ISSN 0976 – 1411

Available online at www.internationaleJournals.com

International eJournals

INTERNATIONAL eJOURNAL OF MATHEMATICS AND ENGINEERING

International eJournal of Mathematics and Engineering 13 (2010) 140-149

www.InternationaleJournals.com

ON THE STABILITY OF AN AMMENSAL-ENEMY SPECIES PAIR WITH UNLIMITED RESOURCES

K.V.L.N.Acharyulu[1] and N.Ch. Pattabhi Ramacharyulu[2]

1. Department of Mathematics, Bapatla Engineering College, Bapatla,A.P.,India.
   E-Mail. kvlna@yahoo.com
2. Former Faculty, Department of Mathematics & Humanities, National Institute of Technology, Warangal – 506004, India

ABSTRACT:

In this paper a mathematical ecological model comprising of two species S_1 and S_2 (enemy) with unlimited resources is investigated. The Ammensal species (S_1), in spite of its natural resources gets adversely effected due to the interaction with the enemy species (S_2).This model is characterized by first order non-linear coupled differential equations. The criteria for its stability of the only existing equilibrium state is discussed .The stability state is obtained only when the death rate is greater than the birth rate of both the species. Comparisons between limited resources and unlimited resources of this model are established.

1) INTRODUCTION

Research in theoretical ecology was initiated by Lotka [11] and by Volterra [16] followed by several mathematicians and ecologists .They contributed their might to the growth of this area of knowledge as reported in the treatises of Meyer [12], Kushing [8], Paul colinvaux [13], Kapur [6,7] etc. The ecological interactions can be brodly classified as Prey – Predation, Competition, Commensalism, Ammensalism, Neutralism and so on. N.C. srinivas [16] studied competitive eco-systems of two and three species with limited and unlimited resources. Later, Lakshminarayan [9], Lakshminarayan and Pattabhi Ramacharyulu [10] studied Prey-predator ecological models with a partial cover for the prey and alternate food for the predator. Recently, some studies on stability analysis of competitive species were carried out by Archana Reddy[3], Pattabhi Ramacharyulu and Gandhi[1] and by Bhaskara Rama Sharma[4,5]., and Pattabhi Ramacharyulu[4].,while Ravindra Reddy[15] investigated mutualism between two species. Phani kumar.N,seshagiriRao.N [14] and Pattabhi Ramacharyulu obtained some results on the stability of a host- a flourishing commensal species pair with limited resources. Acharyulu [1, 2] and Pattabhi Ramacharyulu investigated some results on stability of an enemy and Ammensal species pair with limited resources.

The present investigation relates to an analytical study of Ammensalism between two species. Ammensalism is an ecological relationship between two species where one species (S_1) adversely effects by other species S_2 without being effected by it: S_1 may be referred as the Ammensal species while S_2 the enemy. The following are Some examples of Ammensalism : 1) when penicillum (bread mold), secretes penicillin and kills bacteria, the penicillum does not benefit from killing the bacteria.2) Algal blooms can lead to the death of many species of fish, however the algae do not benefit from the deaths of these individuals.

.The Ammensal species (S_1), in spite of it's natural resources, declines due to the interaction with the enemy species (S_2) which does not effect by S_1. .The model is characterized by a coupled pair of first order non-linear differential equations. The only existing equilibrium point is identified and stability analysis is carried out. It is noticed that the state in which the death rate is greater than the birth rate of both the species' is stable that too under the conditions, stated there in. Fully washed out state and 'the death rate of enemy species is greater than its birth rate' are unstable. The solution curves of linearised perturbed equations and the trajectories are illustrated and conclusions are drawn. The results are compared with the author's earlier work on Ammensal species with limited and/or unlimited resources of the both/either species.

Notation adopted:

N_1, N_2: The populations of the Ammensal (S_1) and enemy (S_2) species respectively at time t
a_1, a_2 : The natural growth rates of S1 and S_2
a_{12} : The Ammensal coefficient.

Further both the variables N_1 and N_2 are non-negative and the model parameters a_1, a_2, and a_{12} are assumed to be non-negative constants. Employing the above terminology, the model equations for a two species Ammensal system are constructed as below.

2) BASIC EQUATIONS

Equation for the growth rate of the Ammensal species (S_1)

$$\frac{dN_1}{dt} = a_1 N_1 - a_{12} N_1 N_2 \tag{2.1}$$

Equation for the growth rate of enemy species (S_2)

$$\frac{dN_2}{dt} = a_2 N_2 \tag{2.2}$$

The system has only one equilibrium state defined by

$$\frac{dN_1}{dt} = 0, \quad \frac{dN_2}{dt} = 0. \tag{2.3}$$

The equilibrium points are obtained as

$$\mathbf{E_1^{(I)}} : \bar{N}_1 = 0; \bar{N}_2 = 0 \quad \text{[Fully washed out state]} \tag{2.4}$$

3). THE STABILITY OF THE EQUILIBRIUM STATES

Now we consider slight deviations U_1 (t) and U_2 (t) over the steady state ($\bar{N}_1, \bar{N}_2$)

$$N_1 = \bar{N}_1 + U_1(t), N_2 = \bar{N}_2 + U_2(t) \tag{3.1}$$

where U_1 (t) and U_2 (t) are so small that the terms other than their first terms can be neglected. Substituting in (2.1) and (2.2) and neglecting products and higher powers of U_1 and U_2 we get

$$\left.\begin{aligned} \frac{dU_1}{dt} &= a_1 U_1 \\ \frac{dU_2}{dt} &= a_2 U_2 \end{aligned}\right\} \tag{3.2}$$

the characteristic Equation for which is $(\lambda - a_1)(\lambda - a_2) = 0$, (3.3)

whose roots a_1, a_2 are both positive. Hence the steady state is **unstable**. By solving (3.2), we get

$$U_1 = U_{10}\, e^{a_1 t} \;;\; U_2 = U_{20}\, e^{a_2 t} \tag{3.4}$$

where U_{10}, U_{20} are initial values of U_1, U_2 respectively.
Showing that both the species increase unboundedly and monotonically.

The solution curves are illustrated in figures (1) to (4) and the conclusions are presented in **TABLE-1.**

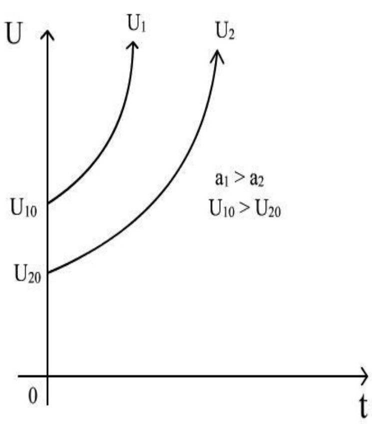

Fig (1)

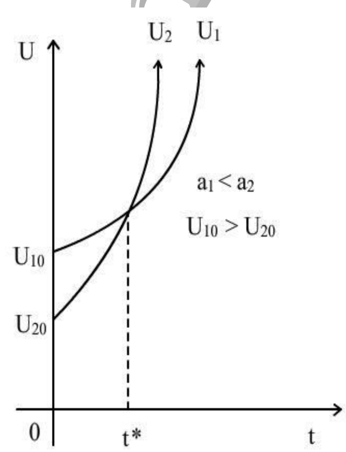

Fig (2)

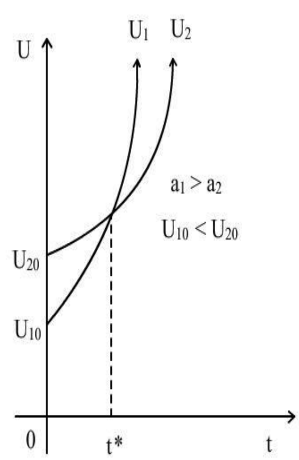

Fig (3)

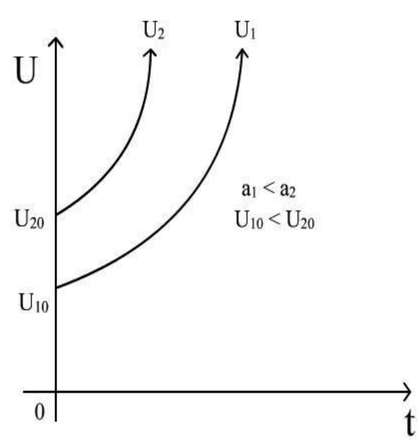

Fig (4)

TABLE- 1

	$a_1 > a_2$	$a_1 < a_2$
$U_{10} > U_{20}$	The Ammensal (N_1) dominates over the enemy (N_2) in natural growth rate as well as its initial population strength.-Fig (1).	The Ammensal (N_1) dominates over the enemy (N_2) in natural growth rate but it's initial strength is less than that of an enemy, and the enemy out numbers the Ammensal till the time-instant $t^* = \frac{1}{a_2 - a_1}\log\left(\frac{U_{10}}{U_{20}}\right)$ after that the dominance is reversed . -Fig (2).
$U_{10} < U_{20}$	The enemy(N_2) dominates over the Ammensal (N_1) in natural growth rate but it's initial strength is less than that of Ammensal and Ammensal out numbers the enemy till the time- instant $t^* = \frac{1}{a_2 - a_1}\log\left(\frac{U_{10}}{U_{20}}\right)$ after that the dominance is reversed.- Fig (3).	The enemy (N_2) dominates over the Ammensal (N_1) all throughout the strength. - Fig (4).

Trajectories of Perturbed Species:

The trajectories (solution curves of 3.2) in the $U_1 - U_2$ plane are given by

$$\left(\frac{U_1}{U_{10}}\right)^{a_2} = \left(\frac{U_2}{U_{20}}\right)^{a_1} \quad \text{and are shown in Fig (5).} \qquad (3.5)$$

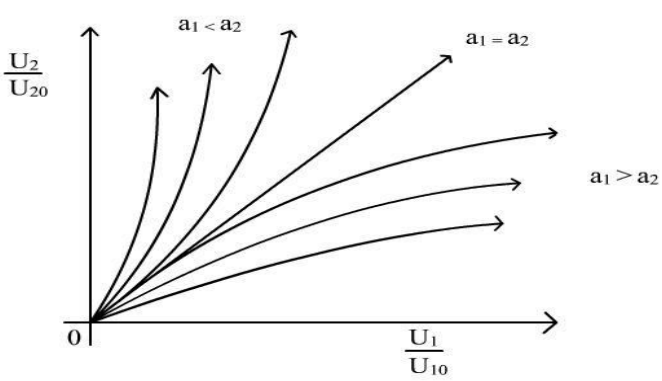

Fig (5)

4)THE DEATH RATE OF ENEMY SPECIES IS GREATER THAN IT'S BIRTH RATE.

Equation for the growth rate of the Ammensal species (S_1)

$$\frac{dN_1}{dt} = a_1 N_1 - a_{12} N_1 N_2 \tag{4.1}$$

Equation for the growth rate of enemy species (S_2)

$$\frac{dN_2}{dt} = -a_2 N_2$$

$\mathbf{E_2}^{(\mathbf{I})}$: $\overline{N}_1 = 0$; $\overline{N}_2 = 0$ [Fully washed out state]

The corresponding linearised perturbed equations are

$$\left.\begin{aligned} \frac{dU_1}{dt} &= a_1 U_1 \\ \frac{dU_2}{dt} &= -a_2 U_2 \end{aligned}\right\} \tag{4.2}$$

The characteristic equation for this system is

$$(\lambda - a_1)(\lambda + a_2) = 0 \tag{4.3}$$

the roots of which are $a_1, -a_2$ and hence the steady state is **unstable.**
By solving (4.2) we get

$$U_1 = U_{10}\, e^{a_1 t};\; U_2 = U_{20}\, e^{-a_2 t} \tag{4.4}$$

The solution curves are illustrated in figures (6) and (7) and the observations are presented in **TABLE- 2**

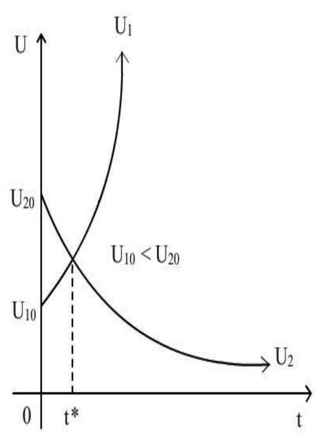

Fig (6)

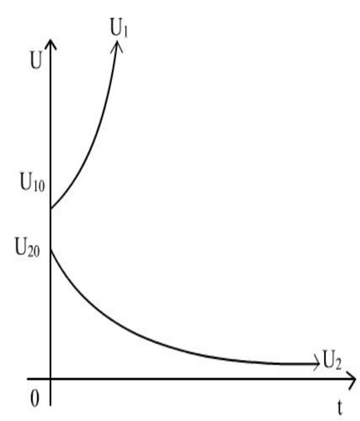

Fig (7)

TABLE – 2

$U_{10} < U_{20}$	The enemy (N_2) while declining, dominates over the the Ammensal (N_1) up to the time -instant t*= $\frac{1}{(a_1+a_2)}\log\left(\frac{U_{20}}{U_{10}}\right)$ there after the Ammensal (N_1) dominates over the enemy (N_2) and the enemy (N_2) declines further.-Fig (6).
$U_{10} > U_{20}$	The Ammensal (N_1) dominates over the the enemy (N_2). The enemy (N_2) declines all throughout.-Fig (7).

Trajectories of Perturbed Species:

The trajectories (solution curves of (4.2)) in $U_1 - U_2$ plane are given by

$$x^{a_2} y^{a_1} = 1 \tag{4.5}$$

where $x = \frac{U_1}{U_{10}}$, $y = \frac{U_2}{U_{20}}$ and are shown in Fig (8)

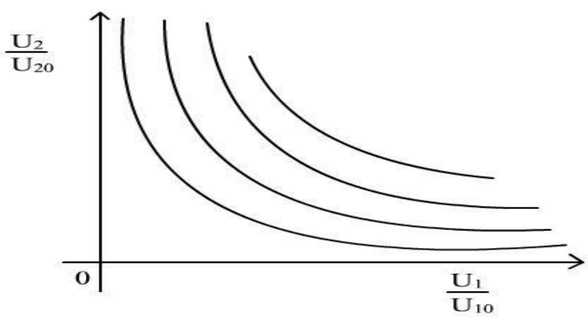

Fig (8)

5) THE DEATH RATE IS GREATER THAN THE BIRTH RATE FOR BOTH SPECIES.

The basic equations are

Equation for the growth rate of the Ammensal species (S_1)

$$\frac{dN_1}{dt} = -a_1 N_1 - a_{12} N_1 N_2 \tag{5.1}$$

Equation for the growth rate of enemy species (S_2)

$$\frac{dN_2}{dt} = -a_2 N_2$$

$E_3^{(I)}$: $\bar{N}_1 = 0,\ \bar{N}_2 = 0$ [Fully washed out state] (5.2)

The corresponding linearised perturbed equations are

$$\left.\begin{aligned} \frac{dU_1}{dt} &= -a_1 U_1 \\ \frac{dU_2}{dt} &= -a_2 U_2 \end{aligned}\right\} \tag{5.3}$$

The characteristic equation for this system is

$$(\lambda + a_1)(\lambda + a_2) = 0 \tag{5.4}$$

the roots of which are $-a_1, -a_2$ i.e. both the roots are negative,

Hence the steady state is **stable**.

By solving (5.3) we get $U_1 = U_{10}\ e^{-a_1 t}$; $U_2 = U_{20}\ e^{-a_2 t}$

The solution curves are illustrated in the following figures. The conclusions are presented in **TABLE- 3**

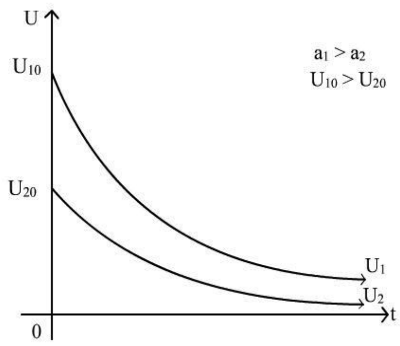

Fig (9)

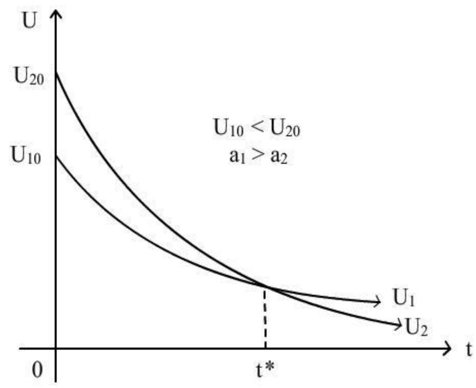

Fig (10)

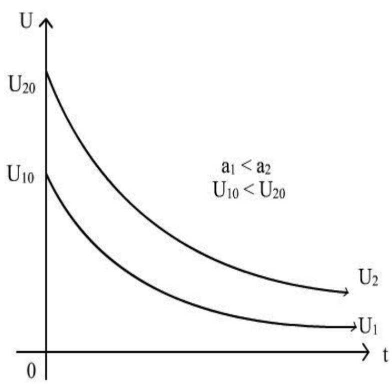

Fig (11)

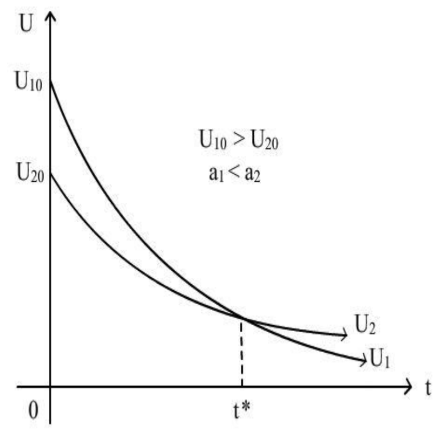

Fig (12)

TABLE- 3

	$a_1 > a_2$	$a_1 < a_2$
$U_{10} > U_{20}$	Both the species decline throughout asymptotically approaching the equilibrium point. However the Ammensal(N_1) dominates over the enemy (N_2). - Fig(9).	The Ammensal (N_1) while declining all throughout, dominates over the enemy(N_2) up to the time - instant t*= $\frac{1}{a_2 - a_1}\log\left(\frac{U_{10}}{U_{20}}\right)$ there after the enemy (N_2) dominates over the Ammensal (N_1) .-Fig(12). .
$U_{10} < U_{20}$	Both N1 and N2 decline. The enemy(N_2) dominates over the Ammensal (N_1) up to the time - instant t*= $\frac{1}{a_2 - a_1}\log\left(\frac{U_{10}}{U_{20}}\right)$ only .- Fig(10).	The enemy (N2) continuous to dominate over the Ammensal (N1) and both the species converge asymptotically to the equilibrium point. -Fig (11).

Trajectories of Perturbed Species:

The trajectories (solution curves of (5.3)) in $U_1 - U_2$ plane are given by

$$x^{a_2} = y^{a_1} \tag{5.5}$$

whee $x = \frac{U_1}{U_{10}}$, $y = \frac{U_2}{U_{20}}$ and are shown in Fig (13)

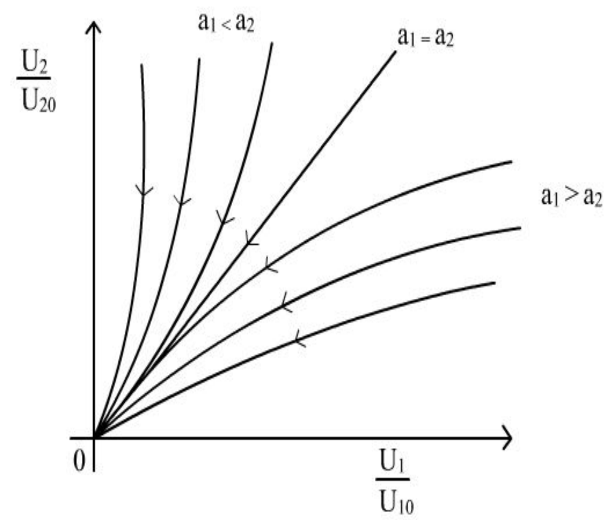

Fig (13)

<u>6) COMPARISIONS BETWEEN THIS MODEL WITH LIMITED AND/OR UNLIMITED RESOURSES OF BOTH/EITHER SPECIES.</u>.

<table>
<tr><th>Equilibrium states</th><th>S_1(LIMITED) S_2 (LIMITED)</th><th>S_1 (LIMITED) S_2 (UNLIMITED)</th><th>S_1 (UNLIMITED) S_2 (LIMITED)</th><th>S_1 (UNLIMITED) S_2 (UNLIMITED)</th></tr>
<tr><td rowspan="2">Fully washed out state [$\bar{N}_1 = 0$; $\bar{N}_2 = 0$]</td><td rowspan="2">unstable</td><td>un stable
(in the case where death rate > birth rate of N_2 species)</td><td>un stable
(in the case where death rate > birth rate of N_2 species)</td><td>un stable
(in the case where death rate > birth rate of N_2 species)</td></tr>
<tr><td>stable
(in the case where death rate > birth rate of the both the species)</td><td>stable
(in the case where death rate > birth rate of both the species)</td><td>stable
(in the case where death rate > birth rate of both the species)</td></tr>
</table>

The enemy survives and the Ammensal is washed out state [$\bar{N}_1 = 0$; $\bar{N}_2 = \frac{a_2}{a_{22}}$]	**unstable** if $\frac{a_1}{a_{12}} > \frac{a_2}{a_{22}}$ **stable** if $\frac{a_1}{a_{12}} < \frac{a_2}{a_{22}}$	Does not exist	**un stable** if $\frac{a_1}{a_{12}} > \frac{a_2}{a_{22}}$ **stable** if $\frac{a_1}{a_{12}} < \frac{a_2}{a_{22}}$	Does not exist
The Ammensal survives and enemy is washed out state. [$\bar{N}_1 = \frac{a_1}{a_{11}}$; $\bar{N}_2 = 0$]	**untable**	**untable** **stable** (in the case where death rate > birth rate of N_2 species)	Does not exist	Does not exist
The co-existent state [$\bar{N}_1 = \frac{a_1 a_{22} - a_2 a_{12}}{a_{11}\, a_{22}}$; $\bar{N}_2 = \frac{a_2}{a_{22}}$]	**stable**	Does not exist	Does not exist	Does not exist

REFERENCES:

[1] Acharyulu. K.V.L.N. & Pattabhi Ramacharyulu. N.Ch., "On the stability of an enemy-Ammensal species pair with limited resources, "International Journal of Applied Mathematical Analysis and Applications" July-Decemder2009, volume 4, No.2, pp.149-161

[2] Acharyulu. K.V.L.N. & Pattabhi Ramacharyulu. N.Ch., "On the stability of an enemy-Ammensal species pair with resources limited for one species and unlimited for the other, "International e Journal of mathematics and Engineering" Volume I, Issue II,pp.1-14;2010

[3] Archana Reddy. R.,On the Stability of Math Models with interacting species in Bio-sciences, Ph.D. Thesis(submitted), J.N.T.U., 2009.

[4] Bhaskara Rama Sharma & Pattabhi Ramacharyulu N.Ch., "Stability Analysis of a TwoSpecies Competitive Eco-System", International Journal of logic based intelligent systems,

[5] Bhaskara Rama Sharma.,Some Mathematical Models in Competitive ecosystems, Ph.D. Thesis(submitted), Dravidian University., 2009.

[6] Kapur. J. N., Mathematical Modellingin Biology and Medicine, AffiliatedEastWest, 1985.

[7] KapurJ.N, MathematicalModelling, Wiley Easter,1985

[8] KushingJ.M., Integro-Differential Equations and Delay Models in Population Dynamics, Lecture Notes in Bio-Mathematics, Vol.20, Springer Verlag, 1977

[9] Lakshmi Narayan K, A Mathematical Study of a Prey-Predator Ecological Model with a partial Cover for the Prey and Alternate Food for the Predator, Ph.D. Thesis, J.N.T.U., 2005.

[10] Lakshmi Narayan K & Pattabhiramacharyulu.N.Ch., "A Prey-Predator Model with Cover for Prey and Alternate Food for the Predator and Time Delay," International Journal of Scientific [11], computing. Vol.1, 2007, pp.7-14.

[11] Lotka A.J., Elements of Physical Biology, Williams & Wilking, Baltimore, 1925.

[12] Meyer W.J., Concepts of Mathematical Modeling Mc. Grawhill, 1985.

[13] Paul Colinvaux A., Ecology, John Wiley, New York, 1986.

[14] Phani kumar N., Seshagiri rao N., & pattami Ramacharyulu N.Ch., "On the stability of a host- a flourishing commensal species pair with limited resources", International Journal of logic based intelligent systems, Vol.3,No.1.January- June 2009.

[15] Ravindra Reddy.B., "A Study on Mathematical Models of ecological Mutualism between two interacting species". Ph.D., Thesis, O.U., 2008.

[16] Srinivas N.C., "Some Mathematical Aspects of Modeling in Bio-medical Sciences" Ph.D Thesis, Kakatiya University, 1991

[17] Volterra V., Leconssen La Theorie Mathematique De La Leitte Pou Lavie, Gauthier-Villars, Paris, 1931.