Available online at www.internationaleJournals.com

International eJournals

International eJournal of Mathematics and Engineering 172 (2012) 1624 – 1634

INTERNATIONAL eJOURNAL OF MATHEMATICS AND ENGINEERING

www.InternationaleJournals.com

Uniform Steady Heat and Moisture Flow in Unsymmetric Laminate

[1] M.Niranjan and [2] D.Rama Murthy

[1]Department of Mathematics, Vasundara Women's Degree college, Plot No.B-8/1, Hi-Tech Electronic Building, Opp: Round Building, ECIL 'X' Roads, Hyderabad – 500 062, India. Email: niruvasundara@yahoo.co.in

[2] Professor of Mathematics(Retd.), Osmania University, Hyderabad – 500007, A.P. India. Email: drmurthy2k1@yahoo.co.in

Authors Personal Copy

ABSTRACT

The study of hygrothermal stress analysis in unsymmetric laminates is important for practical engineering design. Ting.T.C.T, Hsich.M.C and Hwu.C have solved some problems concerning to general composite laminates. Here in this paper we have considered the problem of determination of general solutions for hygrothermal stresses in unsymmetric laminates subjected to uniform heat flow and moisture transfer in the $x_1 - x_2$ plane.

2010 Mathematics subject classification 73.

Keywords and Phrases: Unsymmetric Laminate, Hygrothermal stresses, Heat flux, Anisotropic elastic material, Moisture, Temperature.

1. INTRODUCTION:

Composite laminates are increasingly being used not only in traditional areas like Aerospace, but also in many engineering applications. Some of these applications are the structures under Hygrothermal environment. Although the hygrothermal effects on holes in laminates have been widely discussed, due to mathematical difficulties most of the analytical solutions obtained by Savin.G.N. [5], Florence.A.L and Goodier.P.J.N [1], Hwu.C. [3] and by others are for two-dimensional problems or for mechanical loading conditions for isotropic materials, or for special laminates, not for the general composite laminates under hygrothermal environment. For a unidirectional lamina the coefficients of thermal and moisture expansion, like its other properties, change with direction. Thus the hygrothermal changes result in unequal strains in the longitudinal and transverse directions. Hygrothermal strains do not produce a resultant force or moment when the body is completely free to expand, bend and twist. However, for a composite laminate each individual laminate is not completely force to deform.

The laminate stresses are therefore induced by the constraints placed on its deformation by adjacent lamina. Like the cases of mechanical loading, the existence of holes in laminates will cause high stress concentration around holes under hygrothermal environment. Moreover, the unsymmetry of laminates will cause coupling between stretching and bending, which may complicate the analysis. The study of hygrothermal stress analysis in unsymmetric laminates becomes important for practical engineering design.

Ting.T.C.T [6], Hsich.M.C and Hwu.C [2] solved the hole problems in general composite laminates. Here in this chapter we discuss the problem of determination of general solutions for hygrothermal stresses in unsymmetric laminates disturbed by an elliptical hole subjected to uniform heat flow and moisture transfer in the $x_1 - x_2$ plane.

2. GOVERNING EQUATIONS:

In a fixed rectangular coordinate system x_i, i = 1, 2, 3, let U_i, σ_{ij}, e_{ij}, T, H, q_i and m_i be respectively, displacement, stress, strain, change in temperature, change in moisture content, heat flux and moisture transfer. If the coupling terms between the elastic deformation, heat conduction and moisture transport are neglected, the heat conduction, the moisture diffusion, the strain displacement relation, the constitutive law, the force, heat and moisture equilibrium equations for linear anisotropic elastic materials under static loading and small deformation conditions as given by Nowacki.W [4] are as follows:

$$q_i = -K_{ij}^{t} T_{,j} \qquad (1)$$

$$m_i = -K_{ij}^{h} H_{,j} \qquad (2)$$

$$e_{ij} = \frac{1}{2}\left(U_{i,j} + U_{j,i}\right) \qquad (3)$$

$$\sigma_{ij} = C_{ijks}\, e_{ks} - C_{ijks}\, \alpha_{ks}^{t}\, T - C_{ijks}\, \alpha_{ks}^{h}\, H \qquad (4)$$

$$\sigma_{ij,j} = 0 \qquad (5)$$

$$q_{i,j} = 0 \qquad (6)$$

Author's Personal Copy

$$m_{i,j} = 0 \tag{7}$$

i, j, k, s = 1, 2, 3

Where repeated indices imply summation; C_{ijks}, k^t_{ij}, k^h_{ij}, α^t_{ij} and α^h_{ij} are respectively the elastic constants, heat conduction coefficients, moisture diffusion coefficients, coefficients of thermal expansion and coefficients of moisture expansion. C_{ijks} are assumed to be fully symmetric, i.e.,

$$C_{ijks} = C_{jiks} = C_{ijsk} = C_{ksij} \tag{8}$$

and are positive definite due to the positiveness of strain energy.

K^t_{ij}, K^h_{ij}, α^t_{ij} *and* α^h_{ij} are also assumed to be symmetric. Equations (1) to (7) constitutes 23 partial differential equations interms of three coordinate variables x_i, i = 1, 2, 3. If the deformations are considered to be dependent upon two coordinate variables x_1 and x_2 only, a general solution can be found.

Here we consider a composite laminate composed of layers of various materials. Each layer is assumed to be made of anisotropic materials. If the laminate thickness is smaller than its other dimensions, we can assume the displacement, temperature and moisture content may be assumed to vary linearly through laminate thickness as,

$$U_i(x_1, x_2, x_3) = U_i(x_1, x_2) + x_3\ \beta_i(x_1, x_2)\quad i = 1, 2$$

$$U_3(x_1, x_2, x_3) = W(x_1, x_2) \tag{9}$$

$$T(x_1, x_2, x_3) = T^o(x_1, x_2) + x_3\ T^*(x_1, x_2) \tag{10}$$

$$H(x_1, x_2, x_3) = H^o(x_1, x_2) + x_3\ H^*(x_1, x_2) \tag{11}$$

Where,

Author's Personal Copy

$$\beta_1 = -W_{,1} \quad \beta_2 = -W_{,2} \tag{12}$$

U_1, U_2, W, T^o and H^o are the middle surface displacements, temperature and moisture content and β_i, $i = 1, 2$ are the negative of the slope of the middle surface in the x_1 and x_2 directions. T^* and H^* are the rates of changes in temperature and moisture content.

Based on the assumptions given above and the 23 basic equations for anisotropic materials under hygrothermal condi-tion, we write down the kinematic relations, the constitutive laws and the equilibrium equations for hygrothermal stress analysis of composite laminates as follows:

$$\tilde{q}_i = -K_{ij}^t T,_j^o - K_{ij}^{*t} T,_j^* - K_{i3}^t T^* \tag{13}$$

$$\widetilde{m}_i = -K_{ij}^h H,_j^o - K_{ij}^{*h} H,_j^* - K_{i3}^h H^* \tag{14}$$

$$e_{ij} = \frac{1}{2}\left(U_{i,j} + U_{j,i}\right) \tag{15}$$

$$K_{ij} = \frac{1}{2}\left(\beta_{i,j} + \beta_{j,i}\right) \tag{16}$$

$$N_{ij} = A_{ijkl} \in_{kl} + B_{ijkl} K_{kl} - A_{ij}^t T^o - A_{ij}^h H^o - B_{ij}^t T^* - B_{ij}^h H^* \tag{17}$$

$$M_{ij} = B_{ijkl} \in_{kl} + D_{ijkl} K_{ij} - D_{ij}^t T^o - D_{ij}^h H^o - D_{ij}^t T^o - D_{ij}^h H^* \tag{18}$$

$$N_{ij,j} = 0 \tag{19}$$

$$M_{ij,ij} + P = 0 \tag{20}$$

Author's Personal Copy

$$Q_i = M_{ij,j} \tag{21}$$

$$\tilde{q}_{i,j} + q = 0 \tag{22}$$

$$\widetilde{m}_{i,j} + m = 0, \; i, j, k, l = 1, 2 \tag{23}$$

Where $\in_{ij}$ and k_{ij} denote the midplane strain and plane curvature. N_{ij}, M^{ij} and Q_i denote the stress resultants, bending moments and shear forces. $\tilde{q}_i$ and $\widetilde{m}_i$ denote the heat flux resultant and moisture transfer resultant. A_{ijkl}, B_{ijkl} and D_{ijkl} are respectively the extensional, coupling and bending stiffness tensors. A^t_{ij}, B^t_{ij}, D^t_{ij} and A^h_{ij}, B^h_{ij}, D^h_{ij} are the corresponding tensors for the thermal and moisture expansion coefficients. K^t_{ij}, K^h_{ij} and K^{*t}_{ij}, K^{*h}_{ij} are the coefficients related to the heat conduction and moisture diffusion coefficients. P, q. and m are the lateral distributed load, heat flux and moisture concentration transfer applied on the laminates. Above quantities are defined as,

$$N_{ij} = \int_{-\frac{h}{2}}^{h/2} \sigma_{ij} \, dx_3 \tag{24}$$

$$M_{ij} = \int_{-\frac{h}{2}}^{h/2} \sigma_{ij} \, x_3 \, dx_3 \tag{25}$$

$$Q_i = \int_{-\frac{h}{2}}^{h/2} \sigma_{i3} \, dx_3 \tag{26}$$

$$\tilde{q}_i = \int_{-\frac{h}{2}}^{h/2} q_i \, dx_3, \quad \widetilde{m}_i = \int_{-\frac{h}{2}}^{h/2} m_i \, dx_3 \tag{27}$$

Authors Personal Copy

$$A_{ijks} = \int_{-\frac{h}{2}}^{h/2} C_{ijks}\ dx_3 \tag{28}$$

$$B_{ijks} = \int_{-\frac{h}{2}}^{h/2} C_{ijks}\ x_3\ dx_3 \tag{29}$$

Author's Personal Copy

$$D_{ijks} = \int_{-\frac{h}{2}}^{h/2} C_{ijks}\ x_3^2\ dx_3 \tag{30}$$

$$A_{ij}^{t} = \int_{-\frac{h}{2}}^{h/2} C_{ijks}\ \alpha_{ks}^{t}\ dx_3 \tag{31}$$

$$B_{ij}^{t} = \int_{-\frac{h}{2}}^{h/2} C_{ijks}\ \alpha_{ks}^{t}\ x_3\ dx_3 \tag{32}$$

$$D_{ij}^{t} = \int_{-\frac{h}{2}}^{h/2} C_{ijks}\ \alpha_{ks}^{t}\ x_3^2\ dx_3 \tag{33}$$

$$A_{ij}^{h} = \int_{-\frac{h}{2}}^{h/2} C_{ijks}\ \alpha_{ks}^{h}\ dx_3 \tag{34}$$

$$B_{ij}^{h} = \int_{-\frac{h}{2}}^{h/2} C_{ijks}\ \alpha_{ks}^{h}\ x_3\ dx_3 \qquad (35)$$

$$D_{ij}^{h} = \int_{-\frac{h}{2}}^{h/2} C_{ijks}\ \alpha_{ks}^{h}\ x_3^2\ dx_3 \qquad (36)$$

$$K_{ij}^{t} = \int_{-\frac{h}{2}}^{h/2} K_{ij}^{t}\ dx_3 \qquad (37)$$

$$K_{ij}^{*t} = \int_{-\frac{h}{2}}^{h/2} K_{ij}^{t}\ x_3\ dx_3 \qquad (38)$$

$$K_{ij}^{h} = \int_{-\frac{h}{2}}^{h/2} K_{ij}^{h}\ dx_3 \qquad (39)$$

$$K_{ij}^{*h} = \int_{-\frac{h}{2}}^{h/2} K_{ij}^{h}\ x_3\ dx_3 \qquad (40)$$

in which h is laminate thickness. Since the above basic equations are quite general, it is not easy to find a solution satisfying all these basic equations. Here we consider the case that temperature and moisture distributions depend on x_1 and x_2 only; which occur frequently in engineering applications.

Author's Personal Copy

3. Solutions of the problem when the temperature and moisture content depend on x_1 and x_2 only.

If the temperature and moisture content are assumed to depend on x_1 and x_2 only and the lateral distributed load, heat flux, and moisture concentration transfer applied on the laminates are neglected. i.e. $T^* = H^* = P = q = m = 0$, the basic equations can be simplified as,

$$\tilde{q}_{i,j} = -K^t_{ij}\, T,_{ij} = 0 \tag{41}$$

$$\tilde{m}_{i,j} = -K^h_{ij}\, H,_{ij} = 0 \tag{42}$$

$$N_{ij,j} = A_{ijkl}\, U_{k,ij} + B_{ijkl}\, \beta_{k,ij} - A^t_{ij}\, T,_j - A^h_{ij}\, H,_j = 0 \tag{43}$$

$$M_{ij,ij} = B_{ijkl}\, U_{k,lij} + D_{ijkl}\, \beta_{k,lij} - B^t_{ij}\, T,_{ij} - B^h_{ij}\, H,_{ij} = 0 \quad i, j, k, l = 1, 2 \tag{44}$$

Following the procedure as given by Hsich.M.C and Hwu.C [2], the general solution satisfying the above equations is,

$$T = 2.\mathrm{Re}\left[g'_i(z_i)\right] \tag{45}$$

$$H = 2.\mathrm{Re}\left[g'_h(z_i)\right] \tag{46}$$

$$\tilde{q}_i = -2\,\mathrm{Re}\left[\left(K^t_{i1} + \tau_t\, K^t_{12}\right) g''_i(z_i)\right] \tag{47}$$

$$\tilde{m}_i = -2\,\mathrm{Re}\left[\left(K^h_{i1} + \tau_h\, K^h_{i2}\right) g''_h(z_h)\right] \tag{48}$$

$$U_d = 2\,\mathrm{Re}\left[Af(z) + C_t g_i(Z_t) + C_h g_h(Z_h)\right] \tag{49}$$

Author's Personal Copy

$$\varnothing_d = 2\,\mathrm{Re}\left[Bf(z) + d_t g_i(Z_i) + d_h g_h(Z_h) \right] \tag{50}$$

Where

$$U_d = \begin{bmatrix} U \\ \beta \end{bmatrix},\ \varnothing_d = \begin{bmatrix} \varnothing \\ \psi \end{bmatrix},\ U = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix},$$

$$\beta = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix},\ \varnothing = \begin{bmatrix} \varnothing_1 \\ \varnothing_2 \end{bmatrix},\ \psi = \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix},$$

$$A = \left[a_1\ a_2\ a_3\ a_4 \right],\ B = \left[b_1\ b_2\ b_3\ b_4 \right]$$

$$f(z) = \begin{bmatrix} f_1(z_1) \\ f_2(z_2) \\ f_3(z_3) \\ f_4(z_4) \end{bmatrix},\ Z_k = x_1 + \mu_k\, x_2,\ k=1, 2, 3, 4$$

$$Z_t = x_1 + \tau_t\, x_2,\ Z_h = x_1 + \tau_h x_2 \tag{51}$$

Here Re stands for the real part of a complex number and prime denotes differentiation with respect to its argument. U_d and $\varnothing_d$ are the generalized displacement and stress function vectors.

$\varnothing_1$, $\varnothing_2$ and ψ_1, ψ_2 are the stress functions related to the stress resultants N_{ij}, Shear forces Q_i, effective shear forces V_i and bending moments M_{ij} by

$$N_{ij} = -\varnothing_{i,2}, \quad N_{i2} = \varnothing_{i,1}$$

$$M_{i1} = -\psi_{i,2},\ -\lambda_{i1}\eta$$

$$M_{i2} = -\psi_{i,1}\ -\lambda_{i2}\eta$$

$$Q_1 = -\eta_{,2},\ Q_2 = \eta_{,1} \tag{52}$$

Author's Personal Copy

$$V_1 = -\psi_{2,22}, \; V_2 = \psi_{1,11}$$

Where,

$$\eta = \frac{1}{2}\psi_{k,k} = \frac{1}{2}\left(\psi_{1,1} + \psi_{2,2}\right)$$

and λ_{ij} is the permutation tensor defined as,

$$\lambda_{11} = \lambda_{22} = 0, \; \lambda_{12} = -\lambda_{21} = 1$$

$$f_k\left(Z_k\right), K=1,2,3,4; \quad g_t\left(Z_t\right) \text{ and } g_h\left(Z_h\right)$$

are six holomorphic functions of complex variables Z_k, Z_t and Z_h which will be determined by the boundary conditions set for each particular problem. μ_k, τ_t, τ_h and (a_k, b_k), (c_t, d_t), (c_h, d_h) are respectively, the material eigenvalues and eigenvectors which can be determined by the following eigen relations.

$$K_{11}^t + 2\tau_t \, K_{12}^t + \tau_t^2 \, K_{22}^t = 0 \qquad (53)$$

$$K_{11}^h + 2\tau_h \, K_{12}^h + \tau_h^2 \, K_{22}^h = 0 \qquad (54)$$

$N\xi = \mu\xi$

$$N\eta_t = \tau_t \eta_t + Y_t \qquad (55)$$

$$N\eta_h = \tau_h \eta_h + Y_h \qquad (56)$$

Where N is $a\left(8 \times 8\right)$ real matrix which is the fundamental elasticity matrix for coupled stretching – bending analysis. ξ, η_t and η_h are $\left(8 \times 1\right)$ complex vectors which are composed of the material eigen vectors.

Author's Personal Copy

REFERENCES

1. Florence.A.L. and Goodier P.J.N., "Thermal stress at Spherical cavities and circular Holes in Uniform Heat Flow", ASME, J.Appl.Mech, 26, 1953 PP.293-294.

2. Hsich.M.C and Hwu.C., "Explicit Expressions of the Fundamental elasticity Matrices of Stroh-like formalism for Symmetric laminates" Chin.J.Mech.Ser A, 18, 2002, PP.109-118.

3. Hwu.C, "Thermal Stresses in an Anisotropic Plate Disturbed by an Insulated Elliptic Hole or Crack", ASME.J.Appl. Mech, 57, 1990, PP.916-922.

4. Nowacki.W, "Thermoelasticity" Pergaman Press Ltd., Oxford, 1962.

5. Savin.G.N, "Stress concentration Around Holes", Pergamon press.

6. Ting.T.C.T, "Anisotropic elasticity – Theory and Applications", Oxford Science Publications, New York, 1996.

Author's Personal Copy