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MHD AND CHEMICAL REACTION EFFECTS ON UNSTEADY FLOW PAST AN ACCELERATED ISOTHERMAL INFINITE VERTICAL PLATE

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Abstract

An exact solution to the problem of unsteady hydromagnetic flow past a uniformly accelerated isothermal infinite vertical plate with uniform mass diffusion is presented here, taking in to account the homogeneous chemical reaction of first order. The plate temperature is raised to T_w and the concentration level near the plate is also raised to $C_w^{/}$. The dimensionless governing equations are solved using Laplace-transform technique. The velocity, temperature and concentration fields are studied for different physical parameters like thermal Grashof number, mass Grashof number, Schmidt number, Prandtl number, chemical reaction parameter and time. It is observed that the velocity increases with increasing values of thermal Grashof number or mass Grashof number. It is also observed that the velocity increases with decreasing chemical reaction parameter or magnetic field parameter.

Keywords: accelerated, isothermal, vertical plate, heat and mass transfer, chemical reaction, magnetic field.

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1. Introduction

Hydromagnetic convection plays an important role in petroleum industries, geophysics and in astrophysics. It also finds applications in many engineering problems such as magnetohydrodynamic(MHD) generator, plasma studies, in the study of geological formations, in exploration and thermal recovery of oil, and in the assessment of aquifers, geothermal reservoirs and underground nuclear waste storage sites. MHD flow has applications in metrology, solar physics and in the movement of earth's core. It has applications in the field of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics. Free convection effects on flow past an accelerated vertical plate with variable suction and uniform heat flux in the presence of magnetic field was studied by Raptis *et al* [7]. MHD effects on flow past an infinite vertical plate for both the classes of impulse as well as accelerated motion of the plate were studied by Raptis and Singh[6].

Chemical reactions can be codified as either heterogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems, the reaction is heterogeneous, if it takes place at an interface and homogeneous, if it takes place in solution. In most cases of chemical reactions, the reaction rate depends on the concentration of the species itself. A reaction is said to be of first order, if the rate of reaction is directly proportional to the concentration itself. Chambre and Young [1] have analyzed a first order chemical reaction in the neighborhood of a horizontal plate. Das et al[2] have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Again, mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction studied by Das et al[3]. The dimensionless governing equations were solved by the usual Laplace-transform technique.

Gupta et al [4] studied free convection on flow past an linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method. Kafousias and Raptis[5] extended the above problem to include mass transfer effects subjected to variable suction or injection. Mass transfer effects on flow past an uniformly accelerated vertical plate was studied by Soundalgekar[9]. Again, mass transfer effects on flow past an accelerated vertical plate with uniform heat flux was analyzed by Singh and Singh[8].

Hence, it is proposed to study the first order chemical reaction effects on unsteady flow past a uniformly accelerated isothermal infinite vertical plate subjected to magnetic field. The dimensionless governing equations are solved using the Laplace-transform technique. The solutions are in terms of exponential and complementary error function. Such a study found useful in chemical process industries such as wire drawing, fibre drawing, polymer production and food processing. The results of the study would provide some useful methods in magnetic control of molten iron flow in the steel industry, liquid metal cooling in nuclear reactors and magnetic suppression of molten semi-conducting materials.

2. Analysis

Here the hydromagnetic flow of a viscous incompressible fluid past a uniformly accelerated isothermal infinite vertical plate with uniform mass diffusion in the presence of chemical reaction of first order has been considered. Here the unsteady flow of a viscous incompressible fluid which is initially at rest and surrounds an infinite vertical plate with temperature T_∞ and concentration C'_∞. The *x*-axis is taken along the plate in the vertically upward direction and the *y*-axis is taken normal to the plate. At time $t' \leq 0$, the plate and fluid are at the same temperature T_∞. At time $t' > 0$, the plate is accelerated with a velocity $u = \frac{u_0^3}{\nu} t'$, in its own plane against gravitational field and the temperature from the plate is raised to T_w and the concentration levels near the plate are also raised to C'_w. It is assumed that the effect of viscous dissipation is negligible in the energy equation and there is a first order chemical reaction between the diffusing species and the fluid. A transverse magnetic field of uniform strength B_0 is assumed to be applied normal to the plate. Then under usual Boussinesq's approximation the unsteady flow is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u \qquad (1)$$

$$\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y^2} - K_l C' \tag{3}$$

With the following initial and boundary conditions:

$$\begin{array}{llllll} & u = 0, & T = T_\infty, & C' = C'_\infty & \text{for all} & y, t' \le 0 \\ t' > 0: & u = \dfrac{u_0^3}{\nu} t', & T = T_w, & C' = C'_w & \text{at} & y = 0 \\ & u \to 0, & T \to T_\infty, & C' \to C'_\infty & \text{as} & y \to \infty \end{array} \tag{4}$$

On introducing the following non-dimensional quantities:

$$U = \frac{u}{u_0}, t = \frac{u_0^2 t'}{\nu}, Y = \frac{y u_0}{\nu}$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, Gr = \frac{g \nu \beta (T_w - T_\infty)}{u_0^3}, C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, Gc = \frac{g \nu \beta^* (C'_w - C'_\infty)}{u_0^3} \tag{5}$$

$$M = \frac{\sigma B_0^2 \nu}{\rho u_0^2}, K = \frac{K_l \nu}{u_0^{\ 2}}, Pr = \frac{\mu C_p}{k}, Sc = \frac{\nu}{D}$$

in equations (1) to (4), leads to

$$\frac{\partial U}{\partial t} = Gr\theta + Gc\, C + \frac{\partial^2 U}{\partial Y^2} - M\, U \tag{6}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr} \frac{\partial^2 \theta}{\partial Y^2} \tag{7}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial Y^2} - KC \tag{8}$$

The initial and boundary conditions in non-dimensional quantities are

$$\begin{array}{llllll} & U = 0, & \theta = 0, & C = 0 & for\ all & Y, t \le 0 \\ t > 0: & U = t, & \theta = 1, & C = 1 & at & Y = 0 \\ & U \to 0, & \theta \to 0, & C \to 0 & as & Y \to \infty \end{array} \tag{9}$$

3. Method of solution

The dimensionless governing equations (6) to (8), subject to the initial and boundary conditions (9), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$\theta = erfc\left(\eta \sqrt{\Pr}\right) \tag{10}$$

$$C = \frac{1}{2}\left[\exp(2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc} + \sqrt{Kt}) + \exp(-2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc} - \sqrt{Kt})\right] \tag{11}$$

$$
\begin{aligned}
U = &\left(\frac{t}{2}+c+d\right)\left[\begin{array}{l}\exp(2\eta\sqrt{Mt})erfc(\eta+\sqrt{Mt}) \\ +\exp(-2\eta\sqrt{Mt})erfc(\eta-\sqrt{Mt})\end{array}\right] \\
&-\frac{\eta\sqrt{t}}{2\sqrt{M}}\left[\exp(-2\eta\sqrt{Mt})erfc(\eta-\sqrt{Mt})-\exp(2\eta\sqrt{Mt})erfc(\eta+\sqrt{Mt})\right] \\
&-2c\,erfc(\eta\sqrt{\Pr})-c\exp(at)\left[\begin{array}{l}\exp(2\eta\sqrt{(M+a)t})erfc(\eta+\sqrt{(M+a)t}) \\ +\exp(-2\eta\sqrt{(M+a)t})erfc(\eta-\sqrt{(M+a)t})\end{array}\right] \\
&-d\exp(bt)\left[\begin{array}{l}\exp(2\eta\sqrt{(M+b)t})erfc(\eta+\sqrt{(M+b)t}) \\ +\exp(-2\eta\sqrt{(M+b)t})erfc(\eta-\sqrt{(M+b)t})\end{array}\right] \\
&+c\exp(at)\left[\exp(2\eta\sqrt{a\Pr t})erfc(\eta\sqrt{\Pr}+\sqrt{at})+\exp(-2\eta\sqrt{a\Pr t})erfc(\eta\sqrt{\Pr}-\sqrt{at})\right] \\
&+d\exp(bt)\left[\begin{array}{l}\exp(2\eta\sqrt{Sc(K+b)t})erfc(\eta\sqrt{Sc}+\sqrt{(K+b)t}) \\ +\exp(-2\eta\sqrt{Sc(K+b)t})erfc(\eta\sqrt{Sc}-\sqrt{(K+b)t})\end{array}\right] \\
&-d\left[\exp(2\eta\sqrt{KtSc})erfc(\eta\sqrt{Sc}+\sqrt{Kt})+\exp(-2\eta\sqrt{KtSc})erfc(\eta\sqrt{Sc}-\sqrt{Kt})\right]
\end{aligned}
\tag{12}
$$

Where, $a=\dfrac{M}{Pr-1}$, $b=\dfrac{M-KSc}{Sc-1}$, $c=\dfrac{Gr}{2a\ (1-Pr)}$, $d=\dfrac{Gc}{2b(1-Sc)}$ and $\eta=\dfrac{Y}{2\sqrt{t}}$.

4. Results and Discussion

For physical interpretation of the problem, numerical computations are carried out for different physical parameters Gr, Gc, Sc, Pr, M, K and t upon the nature of the flow and transport. The value of the Schmidt number Sc is taken to be 0.6 which corresponds to water-vapor. Also, the value of Prandtl number Pr are chosen such that they represent water($Pr = 7.0$). The numerical values of the velocity and concentration are computed for different physical parameters like thermal Grashof number, mass Grashof number, chemical reaction parameter, magnetic field parameter, Schmidt number and time are studied graphically.

Figure 1. demonstrates the effect velocity fields for different values of the chemical reaction parameter $(K = 2,8,15)$, $\mathrm{Gr} = \mathrm{Gc} = 2$, $\mathrm{M} = 2$ and $t = 0.2$. It is observed that the velocity increases with decreasing values of the chemical reaction parameter. The trend shows that there is a fall in velocity due to incraaing vales of the chemical reaction parameter.
Figure 2. illustrates the effects of the magnetic field parameter on the velocity when ($M = 0.2,2,5$), $Gr = Gc = 2, K = 8$ and $t = 0.2$. It was observed that the velocity increases with decreasing values of the magnetic field parameter. This shows that the increase in the magnetic field parameter leads to a fall in the velocity. The result agrees with the expectations, since the magnetic field exerts a retarding force on the free convective flow.

The velocity profiles for different values of the thermal Grashof number($Gr = 2,5$), mass Grashof number ($Gc = 5,10$), $K = 15, M = 2$ and $t = 0.2$ are presented in figure 3. It was observed that the velocity increases with increasing values of the thermal Grashof number or mass Grashof number. The velocity velocity profiles for different values of time

$(t = 0.1, 0.2, 0.3)$, $K = 2, Gr = Gc = 2$ and $\mathrm{M} = 2$ are studied and presented in figure 4. It is observed that the velocity increases with increasing values of the time t.

The effect of concentration profiles for different values of the chemical reaction parameter ($K = 0.2, 2, 5, 10$) and time $t = 0.2$ are presented in figure 5. The effect of the chemical reaction parameter is dominant in concentration field.The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the wall concentration increases with decreasing values of the chemical reaction parameter. It is observed that the concentration increases with decreasing chemical reaction parameter.

5. Conclusion

An exact analysis of hydromagnetic flow past a uniformly accelerated isothermal infinite vertical plate with uniform mass diffusion, in the presence of homogeneous chemical reaction of first order has been studied. The dimensionless governing equations are solved by the usual Laplace-transform technique. The effect of different physical parameters like thermal Grashof number, mass Grashof number, chemical recation parameter, magnetic field parameter and t are studied graphically. It is observed that the velocity increases with increasing values of Gr, Gc and t. But the trend is just reversed with respect to the chemical reaction parameter or magnetic field parameter.

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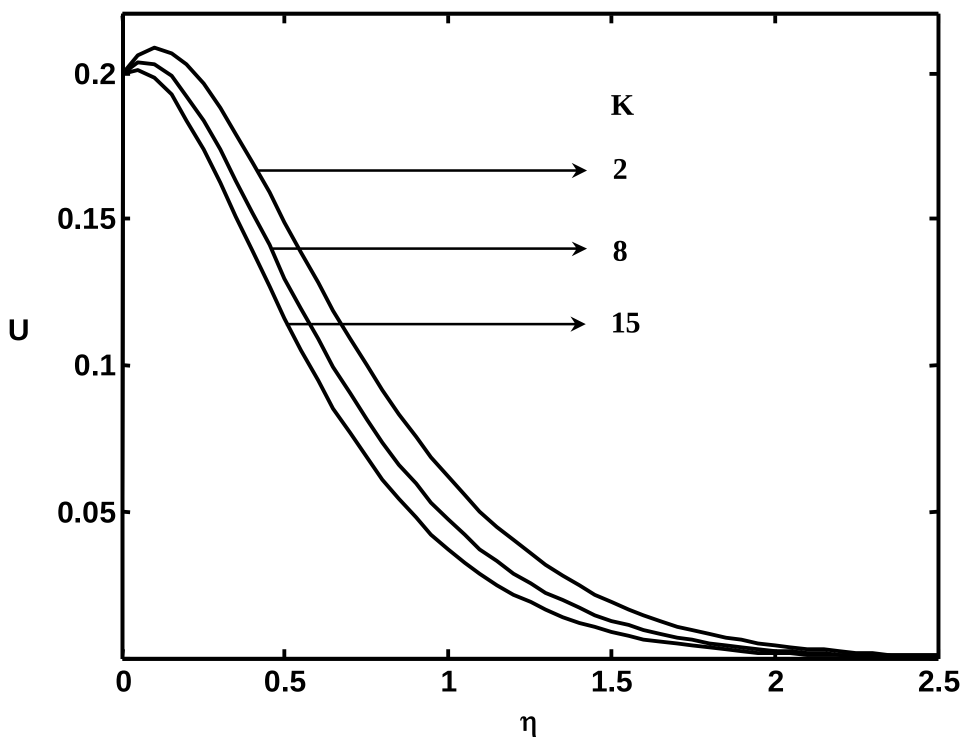

Figure 1. Velocity profiles for different K

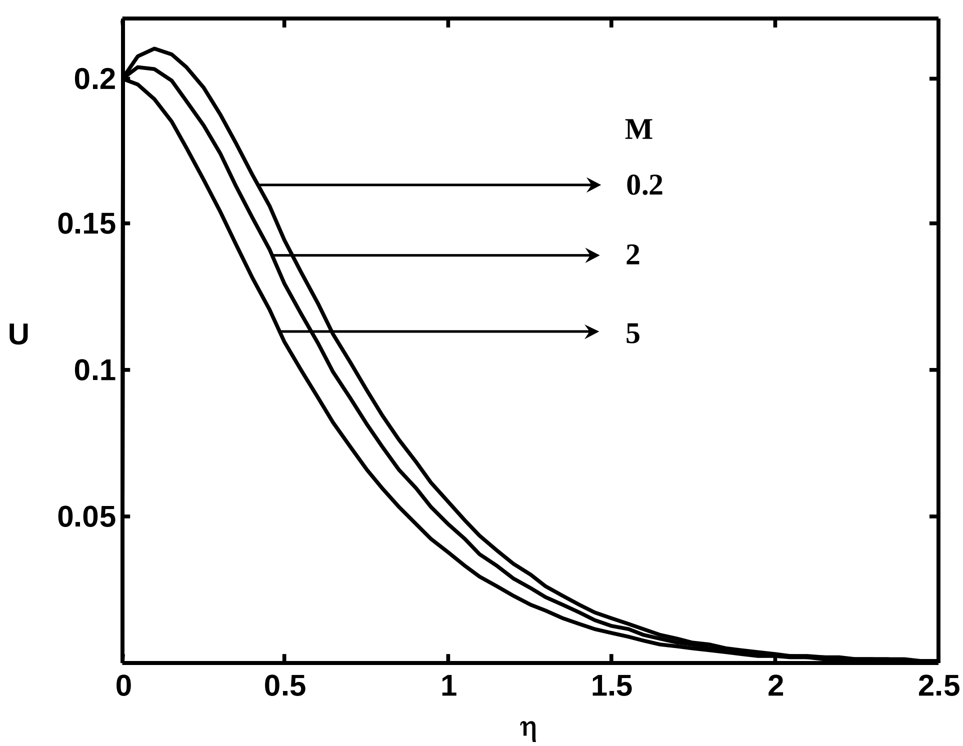

Figure 2. Velocity profiles for different M

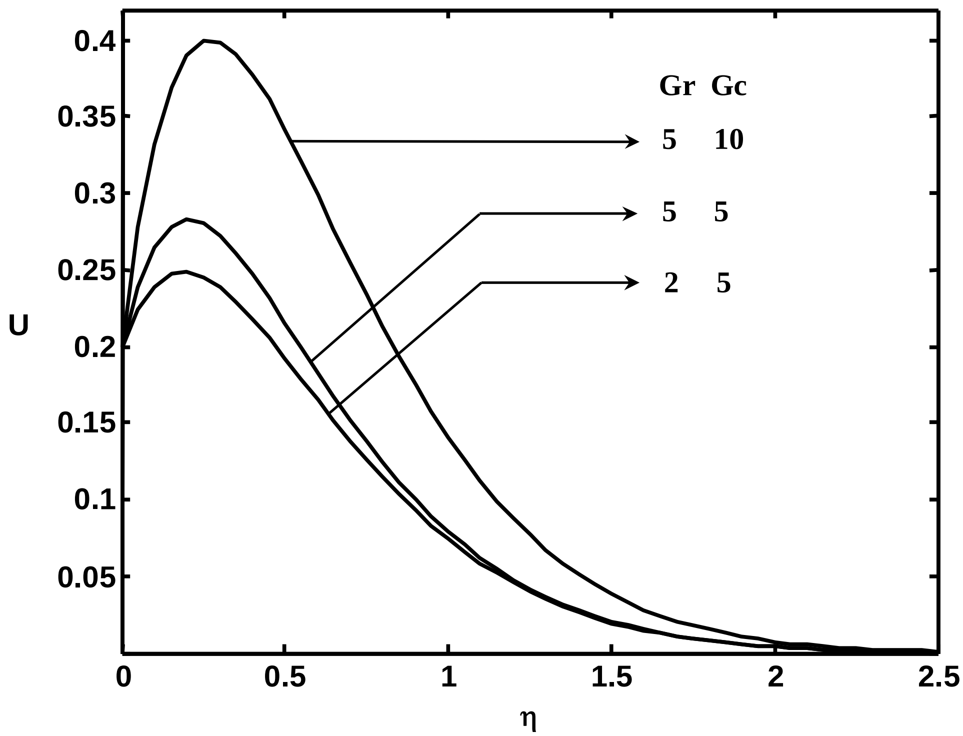

Figure 3. Velocity profiles for different Gr, Gc

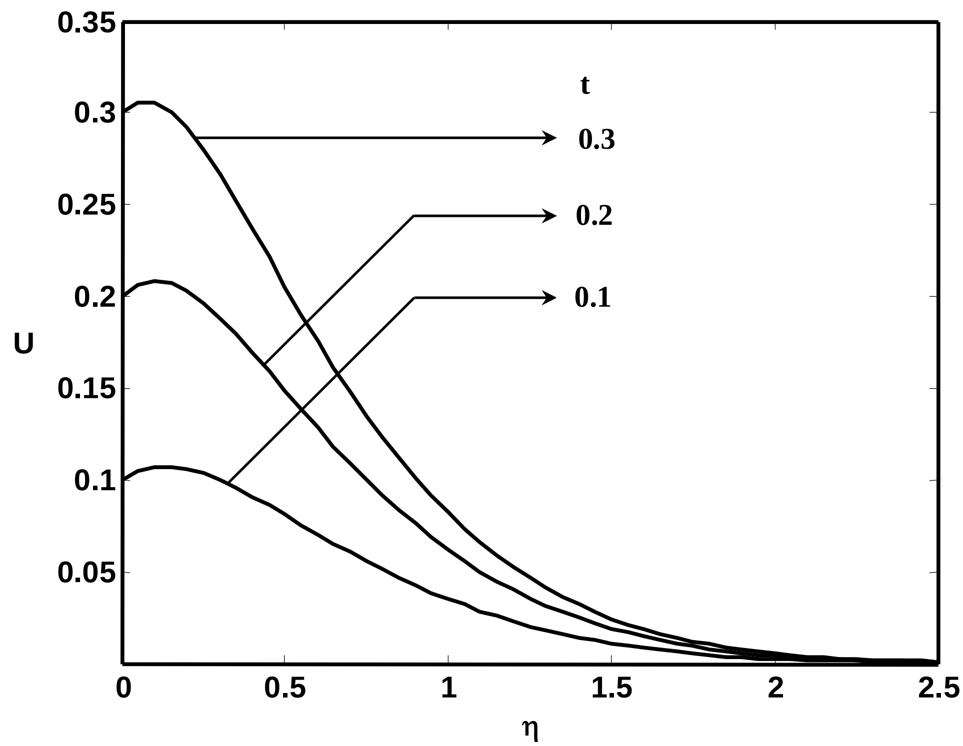

Figure 4. Velocity profiles for different t

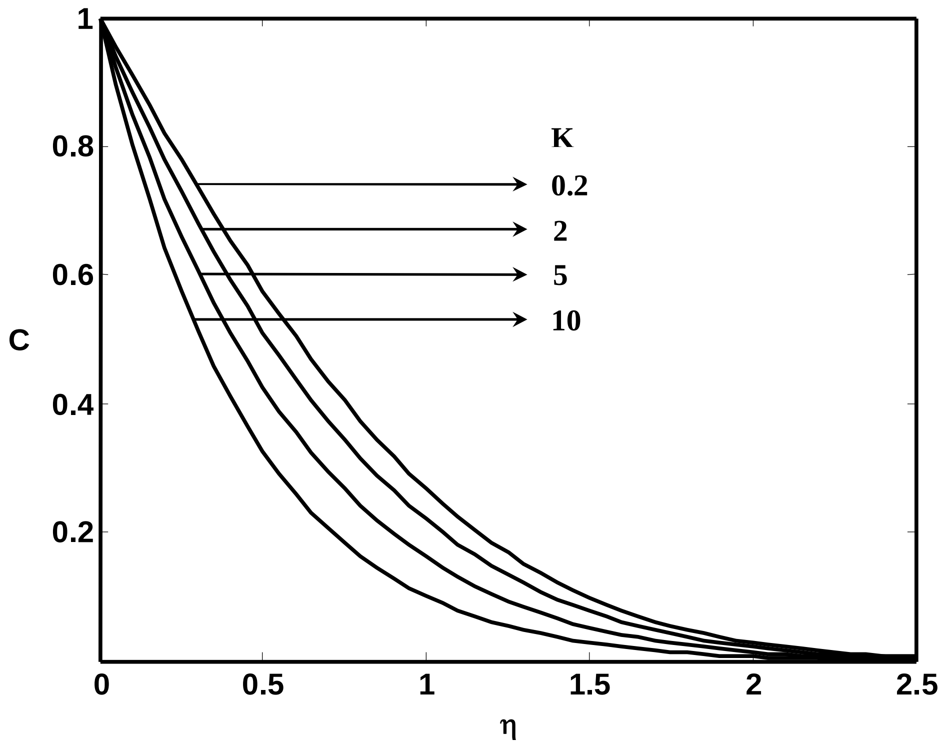

Figure 5. Concentration profiles for different K