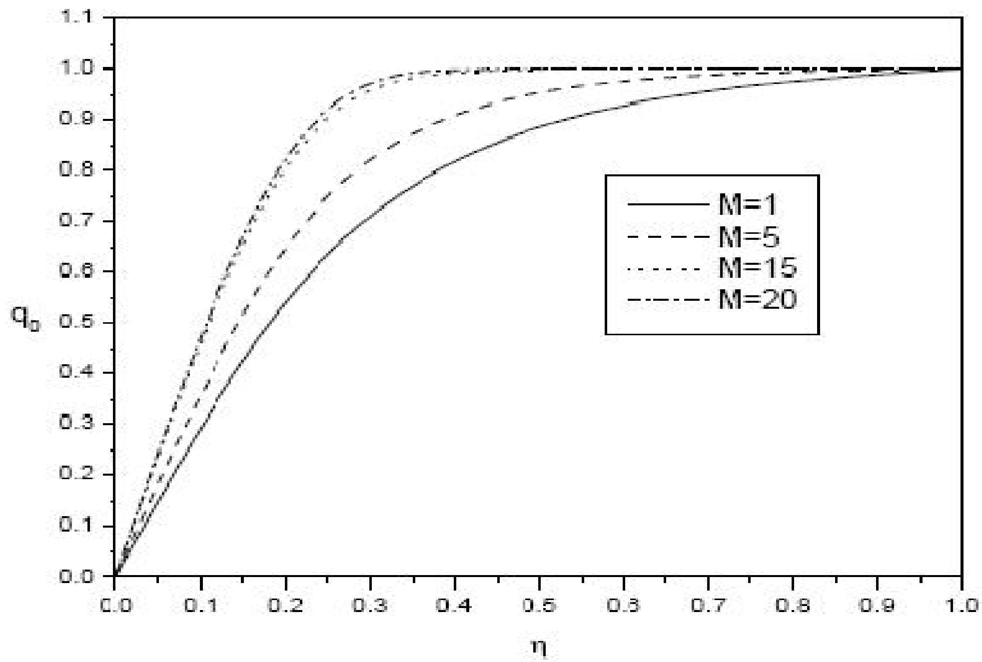


Fig. 4 : Variation of q_1 with the magnetic parameter (M)

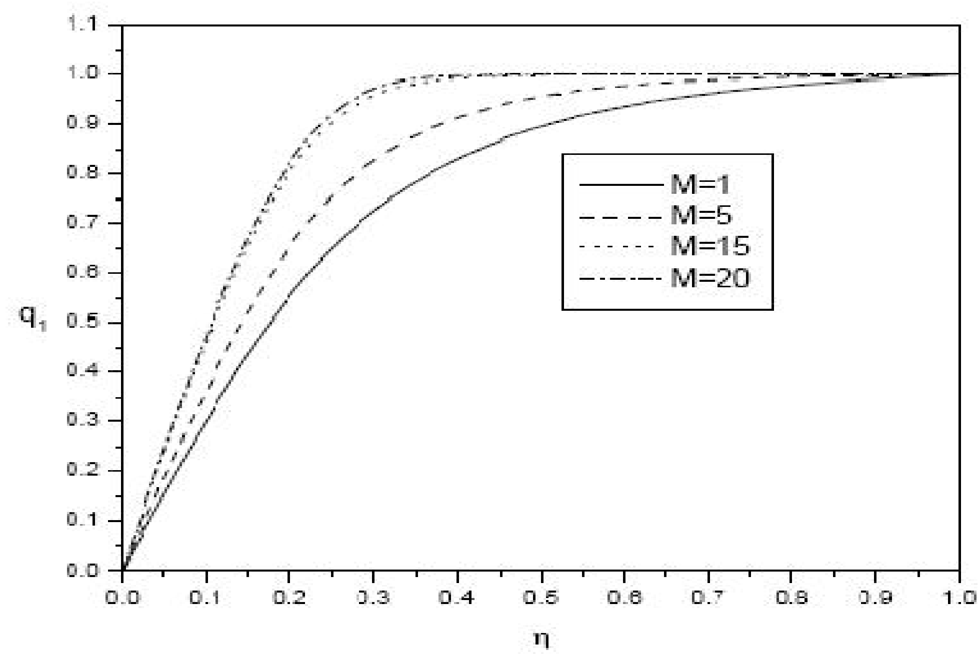


Fig. 5 : Amplitude variation with permeability parameter β

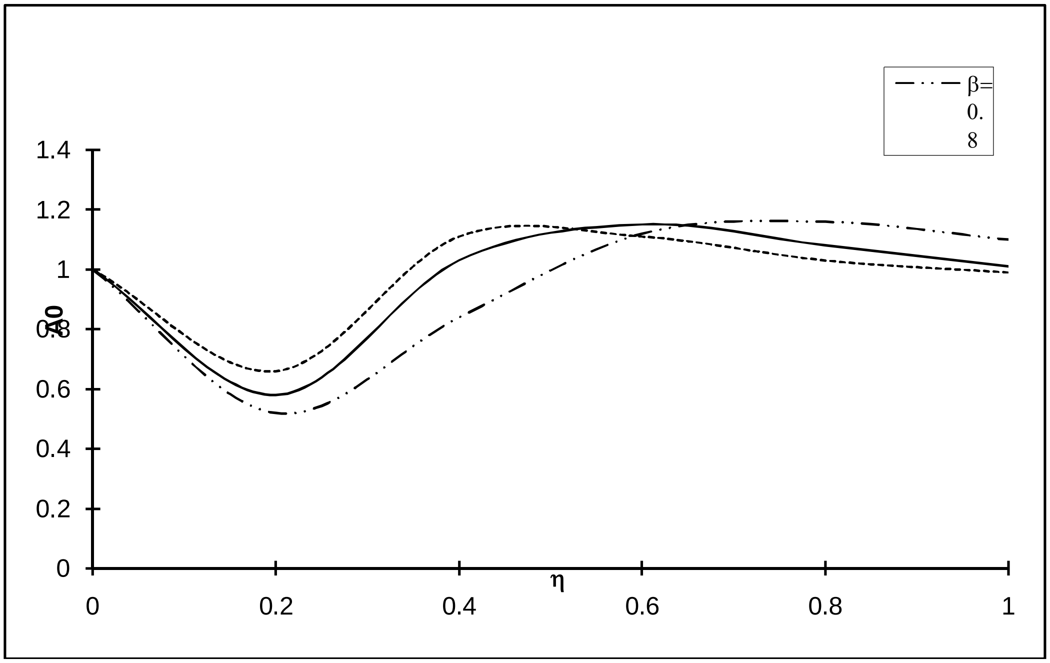


Fig. 6 : Phase angle variation with permeability parameter β

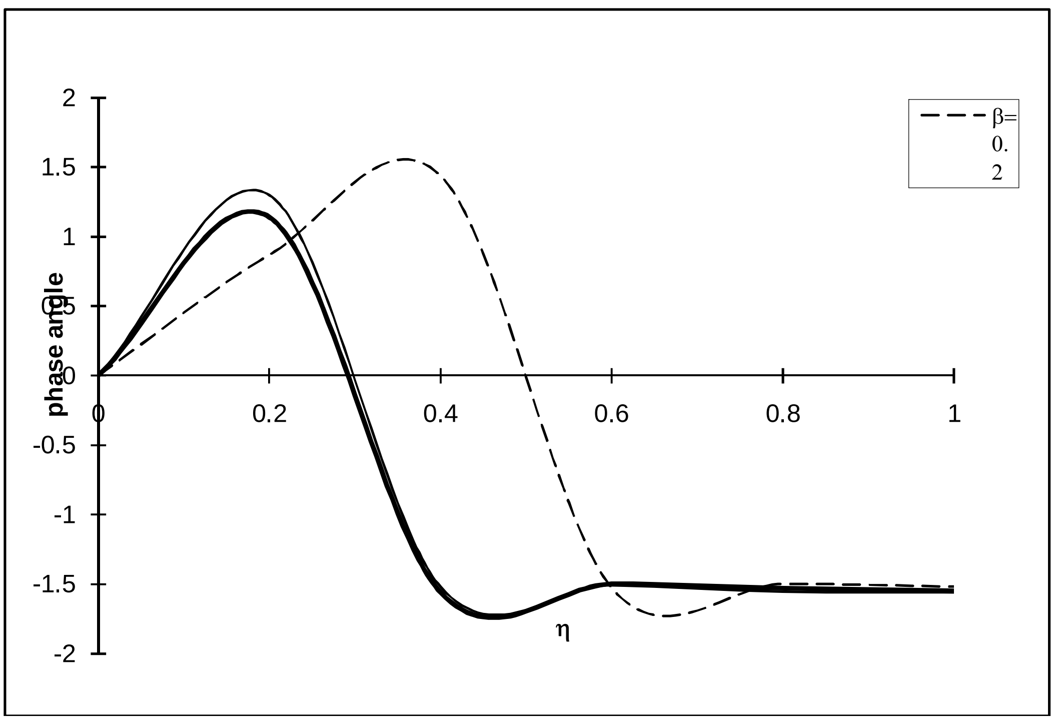


Fig. 7 : Amplitude variation with magnetic parameter M

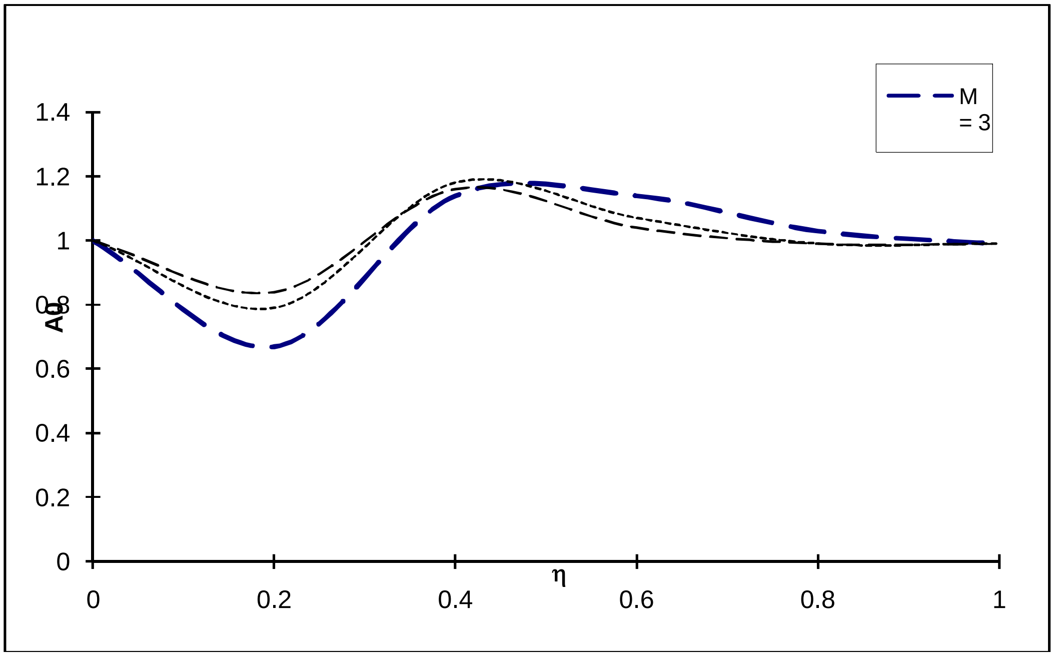
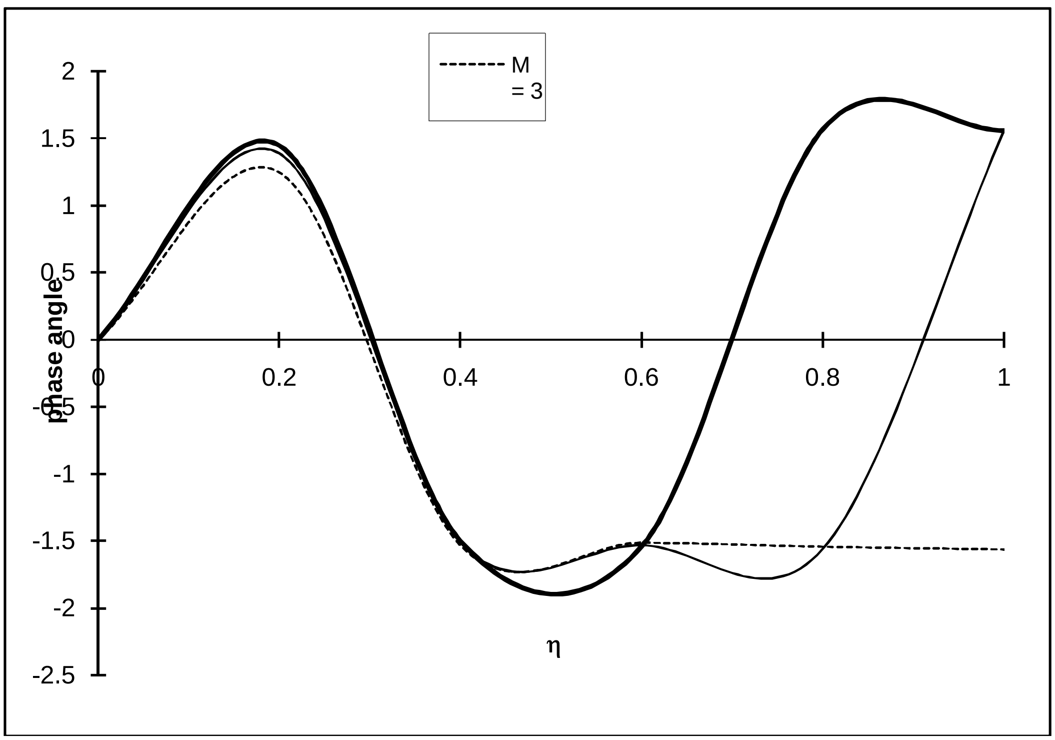


Fig. 8 : Phase angle variation with magnetic parameter M



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