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UNIFORM AND NON-UNIFORM FLOW OF COMMON AXIS CYLINDERS

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ABSTRACT:

Utilizing the uniform and non-uniform fluid flow prospective, the present research work ascertain the stream line, axial velocity, twisting velocity, stream function, complex potential function, velocity potential function, etc. To devise the research work more explicit and elaborative, fluid flow is drawn from the common axis cylinders by using the porous media. The 'Navier – Stokes Equations of Motion' and 'Equation of Continuity' are used for final formulation with some boundary conditions.

KEYWORDS: Navier-Stokes equation, Common axis cylinder, Velocity potential function, Twisting velocity, Stream function.

METHODOLOGY:

we know that the Equation of Continuity is

$$\frac{\partial V}{\partial r}+\frac{V}{r}+\frac{1}{r}\frac{\partial \omega}{\partial \phi}=0 \quad \ldots\ldots \text{(a)}$$

And we know that the governing Navier-stokes equations of motion ,

$$\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial r}+\frac{\omega}{r}\frac{\partial V}{\partial \phi}-\frac{\omega^2}{r}=-\frac{1}{\rho}\frac{\partial P}{\partial r}+V\left[\frac{\partial^2 V}{\partial r^2}+\frac{1}{r}\frac{\partial V}{\partial r}-\frac{V}{r^2}+\frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2}-\frac{2}{r^2}\frac{\partial \omega}{\partial \phi}\right] \ldots\ldots \text{(b)}$$

$$\frac{\partial\omega}{\partial t}+\frac{\partial\omega}{\partial r}+\frac{\omega}{r}\frac{\partial\omega}{\partial\phi}-V\frac{\omega}{r}=\frac{1}{\rho r}\frac{\partial P}{\partial\phi}+V\left[\frac{\partial^2\omega}{\partial r^2}+\frac{1}{r}\frac{\partial\omega}{\partial r}-\frac{\omega}{r^2}+\frac{1}{r^2}\frac{\partial^2\omega}{\partial\phi^2}+\frac{2}{r^2}\frac{\partial V}{\partial\phi}\right]\ldots\ldots(c)$$

$$\frac{\partial U}{\partial t}+V\frac{\partial U}{\partial r}+\frac{\omega}{r}\frac{\partial U}{\partial\phi}-\frac{\omega^2}{r}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+V\left[\frac{\partial^2 U}{\partial r^2}+\frac{1}{r}\frac{\partial U}{\partial r}+\frac{1}{r^2}\frac{\partial^2 U}{\partial\phi^2}\right]\ldots\ldots(d)$$

SOLUTION:

For the flow between two common axis porous circular cylinders,

$$\omega=o \ldots\ldots(e)$$

And for rotational symmetry

$$\frac{\partial}{\partial\phi}()=o\ldots\ldots(f)$$

Under these two conditions equation (c) gives an identity and equation (b), (d) & (a) gives:

$$\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial r}=-\frac{1}{\rho}\frac{\partial P}{\partial r}+V\left[\frac{\partial^2 V}{\partial r^2}+\frac{1}{r}\frac{\partial V}{\partial r}-\frac{V}{r^2}\right]\ldots\ldots(g)$$

$$\frac{\partial U}{\partial t}+V\frac{\partial U}{\partial r}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+V\left[\frac{\partial^2 U}{\partial r^2}+\frac{1}{r}\frac{\partial U}{\partial r}\right]\ldots\ldots(h)$$

And $\frac{\partial V}{\partial r}+\frac{V}{r}=0\ldots\ldots(i)$

Integrating equation (i), we get,

$$Vr=a_1 \ldots\ldots(j)$$

The boundary conditions, if

$$r=2, V=2$$

Then from equation (j)

$$2X2=a_1\, Then, a_1=4$$

Now from equation (j),

$$Vr=4\Rightarrow V=\frac{4}{r}\ldots\ldots(k)$$

From equation (g),

$$o+\frac{4}{r}\left(-\frac{1}{r^2}\right)=-\frac{1}{\rho}\frac{\partial P}{\partial r}+\frac{4}{r}\left[4\left(\frac{2}{r^3}\right)-\frac{4}{r^3}-\frac{4}{r^3}\right]\quad -\frac{4}{r^3}=-\frac{1}{\rho}\frac{\partial P}{\partial r}+\frac{4}{r}\left[\frac{8}{r^3}-\frac{8}{r^3}\right]$$

$$\frac{4}{r^3}=\frac{1}{\rho}\frac{\partial P}{\partial r}\ldots\ldots(l)$$

$$r=2,\frac{\partial P}{\partial r}=2$$

Then,

$$\frac{4}{8}=\frac{1}{\rho}X2,\rho=\frac{1}{4}$$

From equation (l),

$\frac{4}{r^3} = 4\frac{\partial P}{\partial r} \Rightarrow \frac{1}{r^3} = \frac{\partial P}{\partial r}$

Integration on both sides

$P = -\frac{1}{2r^2}$ ………………………………………………………………………… (m)

If, $V = 2, P = -\frac{1}{2r^2}$

And U is independent of time then equation (h) reduces to

$\frac{\partial^2 U}{\partial r^2} = 0,$

$\frac{\partial U}{\partial r} = a_2$

$U = a_2 r + a_3$ ………………………………………………………………………… (n)

The boundary conditions are

$r = 1, U = 1$

$r = 2, U = 2$

From equation (n), $2 = 2a_2 + a_3$

$1 = a_2 + a_3$

On solving, we get $a_2 = 1, a_3 = 0$

Putting these from (n), we get $U = r$

For velocity potential, we have

$U = -\frac{\partial \phi}{\partial r}, V = -\frac{1}{r}\frac{\partial \phi}{\partial \theta}$ When, $U = -\frac{\partial \phi}{\partial r}$

$then, r = -\frac{\partial \phi}{\partial r}, Integration$

$\phi = -\frac{r^2}{2}$ ………………………………………………………………………… (o)

And when, $V = -\frac{1}{r}\frac{\partial \phi}{\partial \theta}$

$\frac{1}{r} = -\frac{1}{r}\frac{\partial \phi}{\partial \theta}$ Integration on both sides, we get,

$\phi = -\theta$ ………………………………………………………………………… (p)

For stream function, we have

$\frac{\partial \psi}{\partial r} = -\frac{1}{r}\frac{\partial \phi}{\partial \theta}, \frac{1}{r}\frac{\partial \psi}{\partial \theta} = \frac{\partial \phi}{\partial r}$

$when, \phi = -\theta and \frac{\partial \psi}{\partial r} = -\frac{1}{r}\frac{\partial \phi}{\partial \theta}, weget$

$\frac{\partial \psi}{\partial r} = \frac{1}{r}\frac{\partial \theta}{\partial \theta} \Rightarrow \frac{\partial \psi}{\partial r} = \frac{1}{r} \Rightarrow \psi = \log r$ ……………………………………………… (q)

$$\phi = -\frac{r^2}{2} \, and \, \frac{1}{r}\frac{\partial \psi}{\partial \theta} = \frac{\partial \phi}{\partial r} \, weget$$

$$\frac{1}{r}\frac{\partial \psi}{\partial \theta} = \frac{\partial}{\partial r}\left(-\frac{r^2}{2}\right)$$

$$\frac{1}{r}\frac{\partial \psi}{\partial \theta} = -2r \Rightarrow \psi = -r^2\theta \quad \ldots\ldots (r)$$

For complex potential,

$$Z = \phi + i\psi$$

From equation (o) and (r), we get

$$Z = -\frac{r^2}{2} - i\left(r^2\theta\right)$$

$$Z = -r^2\left[\frac{1}{2} + i\theta\right] \quad \ldots\ldots (s)$$

And from (p) and (q), we get

$$Z = \omega + i\phi$$

$$Z = -\theta + i\log r \quad \ldots\ldots (t)$$

CONCLUSION:

In this paper we have investigated, the azimuthal velocity, cross flow velocity, pressure, axial velocity, velocity potential, stream function, complex potential, stream line fluid flow of common axis cylinder through porous medium.

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