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Fuzzy Transportation Problem of Trapezoidal Numbers with Methods of Linear Programming Problems

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ABSTRACT

There are various methods, in the literature, for determining the fuzzy optimal solution of a fuzzy transportation problem (transportation problems in which all the parameters are represented by fuzzy numbers).In [1] [2] we solve the Fuzzy transportation problem with the help of dual simplex and two phasemethod.Now we are solving a simple balanced fuzzy transportation problem of trapezoidal number with several methods of linear programming problems with the help of Tora package. We also compare the results of linear programming methods with result of Vogel Approximation Methods. Finally, we give illustrative example and their numerical solutions.

Keywords: Fuzzy Transportation Problem, Trapezoidal Numbers, Methods of linear Programming Problems, Tora package.

INTRODUCTION

Fuzzy transportation problem is a linear programming problem stemmed from a network structure consisting of a finite number of nodes and arcs attached to them. Efficient algorithms have been developed for solving the transportation problem when the cost coefficients and the supply and demand quantities are known exactly. Kaufmann and Gupta [8] first examined the fuzzy transportation problem. One straightforward approach is to apply the existing fuzzy linear programming techniques directly to the fuzzy transportation problem. Several investigators have also discussed using different approaches to solving the problem. Chanas et al. [1] investigate the transportation problem with fuzzy supplies and fuzzy demands and solve them via the parametric programming technique in terms of the Bellman-Zadeh criterion. Chanas and Kuchta [2] proposed a concept of the optimal solution of the transportation problem with fuzzy coefficients and an algorithm determining this solution.

Liu and Kao [9] developed a method to find the membership function of the fuzzy total transportation cost when the unit shipping costs, the supply quantities, and the demand quantities are fuzzy numbers. Jimenez and Verdegay [7] proposed a GA to deal with the fuzzy solid transportation problem in which the fuzziness affects only in the constraint set. They concluded that GA showed a good performance in finding parametric solutions in comparison with nonparametric solutions obtained with other nonlinear solution methods. Lin and Tsai [10] investigated solving the transportation problem with fuzzy demands and fuzzy supplies using a two-stage GA.

This study suggests methods of linear programming approach to solving the transportation problem with fuzzy demands and fuzzy supplies. The numerical solved by two methods. In both methods the coefficients of objective function are in the form of fuzzy numbers and changing problem in linear programming problem then solved by methods of linear programming problem.

Methodology for solving Fuzzy Transportation Problem

Ghatee and Hashemi [3,4,5,6] proposed the fuzzy linear programming formulation of balanced fully fuzzy minimal cost flow problems. In case of balanced fully fuzzy transportation problem, the same fuzzy linear programming formulation will be converted into the following fuzzy linear programming formulation:

Minimize $\sum_{i=1}^{p}\sum_{j=1}^{q} c_{ij} \otimes x_{ij}$

Subject to
$\sum_{j=1}^{q} x_{ij} = a_i$ i= 1,2,3,……p

$\sum_{i=1}^{p} x_{ij} = b_j$j=1,2,3 ….q

$x_{ij}$is a non-negative fuzzy number.

Where

p = Total number of sources;

q = Total number of destinations;

a_i= The fuzzy availability of the product at ithsource;

b_j= The fuzzy demand of the product at jthdestination;

c_{ij}= The fuzzy transportation cost for one unit quantity of the product from ithsource to jthdestination;

x_{ij}= The fuzzy quantity of the product that should be transported from ith source to jth destination (or fuzzy decision variables) to minimize the total fuzzy transportation cost;

$\sum_{i=1}^{p} a_i$= total fuzzy availability of the product;

$\sum_{j=1}^{q} b_{ij}$= total fuzzy demand of the product

$\sum_{i=1}^{p}\sum_{j=1}^{q} c_{ij} \otimes x_{ij}$ = total fuzzy transportation cost.

Arithmetic Operators for solving Numerical

If A = [a_1, a_2, a_3, a_4] and B= [b_1, b_2, b_3, b_4] two trapezoidal fuzzy numbers then the arithmetic operations on A and B as follows:

- Addition $A+B = (a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4)$
- Subtraction $A-B = (a_1-b_4, a_2-b_3, a_3-b_2, a_4-b_1)$
- Multiplication $A \bullet B = (t_1, t_2, t_3, t_4)$

  Where t_1= minimum $\{a_1b_1, a_1b_4, a_4b_1, a_4b_4\}$;

  t_2= minimum$\{a_2b_2, a_2b_3, a_3b_2, a_3b_3\}$;

  t_3= maximum$\{a_2b_2, a_2b_3, a_3b_2, a_3b_3\}$ and

  t_4= maximum$\{a_1b_1, a_1b_4, a_4b_1, a_4b_4\}$.

Ranking of a Fuzzy Number

A ranking function is a function R: F(R)→R, where F(R) isa set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into the real line.

Let $A= (a_1, a_2, a_3, a_4)$ be a trapezoidal fuzzy number then $R(A) = (a_1+a_2+a_3+a_4) / 4$

Numerical Example

To solve the following fuzzy transportation problem starting with initial basic feasible solution obtained by fuzzy Vogel approximation method. Whose fuzzy cost and fuzzy requirement table is given below:

	FD1	**FD2**	**FD3**	**FD4**	**Fuzzy Capacity**
FO1	1,2,3,4	1,3,4,6	9,11,12,14	5,7,8,11	1,6,7,12
FO2	0 ,1,2, 4	-1 ,0 ,1 ,2	5,6,7,8	0,1 ,2,3	0,1,2,3
FO3	3,5,6, 8	5,8,9,12	12,15,16,19	7,9,10,12	5,10,12,17
Fuzzy Demand	5,7,8,10	1,5,6,10	1,3,4,6	1,2,3,4	

Fuzzy transportation problem after ranking

	FD1	**FD2**	**FD3**	**FD4**	**Fuzzy Capacity**
FO1	2.5	3.5	11.5	7.7	6.5
FO2	1.7	0.5	6.5	1.5	1.5
FO3	5.5	8.5	15.5	9.5	11
Fuzzy Demand	7.5	5.5	3.5	2.5	Total 19

Fuzzy transportation problem after VAM

	FD1	**FD2**	**FD3**	**FD4**	**Fuzzy Capacity**
FO1	**1** 2.5	**5.5** 3.5	11.5	7.7	6.5
FO2	1.7	0.5	6.5	**1.5** 1.5	1.5
FO3	**6.5** 5.5	8.5	**3.5** 15.5	**1** 9.5	11
Fuzzy demand	7.5	5.5	3.5	2.5	Total 19

Fuzzy transportation problem after VAM in the form of trapezoidal no.

	FD1	**FD2**	**FD3**	**FD4**	**Fuzzy capacity**
FO1	**[9,0,2,11]** 1,2,3,4	**[1,5,6,10]** 1,3,4,6	9,11,12,14	5,7,8,11	1,6,7,12
FO2	0 ,1,2, 4	-1 ,0 ,1 ,2	5,6,7,8	**[0,1,2,3]** 0,1 ,2,3	0,1,2,3
FO3	**[-6,5,8,19]** 3,5,6, 8	5,8,9,12	**[-18,0,7,25]** 12,15,16,19	**[-2,0,2,4]** 7,9,10,12	5,10,12,17
Fuzzy demand	5,7,8,10	1,5,6,10	1,3,4,6	1,2,3,4	

Transportation cost

(-9,0,2,11)(1,2,3,4)+(1,5,6,10)(1,3,4,6)+(0,1,2,3)(0,1,2,3)+(-6,5,8,19)(3,5,6,8)+(-18,0,7,25)(12,15,16,19)+(-2,0,2,4)(7,9,10,12)
= 123.5

After applying the methods of linear programming problem (Big M, Dual Simplex and Two Phase method) by using Tora Package the transportation cost are 121. The tables of methods are shown on page no. 6 & 7

Conclusion

This study investigated using dual simplex; Two Phase and Big-M method approach to solving the fuzzy transportation problem with fuzzy demands and fuzzy supplies. We have thus obtained an optimal solution for a fuzzy transportation problem with Tora package. The empirical results show that the proposed methods approach outperforms the other methods for solving the fuzzy transportation problem with fuzzy demands and fuzzy supplies. The same approach for solving the fuzzy transportation problems may also be utilized in future studies of operational research.

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BIG-M MTHEOD

Basic	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	sx_{13}	sx_{14}	sx_{15}	sx_{16}	sx_{17}	sx_{18}	sx_{19}	Rx_{20}	Rx_{21}	Rx_{22}	Rx_{23}	solution
z(min)	-1	0	0	-2.2	-5	-2	0	-1	0	-1	0	0	-90	-92	-100	-94	-88.5	-93.5	-84.5	-10	-8	0	-6	121
x3	1	0	1	1	0	1	0	0	0	-1	0	0	0	1	0	0	1	0	0	0	-1	0	0	1
x7	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1.5
x11	-1	0	0	-1	-1	0	0	-1	0	1	1	0	1	0	0	1	0	0	1	-1	0	0	-1	1
x9	1	0	0	0	1	0	0	0	1	0	0	0	-1	0	0	0	0	0	0	1	0	0	0	7.5
x2	0	1	0	0	0	1	0	0	0	1	0	0	0	-1	0	0	0	0	0	0	1	0	0	5.5
Rx22	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	0
x12	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	-1	0	0	0	0	0	0	1	2.5
Lower Bound	0	0	0	0	0	0	0	0	0	0	0	0												
Upper Bound	infinity	infinity	Infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity												
Unrestr,d(y/n)?	n	n	N	n	n	n	n	n	n	N	n	n												

PHASE-II

Basic	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	sx_{13}	sx_{14}	sx_{15}	sx_{16}	sx_{17}	sx_{18}	sx_{19}	Rx_{20}	Rx_{21}	Rx_{22}	Rx_{23}	solution
z(min)	-1	0	0	-2.2	-5.2	-2	0	-1	0	-1	0	0	-5.5	-7.5	-15.5	-9.5	-4	-9	0	block	block	blosk	block	121
x3	1	0	1	1	0	-1	0	0	0	-1	0	0	0	1	0	0	1	0	0	0	-1	0	0	1
x7	0	1	0	0	0	1	0	0	0	1	0	0	0	-1	0	0	0	0	0	0	1	0	0	5.5
x11	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	-1	0	0	0	0	0	0	1	2.5
x9	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1.5
x2	1	0	0	0	1	0	0	0	1	0	0	0	-1	0	0	0	0	0	0	1	0	0	0	7.5
Rx22	-1	0	0	-1	-1	0	0	-1	0	1	1	0	0	-1	1	0	-1	-1	0	0	1	1	-1	1
x12	0	0	0	0	0	0	0	0	0	0	0	0	1	1	-1	1	1	1	1	-1	-1	-1	1	0
Lower Bound	0	0	0	0	0	0	0	0	0	0	0	0												
Upper Bound	infinity	infinity	Infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity												
Unrestr,d(y/n)?	n	n	N	n	n	n	n	n	n	n	n	n												

Dual Simplex

Basic	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	sx_{13}	sx_{14}	sx_{15}	sx_{16}	sx_{17}	sx_{18}	sx_{19}	solution
z(min)	-1	0	0	-2.2	-5.2	-2	0	-1	0	-1	0	0	-4	-9	0	-5.5	7.5	-15.5	-9.5	121
x9	1	0	0	0	1	0	0	0	1	0	0	0	0	0	0	-1	0	0	0	7.5
x12	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	0	0	0	-1	2.5
sx15	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0
x2	0	1	0	0	0	1	0	0	0	1	0	0	0	0	0	0	-1	0	0	5.5
x3	1	0	1	1	0	-1	0	1	0	-1	0	0	1	0	0	0	1	0	0	1
x7	0	0	0	0	1	1	1	-1	0	0	0	0	0	1	0	0	0	0	0	1.5
x11	-1	0	0	-1	-1	0	0	0	0	1	1	0	-1	-1	0	0	-1	-1	0	1
Lower Bound	0	0	0	0	0	0	0	0	0	0	0	0								
Upper Bound	infinity	infinity	Infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity								
Unrestr,d(y/n)?	n	n	N	n	n	n	n	n	n	n	n	n								