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A MONAD AMMENSALISM WITH RESERVE FOR AMMENSAL SPECIES AND HARVESTING FOR THE BOTH THE SPECIES AT VARIABLE RATES

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Abstract

The paper pertains a mathematical model of monad Ammensalism with reserve for Ammensal species and harvesting for the both the species at variable rates in which two species surviving with limited resources and a variable Ammensal co-efficient. The model comprises first order non-linear coupled differential equations .All four equilibrium points are identified. The criteria for asymptotic stability have been established for each of the four equilibrium points. The solutions of the linearised basic equations are found and their trends are exemplified.

Keywords: Equilibrium points, Normal Steady state, stability, harvesting

AMS Classification: 92 D 25, 92 D 40

1) Introduction

Ecology is divided into auto-ecology (the study of individual species) and synecology (the study of group of species). Synecological studies lead to the concept of the ecosystem. An ecosystem is considered as a unit that includes animals, plants and the physical environment in which they live. As such, ecology may also be considered as the study of distribution and abundance of species under a habitat, availing themselves of the same resources. One of the important roles of mathematicians working in areas such as life, medical and social sciences is to evolve new mathematical techniques dealing with complex situations which would arise in nature. Situations in life sciences are quite complex. As such one should have some insight into a situation before attempting to formulate a new mathematical model.

Mathematical models, properly developed, would provide insight into the relations between physical variables and the process influencing the system under study. The resulting interplay between the experimental investigation and theoretical model would be an essential factor in designing experimentally and in the interpretation of data. Mathematical models, properly developed, would provide insight into the relations between physical variables and the process influencing the system under study. The resulting interplay between the experimental investigation and theoretical model would be an essential factor in designing experimentally and in the interpretation of data. Mathematical modeling thus essentially consists of translating real world problems, solving the mathematical problems and interpreting the solutions in the language of real world

Research in theoretical ecology was initiated by Lotka [8] and Meyer [9] followed by several mathematicians and ecologists .They contributed their might to the growth of this area of knowledge as reported in the treatises of Kapur [6] etc. N.C. srinivas [10] studied competitive eco-systems of two and three species with limited and unlimited resources. Later Lakshminarayan and Pattabhi Ramacharyulu [7] studied Prey-predator ecological models with a partial cover for the prey and alternate food for the predator. Recently, Acharyulu [1-5] and Pattabhi Ramacharyulu investigated some results on stability of an enemy and Ammensal species pair with limited resources.

The present study deals with a mathematical model of monad Ammensalism with reserve for Ammensal species and harvesting for the both the species at variable rates in which two species surviving with limited resources and a variable Ammensal co-efficient, based on Monad model. The growth rate equations of the two species are first order non-linear coupled differential equations .In all, four equilibrium points are identified. They are (i) the fully washed out state. (ii)Ammensal is washed out and the enemy remains. (iii) Enemy is washed out while Ammensal survives and (iv) The co-existent state. The criteria for asymptotic stability have been established for each of the four equilibrium points. The co-existent state is always stable. The solutions of the linearised basic equations are obtained and their trends are illustrated.

Notation adopted:

N_1, N_2 : The populations of the Ammensal (S_1) and enemy (S_2) species respectively at time t
a_1 : Natural growth rate of Ammensal(S_1) Species
a_2 : Natural growth rate of Enemy(S_2) Species
a_{11} : Rate of decrease of the Ammensal(S_1) due to insufficient food.
a_{12} : Rate of increase of the Ammensal(S_1) due to inhibition by the enemy.
a_{22} : Rate of decrease of the enemy(S_2) due to insufficient food.
m_1 : (h_1/a_1) is decrease of S_1 due to harvesting.
m_2 : (h_2/a_2) is decrease of S_2 due to harvesting.
K_i : a_i/a_{ii} are the carrying capacities of N_i, i = 1, 2
α, β : coefficients of Ammensalism.
m :The constant characterized by the cover which is provided for the Ammensal species (0<m<1)

The state variables N_1 and N_2 as well as the model parameters a_1, a_2, a_{11}, a_{12}, a_{22}, m_1,m_2, K_1, K_2, α ,m are assumed to be non-negative constants.

Consider α, β as coefficients of Ammensalism, $\beta \neq 0$ .The interaction would be neutral when any one of them is equal to zero. The Ammensalism is low or high according to β >0 or β <0.

The model equations for an Ammensal-enemy ecological model with a variable Ammensal coefficient are given by the following system of non-linear ordinary differential equations.

2 Basic Equations of the Model:

Equation for the growth rate of Ammensal species (S_1):

$$\frac{dN_1}{dt} = N_1\ [a_{11}((1\text{-}m_1)K_1 - N_1) - F(N_2)] \tag{1}$$

Equation for the growth rate of enemy species (S_2):

$$\frac{dN_2}{dt} = a_{22}N_2\,[(1 - m_2)K_2 - N_2] \tag{2}$$

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In the equation (1) the function $F(N_2)$ is the characteristic of the Ammensalism of N_1 with respect to the enemy N_2

$Lt\ F(N_2)$ with the condition that $F(N_2)$ is bounded for large N_2. A reasonably simple choice of $F(N_2)$ is Monad type given by Kapur[6] :

$F(N_2) = \dfrac{(1-m)\alpha N_2}{\beta + N_2}$ where $\alpha = \underset{N_2\to\infty}{Lt} F(N_2)$, α is a parametric characteristic of Ammensalism. Further β ($\neq 0$) is another parameter signifying the strength of the Ammensalism.If β >0 then it is weak Ammensalism and if β <0 then it is strong Ammensalism.

3 Equilibrium States:

The system under this investigation has four equilibrium states.

E$_1$: Fully washed out state: $\overline{N_1} = 0;\ \overline{N_2} = 0$ (3)

E$_2$: The state in which the Ammensal species($\overline{N_1}$) is washed out while the enemy species($\overline{N_2}$) only survives

: $\overline{N_1} = 0;\ \overline{N_2} = (1\text{-m}_2)K_2$ (4)

E$_3$: The state in which the enemy species($\overline{N_2}$) is washed out while the Ammensal species($\overline{N_1}$) only survives: $\overline{N_1} = (1\text{-m}_1)K_1;\ \overline{N_2} = 0$ (5)

E$_4$: The state in which both the species co-exist.

$$\overline{N_1} = \frac{1}{\alpha_{11}}\left((1\text{-m}_1)K_1\alpha_{11} - \frac{\alpha(1\text{-m}_2)K_2}{\beta+(1\text{-m}_2)K_2}\right);\quad \overline{N_2} = (1\text{-m}_2)K_2 \qquad (6)$$

This state may also be called as the "normal steady state":

and this state exists only when $\left((1-m_1)K_1a_{11} - \dfrac{(1-m_1)(1-m_2)\alpha K_2}{\beta+(1-m_2)K_2}\right) > 0$

4 Stability of Equilibrium States:

The basic equations (1) and (2) are linearised to obtain the equation for the

Perturbed state, $\dfrac{dU}{dt} = AU$ (7)

Where $A = \begin{bmatrix} (1-m_1)K_1a_{11} - 2a_{11}\overline{N_1} - \dfrac{(1-m)\alpha\overline{N_2}}{\beta+\overline{N_2}} & -\dfrac{(1-m)\alpha\overline{N_1}}{\beta+\overline{N_2}} - \dfrac{(1-m)\alpha\overline{N_1}\,\overline{N_2}}{(\beta+\overline{N_2})^2} \\ 0 & (1-m_2)K_2a_{22} - 2a_{22}\overline{N_2} \end{bmatrix}$

here N=(N_1, N_2)= $\overline{N} + U$ with U =(U_1, U_2) is a small perturbation over the equilibrium state $\overline{N} = (\overline{N_1}, \overline{N_2})$, U_{10}, U_{20} are initial values of U_1 and U_2 respectively.

4.1 Stability of the Fully Washed Out State (E$_1$):

i.e. $\overline{N_1} = 0;\ \overline{N_2} = 0$

In this case $A = \begin{bmatrix} (1-m_1)K_1a_{11} & 0 \\ 0 & (1-m_2)K_2a_{22} \end{bmatrix}$ (8)

the characteristic roots of which are K_1a_{11} and K_2a_{22}, these are positive.

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Hence the state is **unstable**. The solution curves are

$$U_1 = U_{10} e^{(1-m_1)K_1 a_{11} t} \text{ and } U_2 = U_{20} e^{(1-m_2)K_2 a_{22} t} \qquad (9)$$

and these curves are illustrated in Fig .1 to Fig .4

Case 1: $(1-m_1)a_1 < (1-m_2)a_2$ **and** $U_{10} < U_{20}$

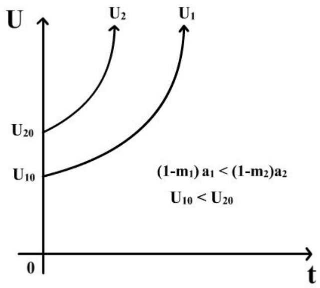

Fig.1

i.e. The enemy(S_2)species dominates the Ammensal(S_1)species in the natural growth rate as well as in its initial population strength. In this case, the enemy species continues to dominate the Ammensal species as shown in Fig .1**.**

case 2: $(1-m_1)a_1 < (1-m_2)a_2$ **and** $U_{10} > U_{20}$

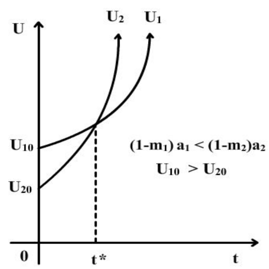

Fig .2

i.e. the enemy(S_2) species dominates the Ammensal (S_1) species in the natural growth rate but its initial strength is less than that of Ammensal species. In this case, the Ammensal species outnumbers the enemy species till the time-instant,

$$t = t^* = \frac{1}{(1-m_2)a_2 - (1-m_1)a_1} \log\left(\frac{U_{10}}{U_{20}}\right).$$

After that the enemy species outnumbers the Ammensal species. This is illustrated in Fig .2**.**

Case 3: $(1-m_1)a_1 > (1-m_2)a_2$ **and** $U_{10} > U_{20}$

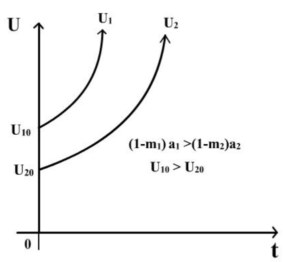

Fig .3

i.e The Ammensal (S_1) dominates over the enemy (S_2) in natural growth rate as well as in its initial population strength. Clearly the Ammensal species continues to be outnumbering the enemy species as shown in Fig .3.

Case 4: $(1-m_1)a_1 > (1-m_2)a_2$ **and** $U_{10} < U_{20}$

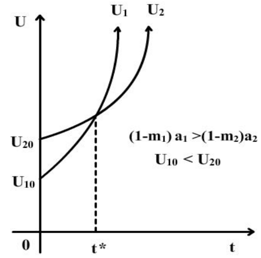

Fig.4

i.e. the Ammensal(S_1)species dominates the enemy(S_2) species in the natural growth rate but its initial strength is less than that of the enemy species. Here the Ammensal outnumbers the enemy till the time- instant

$$\mathbf{t^*} = \frac{1}{(1-m_2)a_2 - (1-m_1)a_1} \log\left(\frac{U_{10}}{U_{20}}\right).$$

After that the dominance is reversed. This is depicted in Fig .4**.**

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4.1A. Trajectories of Perturbed Species:

Further the trajectories (solution curves of (1) and (2)) in the $U_1 - U_2$ plane are given by

$$\left(\frac{U_1}{U_{10}}\right)^{(1-m_2)a_2} = \left(\frac{U_2}{U_{20}}\right)^{(1-m_1)a_1} \qquad (10)$$

And these are illustrated in Fig. 5

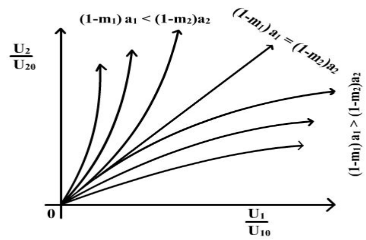


Fig.5

4.2 Stability of the Ammensal Species Washed Out State (E_2):

i.e. $\overline{N_1} = 0;\ \overline{N_2} = (1-m_2)K_2$

In this case we have

$$A = \begin{bmatrix} (1-m_1)K_1a_{11} - \frac{(1-m)\alpha K_2}{\beta+K_2} & 0 \\ 0 & -(1-m_2)K_2a_{22} \end{bmatrix} \qquad (11)$$

the characteristic roots of A are $(1\text{-}m_1)K_1a_{11} - \frac{(1\text{-}m)(1\text{-}m_2)\alpha K_2}{\beta+(1\text{-}m_2)K_2}, -(1\text{-}m_2)K_2a_{22}$

Here two cases would arise

Case (1): **If** $(1\text{-}m_1)K_1a_{11} < \frac{(1\text{-}m)(1\text{-}m_2)\alpha K_2}{\beta + (1\text{-}m_2)K_2}$

Both the roots are negative; hence the steady state is **stable**.

The solution curves: $U_1 = U_{10}e^{\left((1\text{-}m_1)K_1a_{11} - \frac{(1\text{-}m)(1\text{-}m_2)\alpha K_2}{\beta+(1\text{-}m_2)K_2}\right)t}$ and $U_2 = U_{20}\ e^{-(1\text{-}m_2)K_2a_{22}t}$ (12)

are illustrated in Fig .9and Fig .10.

Sub case (i):$U_{10} > U_{20}$.

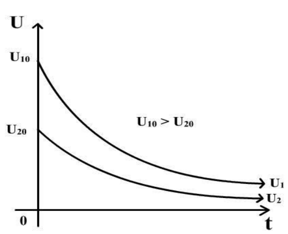


Fig.9

i.e. The Ammensal(S_1) species dominates the enemy (S_2) species in the natural growth rate as well as in its initial population strength.Both the species decline throughout asymptotically approaching the equilibrium point. However the Ammensal dominates over the enemy . (Fig .9)**.**

Sub Case (ii): $U_{10} < U_{20}$

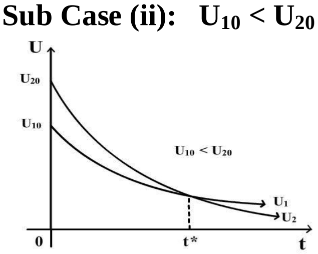


Fig.10

i.e. the Ammensal(S_1) species dominates the enemy (S_2) species in the natural growth rate but its initial strength is less than that of enemy species. Both the species monotonically decline. However the enemy dominates over the Ammensal up to the dominance reversal time-instant

$$t^* = \frac{1}{\left((1\text{-}m_1)a_1 + (1\text{-}m_2)a_2 - \frac{(1\text{-}m)\alpha N_2}{\beta+N_2}\right)} \log\left(\frac{U_{20}}{U_{10}}\right) \quad \text{as}$$

shown in Fig .10**.**

`

4.2A Trajectories of Perturbed species : The trajectories (solutions curves of (12)) in U_1 – U_2 plane are given by

$$\left(\frac{U_1}{U_{10}}\right)\left(\frac{U_2}{U_{20}}\right)^{\frac{-\left[\frac{(1\text{-}m)(1\text{-}m_2)\alpha K_2}{\beta+(1\text{-}m_2)K_2}-(1\text{-}m_1)K_1a_{11}\right]}{(1\text{-}m_2)K_2a_{22}}} = 1; \qquad (13)$$

Let $\frac{(1\text{-}m)(1\text{-}m_2)\alpha K_2}{\beta+(1\text{-}m_2)K_2} - (1\text{-}m_1)K_1a_{11} = M$ and are shown in Fig .11**.**

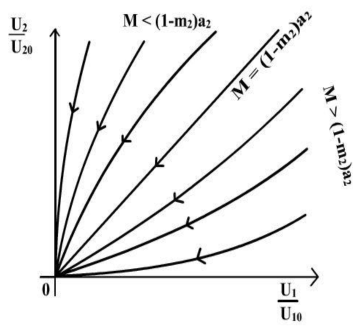


Fig.11

Case (2): if $(1\text{-}m_1)K_1a_{11} > \frac{(1\text{-}m)(1\text{-}m_2)\alpha K_2}{\beta+(1\text{-}m_2)K_2}$

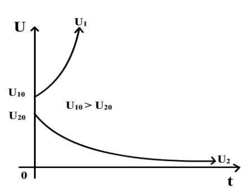
In this case, one of the two roots is positive. Hence the steady state is **unstable.**

We have obtained the solution as

$$U_1 = U_{10}e^{\left((1\text{-}m_1)K_1a_{11} - \frac{(1\text{-}m)(1\text{-}m_1)\alpha K_2}{\beta+(1\text{-}m_2)K_2}\right)t} \text{ and } U_2 = U_{20}e^{-(1\text{-}m_2)K_2a_{22}t} \qquad (14)$$

which are illustrated in Fig.6and Fig.7 with some remarks.

Sub case (i): when $U_{10} > U_{20}$



In this case the Ammensal dominates the enemy in natural growth rate as well as in its initial population strength and the Ammensal species is noted to be going away from the equilibrium point while the enemy species is asymptotic to the equilibrium point as shown in Fig .6**.**

Fig.6

Sub case (ii): when $\mathbf{U_{10} < U_{20}}$

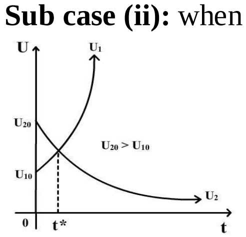


Fig.7

The Ammensal dominates the predator in natural growth rate but its initial strength is less than that of the enemy. Here the enemy outnumbers the Ammensal till the time instant

$$t^* = \frac{1}{((1\text{-}m_1)a_1 + (1\text{-}m_2)a_2 - \frac{(1\text{-}m)\alpha N_2}{\beta + N_2})}\log\left(\frac{U_{20}}{U_{10}}\right)$$ after which the

Ammensal out numbers the enemy. Further the Ammensal species is noted to be going away from the equilibrium point while the enemy species is asymptotic to the equilibrium point as shown in Fig.7.

4.2B Trajectories of Perturbed species:

The trajectories (solutions curves of (14)) in $U_1 - U_2$ plane given

by $$\left(\frac{U_1}{U_{10}}\right)^{\frac{1}{(1-m_1)K_1a_{11}-\frac{(1-m)(1-m_2)\alpha K_2}{\beta+(1-m_2)K_2}}}\left(\frac{U_2}{U_{20}}\right)^{\frac{1}{(1-m_2)K_2a_{22}}}=1; \quad (15)$$

and are shown in Fig.8.

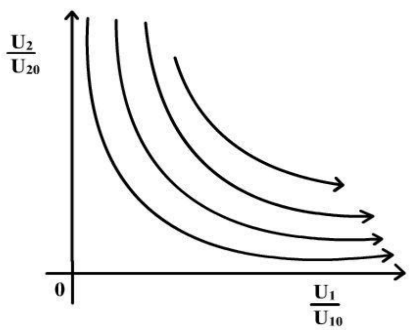


Fig.8

4.3 Stability Of Enemy Species Washed Out State (E_3):

i.e $\overline{N_1}=(1\text{-}m_1)K_1;\ \overline{N_2}=0$

In this case we have $$A=\begin{bmatrix}-(1\text{-}m_1)K_1a_{11} & -\dfrac{(1\text{-}m)(1\text{-}m_1)\alpha K_1}{\beta}\\ 0 & (1\text{-}m_2)K_2a_{22}\end{bmatrix} \quad (16)$$

The characteristic roots of A are $-(1\text{-}m_1)K_1a_{11}$, $(1\text{-}m_2)K_2a_{22}$ and one of these is positive.

Hence the steady state is **unstable**.

The equations (7) yield the solution curves.

$$U_1=\left\{U_{10}+\frac{(1\text{-}m)(1\text{-}m_1)\alpha K_1U_{20}}{\beta\left((1\text{-}m_1)K_1a_{11}+(1\text{-}m_2)K_2a_{22}\right)}\right\}e^{-K_1a_{11}t}-\frac{(1\text{-}m)(1\text{-}m_1)\alpha K_1U_{20}}{\beta\left((1\text{-}m_1)K_1a_{11}+(1\text{-}m_2)K_2a_{22}\right)}e^{K_2a_{22}t} \quad (17)$$

and $$U_2=U_{20}e^{(1\text{-}m_2)K_2a_{22}t} \quad (18)$$

these curves are illustrated in Fig .12 and Fig .13.

Case 1: when $U_{10} < U_{20}$

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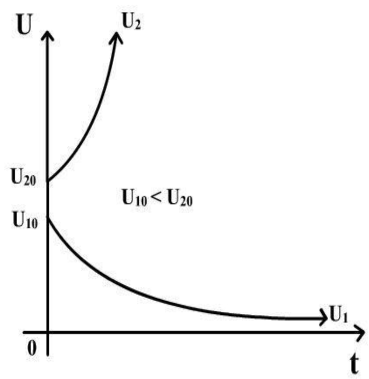

Fig.12

The enemy (S_2) dominates over the Ammensal (S_1) all throughout.We notice that the enemy species is going away from the equilibrium point while the Ammensal species declines all throughout and approaches asymptotically to the equilibrium point Hence the state is **unstable** as shown in Fig .12**.**

Case 2: when $U_{10} > U_{20}$

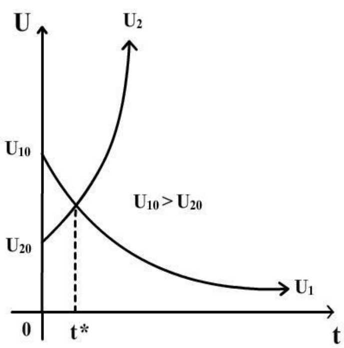

Fig.13

i.e. initially the Ammensal(S_1) species dominates the enemy species.The Ammensal while declining all throughout, dominates over the enemy (S_2) up to the time -instant

t*=

$$\frac{1}{((1-m_1)a_1+(1-m_2)a_2)}\log\left(\frac{U_{10}\beta((1-m_1)a_1+(1-m_2)a_2)+(1-m)(1-m_1)\alpha K_1U_2}{U_{20}(\beta((1-m_1)a_1+(1-m_2)a_2)a_{11}+(1-m)(1-m_1)\alpha K}\right.$$

there after the dominance is reversed as shown in Fig .13**.**

4.3A Trajectories of Perturbed Species: The trajectories of (17) and (18) in the U_1-U_2 plane are given by

$$x+\left(\frac{p_1}{U_{10}}\right)y=\left(1+\frac{p_1}{U_{10}}\right)y^{\frac{-(1-m_1)K_1a_{11}}{(1-m_2)K_2a_{22}}} \qquad (19)$$

where $x=\frac{U_1}{U_{10}}$, $y=\frac{U_2}{U_{20}}$ and $p_1=\frac{(1-m)(1-m_1)\alpha K_1U_{20}}{\beta((1-m_1)K_1a_{11}+(1-m_2)K_2a_{22})}$

These are illustrated in Fig .14**.**

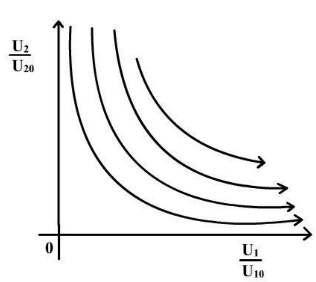

Fig.14

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4.4 Stability of Co-Existent State (E_4):

i.e $\overline{N_1}=\frac{1}{a_{11}}[((1\text{-}m_1)K_1a_{11}-\frac{\alpha(1\text{-}m_2)K_2}{\beta+(1\text{-}m_2)K_2}];\ \overline{N_2}=(1\text{-}m_2)K_2$

In this case we have

$$A=\begin{bmatrix} -\left((1\text{-}m_1)K_1a_{11}-\frac{(1\text{-}m)(1\text{-}m_1)\alpha K_2}{\beta+(1\text{-}m_2)K_2}\right) & -\frac{(1\text{-}m)\alpha}{\beta+(1\text{-}m_2)K_2}\left\{\left(\frac{1}{a_{11}}-\frac{(1\text{-}m_2)K_2}{a_{11}\left(\beta+(1\text{-}m_2)K_2\right)}\right)\left((1\text{-}m_1)K_1a_{11}-\frac{(1\text{-}m)(1\text{-}m_2)\alpha K_2}{\beta+(1\text{-}m_2)K_2}\right)\right\} \\ 0 & \text{-}(1\text{-}m_2)K_2a_{22} \end{bmatrix} \quad (20)$$

the characteristic roots of which are $-\left((1\text{-}m_1)K_1a_{11}-\frac{(1\text{-}m)(1\text{-}m_2)\alpha K_2}{\beta+(1\text{-}m_2)K_2}\right)$, $\text{-}(1\text{-}m_2)K_2a_{22}$ and both of these are negative. Hence the steady state is **stable**. (21)

The equation (7) yields the solution curves.

$$U_1=\left[U_{10}+\frac{P}{\left((1\text{-}m_1)K_1a_{11}-(1\text{-}m_2)K_2a_{22}-\frac{(1\text{-}m)(1\text{-}m_2)\alpha K_2}{\beta+(1\text{-}m_2)K_2}\right)}\right]e^{-\left((1\text{-}m_1)K_1a_{11}-\frac{(1\text{-}m)(1\text{-}m_2)\alpha K_2}{\beta+(1\text{-}m_2)K_2}\right)t}$$

$$-P\frac{e^{-(1\text{-}m_2)K_2a_{22}t}}{\left((1\text{-}m_1)K_1a_{11}-(1\text{-}m_2)K_2a_{22}-\frac{(1\text{-}m)(1\text{-}m_2)\alpha K_2}{\beta+(1\text{-}m_2)K_2}\right)}$$

and $U_2=U_{20}e^{-(1\text{-}m_2)K_2a_{22}t}$ (22)

where $P=\frac{(1\text{-}m)\alpha U_{20}}{\beta+(1\text{-}m_2)K_2}\left\{\left(\frac{1}{a_{11}}-\frac{(1\text{-}m_2)K_2}{a_{11}\left(\beta+(1\text{-}m_2)K_2\right)}\right)\left((1\text{-}m_1)K_1a_{11}-\frac{(1\text{-}m)(1\text{-}m_2)\alpha K_2}{\beta+(1\text{-}m_2)K_2}\right)\right\}$

and these curves are illustrated in Fig .15 and Fig.16 in the two sub cases that would arise.

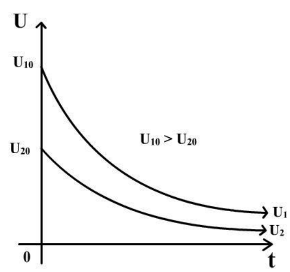

Fig.15

Sub Case (i): $U_{10} > U_{20}$

The Ammensal(S_1) dominates the enemy(S_2) in natural growth as well as in its initial population strength. In this case the Ammensal continues to outnumber the enemy as shown in Fig.15**.** However both converge asymptotically to the equilibrium point.

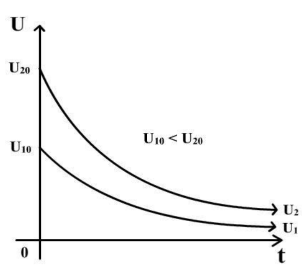

Fig.16

Sub Case (i): $U_{10} < U_{20}$

The enemy (S_2) dominates the Ammensal (S_1) in natural growth as well as in the initial population strength. In this case the enemy throughout outnumbers the Ammensal as shown in Fig.16**.** However both converge asymptotically to the equilibrium point.

4.4A. Trajectories Of Perturbed Species:

The trajectories of (21) and (22) in the U_1-U_2 plane are given by

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$$x+Wy=\left(W+1\right)y^{\left(\frac{(1-m_1)K_1a_{11}-\frac{\alpha(1-m_2)K_2}{\beta+(1-m_2)K_2}}{(1-m_2)K_2a_{22}}\right)} \qquad (23)$$

where $x=\dfrac{U_1}{U_{10}}$, $y=\dfrac{U_2}{U_{20}}$, $W=\dfrac{P}{U_{10}\left((1\text{-}m_1)K_1a_{11}-(1\text{-}m_2)K_2a_{22}-\dfrac{\alpha(1\text{-}m_2)\,K_2}{\beta+(1\text{-}m_2)K_2}\right)}$

$$P=\frac{(1\text{-}m)\alpha\,U_{20}}{\beta+(1\text{-}m_2)K_2}\left\{\left(\frac{1}{a_{11}}-\frac{(1\text{-}m_2)K_2}{a_{11}\left(\beta+(1\text{-}m_2)K_2\right)}\right)\left((1\text{-}m_1)K_1a_{11}-\frac{(1\text{-}m)(1\text{-}m_2)\alpha\,K_2}{\beta+(1\text{-}m_2)\,K_2}\right)\right\}$$

and Let $Q=(1\text{-}m_1)K_1a_{11}-\dfrac{(1\text{-}m)\,(1\text{-}m_2)\alpha K_2}{\beta+(1\text{-}m_2)K_2}$.

these are illustrated in Fig .17**.**

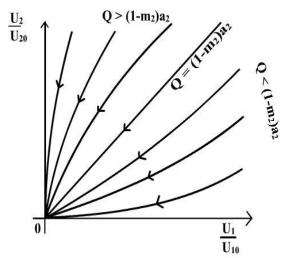

Fig.17

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