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WAVE PROPAGATION IN PSEUDO-COSSERAT CONTINUUM IN A SPECIAL CASE

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Abstract : In this paper it was shown that the waves in pseudo-Cosserat Continuum of large region propagate with two different velocities in a special case. We observed that these two waves correspond to irrotational and equivoluminal waves of classical theory of elasticity. Further these velocities are obtained as a particular case of this paper.

[Key words: pseudo-Cosserat – Continuum – Waves in a Special case]

Introduction

The Cosserat theory of elasticity incorporates a local rotation of points as well as deformation in classical elasticity and a couple stress (a torque for unit area) as well as the force stress is referred to simply as 'stress' in classical elasticity in which there is no other kind of stress. The concept of couple stress is introduced by Voigt [1]. Theories incorporating couple stresses were developed using the full capabilities of modern continuum mechanics. One of the ways to take into account the couple-stress is to use the model of pseudo-Cosserat Continuum [2], which is based on the assumption that the displacement vector $\vec{u}$ of the medium points is related to the vectors of small rotations $\vec{w}$ by the equation

$$\vec{w} = \frac{1}{2} \nabla \text{ x } \vec{u}$$

Hence, for consideration of pseudo-Cosserat Continuum we have one independent kinematic unknown ($\vec{u}$). However the asymmetric part of the stress and the symmetric part of the couple stress can not be derived directly from the physical equations, which was the reason why Eringen [3] named the theory of the pseudo-Cosserat continuum theory as the theory of undefined couple-stresses.

In this paper it is shown that there exists two different types of waves in an infinite pseudo-Coserat continuum when $\nabla \text{ x } \vec{u} = 0$ and these waves are correspond to irrotational and equivolumnal waves of classical theory of elasticity [4].

The velocity of irrotational and equivoluminal waves of classical elasticity are obtained as a particular case of this paper.

Formulation of Problem :

The equations of motion of cosserat continuum [5] under the absence of body forces and body couples in terms of displacement.

Vector $\vec{u}$ and the rotation vector $\vec{w}$ are given by

$$(\lambda+2\mu)\nabla\left(\nabla.\vec{u}\right)-(\mu+\alpha)\nabla\mathrm{x}\nabla\mathrm{x}\,\vec{u}+2\alpha\ \nabla\mathrm{x}\,\vec{w}=\rho\frac{\partial^2\vec{u}}{\partial t^2} \quad (1)$$

$$(\beta+2\gamma)\nabla\left(\nabla.\vec{w}\right)-(\gamma+\in)\nabla\mathrm{x}\nabla\mathrm{x}\,\vec{w}+2\alpha\ \nabla\mathrm{x}\,\vec{u}-4\alpha\,\vec{w}=\rho j\frac{\partial^2\vec{w}}{\partial t^2} \quad (2)$$

where λ, μ are Lame's constants: $\alpha, \beta, \gamma, \in$ are physical constants of the material in the context of the Cosserat Continuum, $\vec{u}$ is the displacement vector, $\vec{w}$ is the rotation vector, ρ is the density and j is the micro-inertia.

The displacement vector $\vec{u}$ can be represented as a sum of two vectors, one of which is sole nolidal and the other irrotational.
This leads the consideration of the case

$$Curl\ u = 0 \quad (3)$$

The Psedeo-Cosserat Continuum is based on the assumption

$$\vec{w}=\frac{1}{2}\nabla\mathrm{x}\,\vec{u} \quad (4)$$

In view of (3) and (4) the equation of motion (1) and (2) reduce to

$$(\lambda+2\mu)\nabla\left(\nabla.\vec{u}\right)-(\mu+\alpha)\nabla\mathrm{x}\nabla\mathrm{x}\,\vec{u}=\rho\frac{\partial^2\vec{u}}{\partial t^2} \quad (5)$$

Solution of the Problem:

We assume that the region of Pseudo-Cosserat Continuum is large so that the effects of the boundary can be disregarded.

We decompose the vector $\vec{u}$ into scalar potential G and vector potential $\vec{H}$ as follows

$$\vec{u}=\nabla\mathrm{G}+\nabla\mathrm{x}\,\vec{\mathrm{H}} \quad (6)$$

Substituting (6) in (5) and using the following vector identities

$$\begin{aligned}&\nabla\mathrm{x}\nabla\mathrm{Q}=0\\&\nabla.\nabla\mathrm{x}\,\vec{\mathrm{P}}=0\end{aligned} \quad (7)$$

The equation (5) reduces

$$(\lambda+2\mu+\alpha)\nabla\left(\nabla^2\mathrm{G}\right)+(\mu+\alpha)\nabla\mathrm{x}\nabla^2\,\vec{\mathrm{H}}=\rho\nabla\ddot{\mathrm{G}}+\rho\nabla\mathrm{x}\,\ddot{\vec{\mathrm{H}}} \quad (8)$$

where the superposed dot on a symbol indicates differentiation with respect to time t.

The equation (8) gives two wave equations

$$\nabla^2\mathrm{G}=\frac{1}{\mathrm{C}_1^{\,2}}\ddot{\mathrm{G}} \quad (9)$$

$$\nabla^2 \vec{H} = \frac{1}{C_2^{\,2}} \ddot{\vec{H}} \qquad (10)$$

where C_1, C_2 are the velocities of the waves and they are given by

$$C_1^{\,2} = \frac{\lambda + 2\mu + \alpha}{\rho} \qquad (11)$$

$$C_2^{\,2} = \frac{\mu + \alpha}{\rho} \qquad (12)$$

The velocity of propagation of wave given by (9) corresponds to the velocity of propagation of irrotational wave of classical elasticity. Further it reduces to $\sqrt{\frac{\lambda + 2\mu}{\rho}}$ the velocity of irrotational wave of classical case when $\alpha \to 0$.

The velocity of propagation C_2 corresponds to the velocity of equivoluminal wave of classical elasticity and it reduces to the classical result $\sqrt{\frac{\mu}{\rho}}$ when $\alpha \to 0$.

Thus, in the interior of a Pseduo-Cosserat Continuum which large region waves may be propagated with two different velocities C_1 and C_2 in the case which we considered. Further, these two waves are correspond to the irrotational and equivoluminal waves of classical theory of elasticity.

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